THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS
School of Engineering
Department of Electrical and Computer Engineering

332:224 Principles of Electrical Engineering II Laboratory

Experiment 6

Fourier Series Analysis

1 Introduction

Objectives

- The aim of this experiment is to study the Fourier series of certain common waveforms

Overview

This experiment treats the subject of Fourier series. The form and coefficients are introduced in section 2 as is the specific form it attains for various waveforms. A circuit tuned at 10Khz is then utilized to pull out one by one the harmonics of certain periodic waveforms in section 4. The experimentally determined coefficients are then compared with the theoretical ones.

Authored by P. Sannuti
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by P. Panayotatos

2 Theory

2.1 Overview of Fourier Series Analysis[1]

A periodic function **f(t)** can be represented by an infinite sum of sine and/or cosine functions that are harmonically related. That is, the frequency of any trigonometric term in the infinite series is an integral multiple, or *harmonic*, of the fundamental frequency of the periodic function. Thus, given f(t) is periodic (e.g. square wave, triangular wave, half rectified wave, etc.), then f(t) can be represented as follows:

$$f(t) = a_o + \sum_{k=1}^{\infty}\left[a_k \cos\left(k\omega_o t\right) + b_k \sin\left(k\omega_o t\right)\right]$$

where k is the integer sequence 1,2,3, ... ,

a_0, a_k, and b_k are called the Fourier coefficients, and are calculated from f(t),

$\omega_0 = 2\pi/T$ is the fundamental frequency of the periodic function f(t) with period T,

and $k\omega_0$ is known as the *k-th harmonic* of f(t).

a_0, a_k, and b_k can be found as follows:

$$a_o = \frac{1}{T}\int_{t_o}^{t_o+T} f(t)dt$$

$$a_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\cos(k\omega_o t)dt$$

$$b_k = \frac{2}{T}\int_{t_o}^{t_o+T} f(t)\sin(k\omega_o t)dt$$

2.2 The effect of symmetry on the Fourier coefficients

2.2.1 Even-function symmetry

A function is defined as even if f(t) = f(-t). The equations for the Fourier coefficients reduce to:

$$a_o = \frac{2}{T}\int_{0}^{T/2} f(t)dt \quad , \quad a_k = \frac{4}{T}\int_{0}^{T/2} f(t)\cos(k\omega_o t)dt \quad , \; b_k = 0 \text{ for all k.}$$

[1] The subject is treated in detail in section 16.1 of the text.

2.2.2 Odd-function symmetry

A function is defined as odd if f(t) = -f(-t); then the Fourier coefficients are given by:

$a_o = 0$, $a_k = 0$ for all k, and $$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin(k\omega_o t)dt$$

2.2.3 Half-wave symmetry

A periodic function possesses half-wave symmetry if it satisfies the constraint f(t) = -f(t-T/2). The expression for the Fourier coefficients are:

$a_o = 0$, $a_k = 0$, for k even, $a_k = \frac{4}{T}\int_0^{T/2} f(t)\cos(k\omega_o t)dt$ for k odd,

$b_k = 0$ for k even, and $b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin(k\omega_o t)dt$ for k odd.

2.2.4 Quarter-wave symmetry

A periodic function possesses quarter - wave symmetry if:
1- it has half - wave symmetry, and
2- it has symmetry about the midpoint of the positive and the negative half cycles.

2.2.4.a If a periodic function is even, and has quarter-wave symmetry, the Fourier coefficients are:

$a_o = 0$ because of half-wave symmetry;
$a_k = 0$ for k even, because of the half-wave symmetry;

$a_k = \frac{8}{T}\int_0^{T/4} f(t)\cos(k\omega_o t)dt$ for k odd;
$b_k = 0$ for all k, because f is even.

2.2.4.b If a periodic function is odd, and has quarter-wave symmetry, the Fourier coefficients are:

$a_o = 0$ because f is odd; $a_k = 0$ for all k, because f is odd;
$b_k = 0$ for k even, because of half-wave symmetry;

$$b_k = \frac{8}{T}\int_0^{T/4} f(t)\sin(k\omega_o t)dt \text{ for k odd.}$$

2.3 Fourier series of some waveforms

Let A denote the peak amplitude of each waveform.

(a) Balanced odd square waveform: $f(t) = \frac{4A}{\pi}\sum_{k=1,\text{odd}}^{\infty}\frac{1}{k}\sin\left(k\omega_o t\right)$

(b) Balanced odd triangular waveform: $f(t) = \frac{8A}{\pi^2}\sum_{k=1,\text{odd}}^{\infty}\frac{1}{k^2}\sin\frac{k\pi}{2}\sin\left(k\omega_o t\right)$

(c) Half-wave rectified sine wave with f(0)=0:

$$f(t) = \frac{A}{\pi} + \frac{A}{2}\sin\left(\omega_o t\right) - \frac{2A}{\pi}\sum_{k=2,\text{even}}^{\infty}\frac{1}{k^2-1}\cos\left(k\omega_o t\right)$$

2.4 RMS values vs. peak values for some waveforms

Let A denote the peak amplitude of each wave form.
(a) Sinusoidal wave: RMS value is $A/\sqrt{2}$
(b) Balanced square wave: RMS value is the same as the peak value A.
(c) Balanced triangular wave form: RMS value is $A/\sqrt{3}$
(d) Half-wave rectified sine wave: RMS value is A/ 2.

Mean Square value of any wave in terms of its Trigonometric Fourier Coefficients is given by the sum of (Square of DC Value + Half of the square of the amplitude of the fundamental and of each harmonic).

2.5 Anti-Resonant Filter (A Tuned Circuit)

A circuit sometimes used for simple filtering consists of a coil shunted by a capacitor, the coil of course having some resistance as well as inductance, as shown in fig. 1.

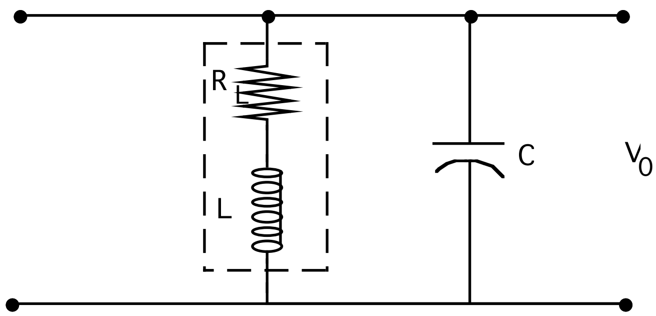

Fig. 1 An Anti-Resonant Filter

At $\omega_o = (1/LC)^{1/2}$, the impedance of the parallel combination is

$$Z(j\omega_o) = (\omega_o{}^2 L^2/R_L) - j\omega_o L = \omega_o L(Q_n - j1) \qquad (1)$$

where

$$Q_n = \omega_o L/R_L \qquad (2)$$

It follows that for $Q_n > 10$, the impedance is essentially the pure resistance

$$Z(j\omega_o) = Q_n \omega_o L = Q_n{}^2 R_L \qquad (3)$$

The high impedance is a result of the anti-resonance phenomenon and the magnitude of the impedance decreases as ω departs from ω_o in value; the sharpness of the peak depends on Q. As a consequence, if a practical L-C parallel circuit is fed from a constant current source, the voltage developed at or near the anti-resonant frequency can be much higher than at some other frequency. Hence, if the signal contains a mixture of discrete frequencies (as in the case of Fourier components of a periodic wave), the circuit has the ability to select one of these frequencies and effectively reject the others.

3 Prelab Exercises

3.1 Derive Eqs. 1 & 3 in Section 2.5

4 Experiments

Suggested Equipment:

Tektronix FG 501A 2MHz Function Generator[2]
Tektronix DC 504A Counter-Timer
HP 54600A or Agilent 54622A Oscilloscope
Keithly 179A TRMS Multimeter
LS-400A Inductance Substituter Box
500 KΩ Resistor, pn diode
Protoboard
Other circuit elements to be determined by the student.

4.1 Square Wave Components

Arrange a parallel L-C circuit and connect to the function generator through a series resistance of about 500 KΩ, as shown in fig. 3. Let L = 25 mH and C be the value that corresponds to an anti-resonant frequency of 10 KHZ. The voltage source V_s in series with the resistance R is equivalent to a current source V_s /R in parallel with the resistance R.

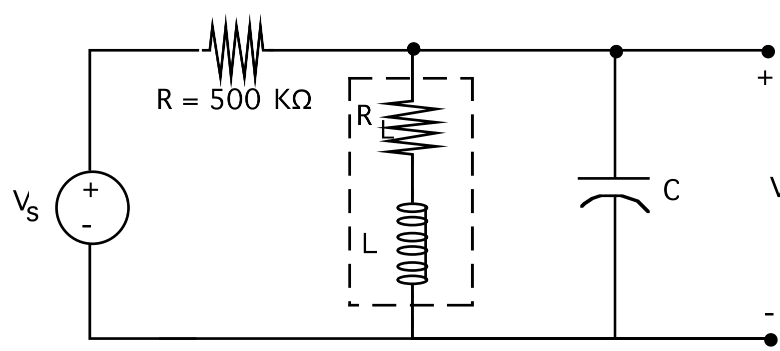

Fig. 3 A Tuned L-C Circuit

Display both input and output on the screen of the oscilloscope. With a square wave input, vary the frequency in the neighborhood of 10 KHz. You should see a sine wave appear on the output channel. This is the fundamental component of the Fourier series of the square wave and the amplitude is maximum when you are right on the anti-resonant frequency.

(a) Set f = 10 KHz as closely as you can using the frequency counter.
(b) Tune the circuit for maximum V_0 by adjusting C and/or L. Measure L, R_L, and C.
(c) Measure V_0 using the DVM.
(d) Adjust the generator output until the DVM reads 0.5 V.
Read the corresponding input voltage. V_{in}=________________V

[2] **NOTE:** The oscillator is designed to work for a very wide range of frequencies but may not be stable at very low frequencies, say in the order of 100 Hz or 200Hz. To start with it is a good idea to have the circuit working at some mid-range frequency, say in the order of 1K Hz or 2K Hz, and then change the frequency slowly as needed.

(e) Prepare a table (see the last page for a sample table) in which to record k (the order of the harmonic), f (in KHz) and V_0 (the RMS voltage of the harmonic.) Keep the input voltage exactly at the same level as in part (d) for all the following readings.

k	f (Hz)	$V_{o,RMS}$ (V)

(f) In order to measure the m-th harmonic output where m is an integer, we need a filter which passes that m-th harmonic frequency and rejects all the other frequencies. Since we have a filter with a fixed resonant frequency of 10 KHz, when we change the fundamental frequency of the input wave to 10/m KHz, the m-th harmonic of it would be 10 KHz, thus the output of the filter corresponds to the m-th harmonic content of the input wave.
Reduce the frequency gradually until at about 10/3 KHz you see the third harmonic component appear on the output channel.
(g) Adjust the frequency for peak amplitude of the harmonic, and record the three variables in your table as above.
(h) Repeat this procedure for each harmonic at least to the seventh.

4.2 Triangular Wave Components

Repeat Section 4.1 using a triangular input.

k	f (Hz)	$V_{o,RMS}$ (V)

4.3 Half Rectified Sine Wave Components (Optional[3])

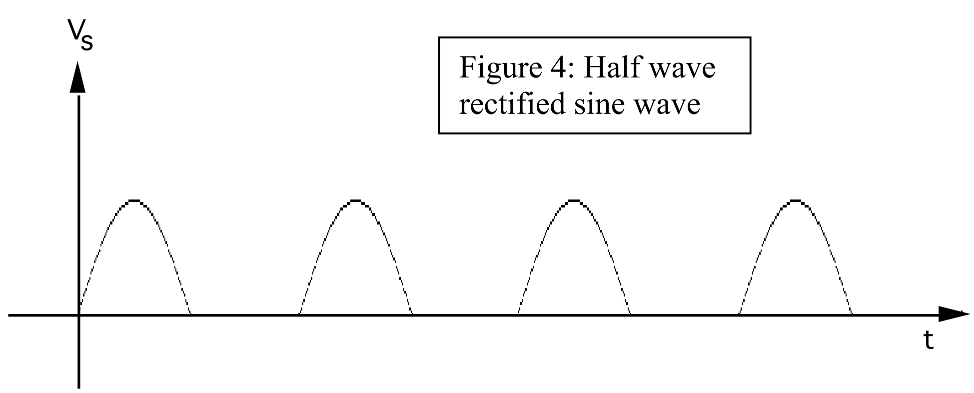

Figure 4: Half wave rectified sine wave

[3] If you do this, this part of thee lab report must be done similar to the one for Square Wave.

In order to generate a half rectified sine wave, a **pn diode** is placed in series with the **500 KΩ** resistor as shown in fig. 5. The diode works as follows:
When the input voltage is positive, the diode conducts (behaves as a closed switch), and when it is negative, the diode does not conduct (behaves as an open switch). Therefore, with a sinusoidal input, the diode conducts on the positive half cycle, and stops conducting on the negative half cycle. Hence the half rectified sine wave gets generated and is now the input to the tuned **L-C** circuit.

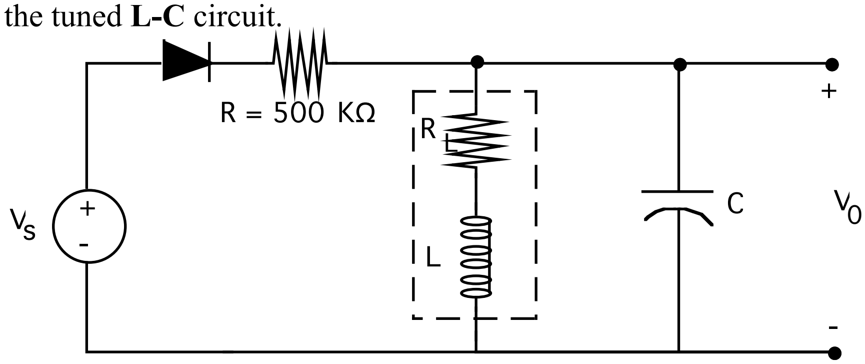

Fig. 5

Repeat Section 4.1 using a sinusoidal input to the circuit shown in fig. 5. Continue to the eighth harmonic only.

k	f (Hz)	$V_{o,RMS}$ (V)

5 Report

5.1 (a) Using Eq. 3, find Z(jω) using the values of L, R_L, and C obtained in Section 4.1.
(b) If the input to the circuit of fig. 3 is a sinusoid of RMS value equal to B volts, derive an expression for the RMS output voltage V_0.

5.2 Derive the Fourier series of:
(a) a square wave,
(b) a triangular wave, and
(c) a half rectified sine wave (optional).
Assume that the peak amplitude of every wave is A volts. Make sure your derivations yield coefficients consistent with those given in Section 2.3.

5.3 Using the measured RMS value of the square wave in Section 4.1(d), do the following :
(a) Find the peak amplitude of the input wave. (Note that the RMS value of a square wave is the same as its peak value.)
(b) Use the derived Fourier series in 5.2 to find the theoretical peak amplitudes of the first four harmonics of the input wave.
(c) Determine the RMS values of these harmonics.

5.4 Using the measured RMS value of the triangular wave in section 4.2(d), repeat parts (a) through (c) of 5.3 with a triangular input.

5.5 Using the peak and RMS values of the sine waves calculated in 5.3, and the results of 5.1(b), determine the peak value of the output for the first, third, fifth, and seventh harmonic of the input square wave.

5.6 Using the peak and RMS values of the sine waves calculated in 5.4, and the results of 5.1(b), determine the peak value of the output for the first, third, fifth, and seventh harmonic of the input triangular wave.

5.7 Prepare a table as shown in fig. 6.

Fundamental Frequency	Harmonic Order	Square wave V_0 measured in Section 4.1	Square wave V_0 Calculated in item 5.5	Triangular wave V_0 measured in Section 4.2	Triangular wave V_0 Calculated in item 5.6
	First				
	Third				
	Fifth				
	Seventh				

Fig. 6 A Table for 5.7

5.8 Submit a table from Section 4.3 (Optional).

5.9 A series RLC circuit is to be used as a notch filter to eliminate a bothersome 60 Hz hum in the audio channel. The signal source has a Thévenin resistance of 600 ohms. Select values of L and C so the upper cutoff frequency is below 200 Hz (Optional, extra credit if done properly).

5.10 Prepare a summary.