

332:224 Principles of Electrical Engineering II Laboratory

Experiment 5

*Step response of an RLC series circuit*

**1 Introduction**

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- Objectives**
- To study the behavior of an underdamped RLC Series Circuit for different damping coefficients

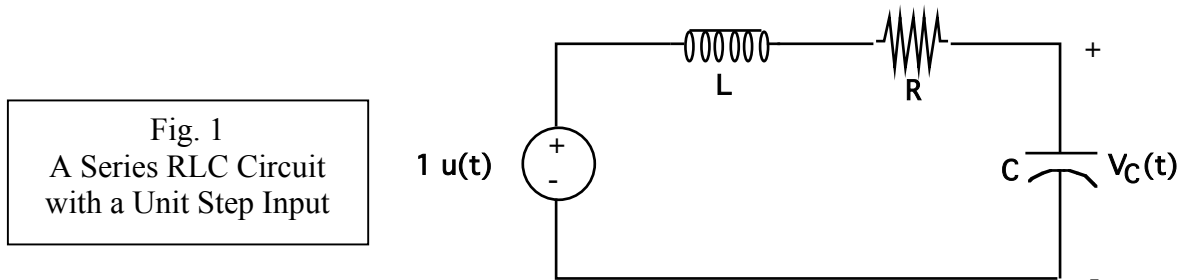
**Overview**

This experiment is a study of the step response of an underdamped RLC series circuit. The natural frequency is chosen and that determines the values of L and C. Then the damping coefficient is varied by changing the value of R.

A short theory is presented. The prelab exercise is rudimentary but crucial for the performance of the experiment which includes comparison of experimentally determined and theoretically derived waveform maxima and minima.

## 2 Theory<sup>1</sup>

Consider an RLC series circuit subject to a unit step voltage as shown in fig. 1.



For a second order linear differential equation with step function input

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y = Au(t)$$

the step response is the general solution for  $t \geq 0$ . This step response can be partitioned into forced and natural components. Natural response is the general solution of the homogeneous equation (with  $u(t) = 0$ ), while the forced response is a particular response of the above equation.

Following the notation of the textbook, the following notation is used for a second order circuit:

- $\alpha$  = The damping factor
- $\zeta$  = The damping coefficient
- $\omega_n$  = The damped frequency
- $\omega_o$  = The natural frequency

For the RLC series circuit of fig. 1, the expression for the capacitance voltage in the oscillatory (*underdamped*) case is

$$v_C(t) = 1 + Ke^{-\alpha t} \cos(\omega_d t + \phi) \quad (1)$$

where  $\alpha = R/2L = \zeta\omega_o$  and

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{1/2} = \omega_o(1 - \zeta^2)^{1/2}$$

<sup>1</sup> A more detailed treatment can be found in the text starting with section 8.4.

$$\text{Also, } \omega_o = \frac{1}{\sqrt{LC}} \quad \text{and} \quad \zeta = \frac{R}{2} \sqrt{\frac{C}{L}}$$

In equation (1) for  $\mathbf{v}_C(\mathbf{t})$ , the term  $Ke^{-\alpha t} \cos(\omega_d t + \phi)$  represents the natural response and the unity represents the particular solution.

Consider the initially quiescent state (both the initial inductance current and the initial capacitor voltage are zero). In this case,

$$\phi = -\tan^{-1}\left(\frac{\alpha}{\omega_d}\right) = -\tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right) \quad (2)$$

$$K = -\frac{1}{\cos \phi} = -\frac{1}{\sqrt{1-\zeta^2}} \quad (3)$$

The maxima and minima occur alternately when

$$\tan(\omega_d t + \phi) = -\alpha/\omega_d, \text{ i.e. } \omega_d t = k\pi; k = 0, 1, 2, \dots \quad (4)$$

From the above, the maximum and the minimum values of  $\mathbf{v}_C$  are given by

$$v_c(k\pi / \omega_d) = 1 - (-1)^k \exp\left[-\frac{k\pi\zeta}{\sqrt{1-\zeta^2}}\right] \quad k = 0, 1, 2, \dots \quad (5)$$

(odd values of  $\mathbf{k}$  give the maxima)

### 3 Prelab Exercises

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**3.1** For  $L = 100\text{mH}$ , compute the value of  $C$  needed for the natural frequency  $\omega_o = 2\pi f_o = 10^4\pi$ .

To vary the damping coefficient  $\zeta$ , vary the value of  $R$ . Compute the values of  $R$  for each of the six different values of the damping coefficient  $\zeta = 0.1, \zeta = 0.2, \zeta = 0.4, \zeta = 0.6, \zeta = 0.8$ , and  $\zeta = 1.0$ , (see [Table 1](#) under section 4.4).

**3.2** From [Eq. 5](#), calculate and fill the theoretical values of the maxima and minima in [Table 1](#) under section 4.4.

## 4 Experiments

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Suggested Equipment:

TEKTRONIX FG 501A 2MHz Function Generator  
 HP 54600A or Agilent 54622A Oscilloscope  
 DC 504A Counter-Timer  
 Protek Model B-845 Digital Multimeter  
 Resistance/Capacitance Box  
 Inductance Box

**NOTE:** The oscillator is designed to work for a very wide range of frequencies but may not be stable at very low frequencies, say in the order of 100 Hz or 200Hz. To start with it is a good idea to have the circuit working at some mid-range frequency, say in the order of 1K Hz or 2K Hz, and then change the frequency slowly as needed.

Any function generator used has internal resistance. Also, the inductor has internal resistance. Both need to be determined.

### 4.1 Function generator resistance

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The internal resistance of the function generator will affect the damping of an RLC circuit to which it is connected. Check the resistance in the following way:

- a- With a sine wave output, set the open circuit voltage to some convenient value, say 1V.
- b- Connect a pure variable resistance load (potentiometer) thus forming a voltage divider. Adjust R until the terminal voltage falls to one-half the open circuit value. At this point the two resistances of the voltage divider have to be equal. Therefore, the resistance of the potentiometer should now be equal to the internal resistance of the function generator. Disconnect the potentiometer from the circuit and measure its resistance.

### 4.2 Inductor internal resistance

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Use the digital ohmmeter to measure the internal resistance of the inductor used. The values of R that should be used in the series RLC circuit are those computed in [Section 3](#) (Pre-lab exercise) *minus the internal resistance of the function generator* and *minus the internal resistance of the inductor*.

### 4.3 Step response of RLC branch

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Arrange a series RLC series circuit using values calculated for  $\zeta = 0.1$  in [Section 3](#) (Pre-lab exercise). Set the scope sweep rate to 0.5ms/div (CAL), sensitivity to 500mV/div. With a

square wave input to the circuit, set  $f = 250\text{Hz}$  and increase the amplitude of the input until you see the typical oscillatory response to a step input. (Instead of a step input, a square wave of low enough frequency is used so that the repetitive wave form of the capacitor voltage can be easily plotted on the scope). Adjust the oscillator voltage so that the steady state value of the capacitor voltage is 1V corresponding to the particular solution for a unit step input. Download the waveform.

#### 4.4 Measurement of peaks

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Read the values of the maxima and the minima based on a unit step input. (Use the cursor). Repeat 4.3 for different values of  $\zeta$ , fill all the values in Table 1 and compare with the theoretical values that were calculated in pre-lab exercise 3.2. In each case download the waveform.

$\zeta$	1st Max	1st Min	2nd Max	
0.1				Measured
				Theoretical
0.2				Measured
				Theoretical
0.4				Measured
				Theoretical
0.6				Measured
				Theoretical
0.8				Measured
				Theoretical
1.0				Measured
	Critically Damped			Theoretical

Table 1 Unit step response of 2<sup>nd</sup> order system

#### 4.5 Natural frequency

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Change the sweep to 0.1ms/div and reduce damping as much as possible by reducing the value of R. Notice that the period of the oscillation is close to 0.2ms which corresponds to the natural frequency  $\omega_0$  used to determine L and C values.

## 5 Report

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- 5.1 Derive Equation 1 in Section 2 for the underdamped case of a series RLC circuit.
- 5.2 Derive and prove Equation 4 in Section 2.
- 5.3 In pre-lab exercise 3.2 you have already filled the theoretical values of the maxima and minima in Table 1. Compare them now with the experimental values. Print out the downloaded waveforms and label them appropriately.
- 5.4 Prepare a summary.