Overview

- We consider sparse sensing-based distributed MIMO radars, which exploit the sparsity of the targets in the space to achieve good target estimation performance of MIMO radars but with fewer measurements.
- In the model of sparse sensing-based distributed MIMO radars, the sensing matrix is block-diagonal and the sparse vector to be recovered consists of equal-length sub-vectors that have the same sparsity profile.
- This paper develops the theoretical requirements and performance guarantees for the application of block sparse recovery technique in this context.
- The results confirm that exploiting the block sparsity of the target vector can reduce the number of measurements needed for target estimation, or can result in improved target estimation for the same number of measurements.

System Model

The location-speed space is discretized by Θ with N grid points. For the (ij)-th TX/RX pair, the signal vector at j-th RX from P pulses due to the transmission of i-th TX

\[ \mathbf{x}_{ij} = \mathbf{s}_{ij} + \mathbf{n}_{ij}, \quad \forall i \in \mathbb{N}_+, j \in \mathbb{N}_+ \]

where \( \mathbf{s}_{ij} = [s_{ij}^1, \ldots, s_{ij}^N]^T \) with \( s_{ij}^k \) being non-zero only if there is a target at the n-th grid point; and

\[ \mathbf{s}_{ij} = \begin{bmatrix} \mathbf{x}_{ij1} \mathbf{e}^{j2\pi f_{ij1}^d L} & \cdots \mathbf{x}_{ijN} \mathbf{e}^{j2\pi f_{ijN}^d L} \\ \vdots \cdots \vdots \vdots \end{bmatrix}_{(LP \times N)} \]

where \( \mathbf{x}_{ij} \) is a vector contains L samples of the i-th waveform shifted by \( f_{ij}^d \), \( n_{ij} \) represents noise, \( f_{ij}^d \) and \( f_{ij}^t \) denote the delay time and Doppler frequency. The transmitted waveforms are Gaussian signals with variance \( \sigma_0^2 \).

Stacking the received samples into a vector \( \mathbf{z} \), we get

\[ \mathbf{z} = [\mathbf{z}_{11}^T, \ldots, \mathbf{z}_{M L M}^T]^T = \Psi \mathbf{s} + \mathbf{n} \quad (1) \]

where \( \mathbf{s} = [\mathbf{s}_{11}^T, \ldots, \mathbf{s}_{M N M}^T]^T \), \( \mathbf{n} = [\mathbf{n}_{11}^T, \ldots, \mathbf{n}_{M N M}^T]^T \) and \( \Psi = \text{diag}(\mathbf{s}_{11}, \ldots, \mathbf{s}_{M N M}) \).

The vector \( \mathbf{s} \) is a concatenation of \( M \times M \) sub-vectors that share the same sparsity profile, and have exactly \( K \) nonzero entries each. \( s \) lies in \( \mathbb{A}_K^N \) defined by \( \mathbb{A}_K^N \equiv \{ \mathbf{s} \in \mathbb{C}^{N M \times M}; |\text{supp}(\mathbf{s}_1)| = \cdots = |\text{supp}(\mathbf{s}_M)|, |\text{supp}(\mathbf{s}_1)| \leq K \} \n
Sparse Signal Recovery

- By directly applying the L-OPT in [1], we have

\[ \min_{\mathbf{s}} \sum_{n=1}^{N} \| s_{[n]} \|_2 \quad \text{s.t.} \quad \| z - \mathcal{P}_M(\Psi) \mathcal{P}_R(\mathbf{s}) \|_2 \leq \epsilon \]

where

- \( \mathcal{P}_R(\mathbf{s}) = [s_{[1]}, \ldots, s_{[N]}] \)
- \( \mathcal{A}_K^N \equiv \{ \mathcal{P}_R(\mathbf{s}) \in \mathbb{A}_K^N \} \)
- \( \mathcal{P}_M(\Psi) \) is permutation of columns of \( \Psi \).

Definition 1: Matrix \( \Psi \) satisfies the RIP over \( \mathbb{A} \) with \( \delta_K \), or equivalently the \( \mathcal{A} \)-RIP \( (K, \delta_K) \), if for every \( \mathbf{x} \in \mathbb{A} \) it holds that \( (1 - \delta_K \| K \|_2^2) \leq \| \mathbf{x} \|_2^2 \leq (1 + \delta_K \| K \|_2^2) \).

Theorem 1: For any \( \delta_K \in (0, 1) \), there exist \( c_1 \) and \( c_2 \) such that \( \Psi \) satisfies the \( \mathcal{A} \)-RIP \( (K, \delta_K) \) with probability exceeding

\[ 1 - \exp \left( -c_1 (L - 1)/K^2 \right) \]

where \( L \geq c_2 K^2 \log(N M M) + 1 \).

Sketch of proof: Under condition (4), the bounds on the off-diagonal entries from case (ii) and (iii) are unified by

\[ \Pr \left( \| \mathcal{P}_R \mathcal{P}_M(\Psi) \|_2 > t \right) \leq 4 \exp \left( - \frac{L}{16} (\delta_K^2) \right) \]

Numerical Results

Measurement Matrix Satisfying \( \mathcal{A}_1 \)-RIP

- The Gram of \( \Psi \) is denoted by \( \mathbf{G} = \text{diag}(\mathbf{G}_{11}, \ldots, \mathbf{G}_{M M}) \)
- The Gram of \( \mathbf{G} \) is satisfied by \( \mathcal{A}_1 \)-RIP \( (K, \delta_K) \) if

\[ \| \mathbf{G}_{ij} \|_2 \leq 1 \quad \text{and} \quad \| \mathbf{G}_{ij} \|_2 \leq 1 \]

To bound the entries of \( \mathbf{G}_{ij} \), we have three cases

- Case (i): \( n = m, i.e., diagonal entries \mathbf{G}_{ij}(n) \)

\[ \Pr \left( \| \mathbf{G}_{ij}(n) - 1 \| > t \right) \leq 2 \exp \left( -Lt^2/16 \right) \]

- Case (ii): \( \tau_{ij}^m \neq \tau_{ij}^m, i.e., \text{off-diagonal entries,} \)

\[ \Pr \left( \| \mathbf{G}_{ij}(n, m) - 1 \| > t \right) \leq 4 \exp \left( -\frac{(L - 1)t^2}{8 + 4t} \right) \]

- Case (iii): \( \tau_{ij}^m = \tau_{ij}^m \neq \tau_{ij}^m \), we have

\[ \| \mathbf{G}_{ij}(n, m) \|_2 \leq \frac{\mathbf{q}_{ij}^m}{L \sigma_0^2} \]

Denoting the second multiplier as \( \phi_{ij}^m / \mathbf{G}_{ij}(n, m) \) can be viewed as a squared norm of a Gaussian vector. Applying Lemma 5 [2], we have

\[ \Pr \left( \| \mathbf{G}_{ij}(n, m) > t \right) \leq \exp \left( -\frac{L}{16} (t/\tau_{ij}^m - 1)^2 \right) \]

where \( \gamma_{ij} = \sup \{ \phi_{ij}^m / \mathbf{G}_{ij} \} \)

\[ \mathbf{S}_2 = \{(m, n) | m, n \leq N, \tau_{ij}^m = \tau_{ij}^m \neq \tau_{ij}^m \} \]

References
