THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:224 Principles of Electrical Engineering II Laboratory

Experiment 4

The R-C series circuit

1 Introduction

Objectives

- To study the behavior of the R-C Series Circuit under different conditions
- To use different methods for the determination of the RC time constant from experimental results

Overview

The aim of this experiment is to study the R-C Series Circuit under different conditions by observing input and output waveforms and studying their interrelation.
In particular the following are explored:
(a) *Natural response of an R-C Circuit*: The capacitor is charged to a certain value and its decay is observed as a function of time. Such a decay is known as the *natural response* of the R-C Circuit as there are no forcing inputs applied to the circuit.
(b) *Response of an R-C Circuit to a periodic square wave input*: When the input to a circuit is a periodic signal (wave), the output voltage is a periodic wave as well but not necessarily of the same waveform as that at the input. The output voltage across the capacitance is studied for a square wave input.
(c) *Differentiation And Integration Properties*: Under certain appropriate conditions, an R-C Circuit can function approximately as an integrating circuit or as a differentiating circuit. These properties are also observed.

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Latest revision: December 16, 2005, 2004
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2 Theory

2.1 The Natural Response of an R-C Circuit[1]

Consider the R-C circuit of fig. 1 where the source voltage V_s is a DC voltage source. Assuming that the switch has been closed for a long period of time, the circuit has reached a steady state condition. In steady state, the capacitor has been charged to

$$V_g = V_s R/(R+R_i)$$

Therefore, when the switch opens, at t = 0, the initial voltage on the capacitor is V_g volts. With the capacitor so charged, it would be desirable to compute the natural response of the R-C Series Circuit shown in fig. 2.

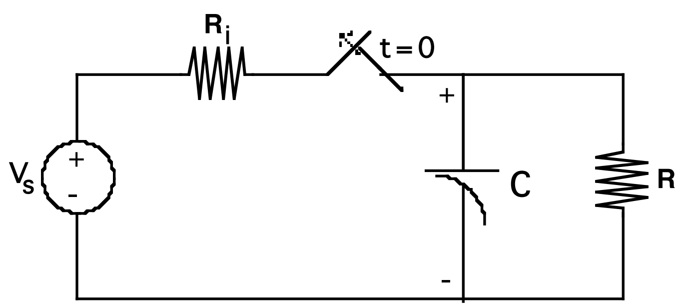

Fig. 1

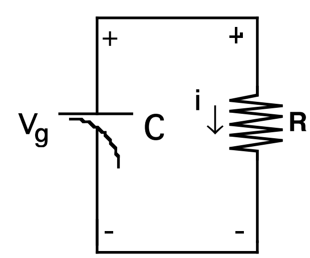

Fig. 2

Analysis of the circuit in fig. 2 yields the general solution

$$v_c(t) = v_c(0)\, e^{-t/RC}, \quad t \geq 0$$

With initial condition: $v_c(0^-) = v_c(0^+) = V_g = V_s R/(R+R_i)$ from which

$$v_c(t) = V_g\ e^{-t/\tau}, \qquad (1)$$

where $\tau = RC$ is the time constant. This means that the natural response of the R-C circuit is an exponential decay of the initial voltage. The rate of this decay is governed by the time constant RC. The graphical plot of Eq. 1 is given in fig. 3 where the graphical interpretation of the time constant is also shown.

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[1] A more detailed description can be found in section 7.2 of the text.

The time constant $\tau = RC$ can be measured in several ways. Assuming that R and C are not known, τ can be measured from the discharge data of the capacitor as will be seen in section 5 below.

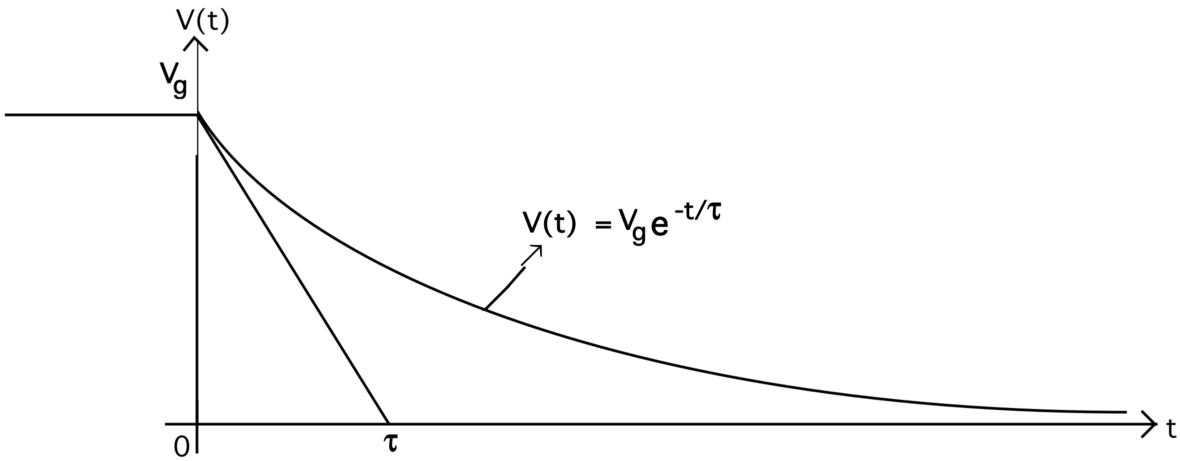
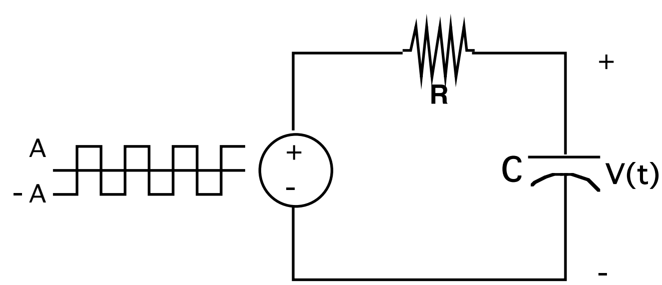


Fig. 3 Exponential Decay Of An R-C Circuit

2.2 Square Wave Response[2]

Let the source voltage V_s be a square wave of frequency f and amplitude A, applied to the R-C circuit as shown in fig. 4.

Fig. 4 A square wave input applied across a series R-C Circuit



Since the input is a periodic wave, the output voltage across the capacitor is also a periodic wave, albeit not a square wave. In each period, the output voltage across the capacitor consists of two parts:

- During the half period when the input is a positive constant, the capacitor gets charged exponentially. Hence the output voltage v(t) during this half period is an *exponentially increasing* signal. At the end of this half period, v(t) has attained a certain positive peak value.

[2] A detailed description of the step response can be found in section 7.3 of the text.

- During the half period when the input is a negative constant, the capacitor gets discharged exponentially. Hence the output voltage v(t) during this half period is an *exponentially decreasing* signal. By the end of this half period, v(t) has attained a certain negative peak value.

The difference between the positive and negative peaks is called the *peak-to-peak voltage of the capacitor*. It can be shown that

$$V_{CPP} = V_{PP}\,(1-e^{-K})/(1+e^{-K}) \qquad (2)$$

where V_{CPP} is the peak-to-peak voltage of the capacitor,
V_{PP} is the peak-to-peak voltage of the input square wave, and
K is a number such that $K\tau$ is the input square wave half-period when $\tau = RC$.

2.3 Differentiation and Integration Properties

Consider the loop equation of the R-C series circuit,

$$v_R(t) + v_C(t) = v(t),$$

where v(t) is the source voltage v_s which could be any time varying signal.

If $v_C(t)$ is kept small with respect to $v_R(t)$, then $v(t) \approx v_R(t) = R\,i(t),$ and

$$v_c(t) = \frac{1}{C}\int i(t)dt = \frac{1}{RC}\int v_R(t)dt \approx \frac{1}{\tau}\int v(t)dt$$

i.e. the capacitor voltage is very closely proportional to the integral of the source voltage. If v(t) is a periodic function, $v_C(t)$ can be kept small by making the period $T \ll \tau$. In this way v_C never gets time to grow large.

On the other hand, when the period $T \gg \tau$, v_C tends to follow v(t) almost exactly. In such a case,

$$v_R(t) = Ri(t) = RC\frac{dv_C(t)}{dt} \approx \tau\frac{dv(t)}{dt}$$

i.e. the resistor voltage is very closely proportional to the derivative of the source voltage.

3 Prelab Exercises

3.1 Following the discussion in section 7.2 of the textbook, write down the differential equation of the series R-C circuit in the absence of any forcing input. Then, explain or derive equation (1) in your own way.

3.2 For $R = 10\text{M}\Omega$ and $C = 15\mu\text{F}$, determine the expected time constant $\tau = RC$.

3.3 Equation (2) for V_{CPP} is rather difficult to prove at this time. Take it as a challenge to derive it as you learn increasingly more on the topic of differential equations.

3.4 Explain in your own words why an R-C series circuit can act approximately as an integrator as well as a differentiator and under what conditions.

4 Experiments

Suggested Equipment:

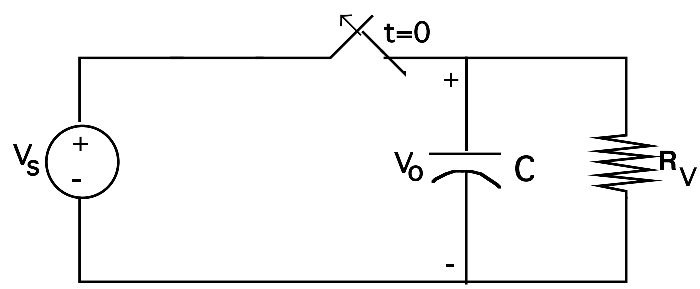
TEKTRONIX FG 501A 2MHz Function Generator
HP 54600A or Agilent 54622A Oscilloscope
Protek Model B-845 Digital Multimeter
TEKTRONIX DC504A Counter-Timer
TEKTRONIX P503A Dual Power Supply
100Ω, 10KΩ, 100KΩ, Resistors
0.001µF, 0.01µF, 1µF, 15µF Capacitors
Breadboard
One 3.5" diskette

NOTE: The oscillator is designed to work for a very wide range of frequencies but may not be stable at very low frequencies, say in the order of 100 Hz or 200Hz. To start with it is a good idea to have the circuit working at some mid-range frequency, say in the order of 1K Hz or 2K Hz, and then change the frequency slowly as needed.

4.1 Time Constant of an R-C Circuit

Construct the R-C circuit shown in fig. 5.

Fig. 5 Natural Response of an R-C Circuit



Let the source voltage V_s be a DC voltage of 10 V, C = 15µF, and R_V = 10MΩ is the internal resistance of the DVM. Neglect the internal resistance R_i of the source. Then, when the switch is closed, the capacitor charges quickly to the source voltage V_s.

At t = 0, the switch opens, and the voltage source gets disconnected from the R-C circuit. The capacitor will now discharge through the internal resistance of the DVM. Using a timer or a stopwatch, record the DVM readings for an interval of 5 minutes taking data in 15 second intervals. Repeat the same procedure until you are assured that you have a representative set of data. Fill Table 1 with the data and use the set of data that, in your opinion, corresponds to the run that is mostly representative of the capacitor discharge.

T (min)	v(t) V	v(t) V	v(t) V	v(t) V	v(t) V	f(t)=v(t)/v(0)	ln(f(t))
0.0							
0.15							
0.30							
1.00							
1.15							
1.30							
1.45							
2.00							
2.15							
2.30							
2.45							
3.00							
3.15							
3.30							
3.45							
4.00							
4.15							
4.30							
4.45							
5.00							

Table 1 Capacitor Discharge Data

4.2 Square Wave Response

a- Take $C = 0.01\mu F$ and use a resistor R so that one-half period of a 1.00KHz square wave will be 5τ where the time constant $\tau = RC$. Arrange such an R-C circuit (fig. 4) with the function generator as the source. Observe the function generator output on Channel 1 and v_C on Channel 2. R =______________Ω.

b- Set the generator to square wave and set f = 500Hz[3]. Set the scope sensitivities to 1 volt/div (CAL), AC input. Set sweep to 0.5ms/div (CAL). Adjust the generator amplitude for 6 V peak-to-peak (V_{PP} = 6 V, i.e. the square wave amplitude A=3 V) and center both images vertically on the screen.

c- Since f = 500Hz, a half period is 10τ. Notice that the capacitor has time to fully charge on each half-cycle (see eq. (2) and note that e^{-10} is negligible.)
Change the sweep to 0.1ms/div. Now a half period is 10 divisions wide.
Let us write a theoretical expression for the capacitor voltage v_C. Taking the origin of time, t =0, when the square wave jumps from -A to A, we can determine the capacitor voltage v_C during the half cycle that follows t =0 as

$$v_C(t) = A - 2A\, e^{-t/\tau}$$

Verify that the plot on the screen follows the above equation. This can be achieved by checking whether four or so representative points of $v_C(t)$ on the screen are as predicted by the equation. Download the waveform for your report.

d- Increase R and decrease C by a factor of 10, and verify that the circuit still behaves as in part c. Download the waveform for your report.

e- From their original values, decrease R and increase C by a factor of 100 and again verify that the circuit behavior is substantially the same as in part c. However you may notice that the 50-ohm internal impedance of the function generator causes some distortion at the beginning of each half-cycle when the current is large. Download the waveform for your report.

f- Return to the original values of R and C and adjust the amplitude for V_{PP} = 6.0 if necessary. Change the frequency to 2.00KHz (check frequency again). The half period is now 2.5τ. Measure the peak to peak capacitor voltage V_{CPP} and keep this value for the Report.

V_{CPP} =______________V.

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[3] Because of the inaccuracy of the oscillator frequency knobs of the function generator, it is essential to *actually measure* the exact frequency or period of the signal generated. Both the counter and the digital oscilloscope can be used for this purpose. Either should be used *in all experiments where a frequency or a period reading is required*, in order to obtain a correct and accurate result. In this case, use the counter.

4.3 Integration and Differentiation

4.3.1 Integration of a square wave:
Choose a frequency at which you are satisfied with the performance of the circuit as an integrator (adjust sensitivity and sweep rate as needed). Measure the peak to peak capacitor voltage V_{CPP} and the frequency of the waveform using the frequency counter. Study the relation of the function and its integral and download the scope image for your report.

4.3.2 Integration of a triangular wave:
With the circuit functioning well as an integrator, switch to triangular input. Adjust the generator output to 6 V peak-to peak. Study the relation of the function and its integral and copy the scope image for your report.
The image of the integral on the scope may look like a sine wave but in fact it is parabolic. To prove this, make adjustments with the sweep, sensitivities, and variables until the image spans 8 divisions peak-to-peak and the half period is 4 divisions wide. Notice that the amplitude at 1/8 period is 3/4 of peak instead of 0.707 of peak as it would be in a sine wave. Download the waveform.

4.3.3 Integration of a sine wave:
Change to sine wave input. Set the sensitivity variables and the sweep variable to CAL. Study the image and download as before.

4.3.4 Differentiation of a triangular wave:
Return to triangular wave. Interchange the capacitor and the resistor in the circuit. Channel 2 should now be receiving v_R. Adjust the sensitivity of the scope so v_R becomes visible. Reduce frequency gradually, making changes in scope sensitivity and sweep rate as you go, until you are satisfied with the performance of the circuit as a differentiator.
Measure the peak to peak resistance voltage V_{RPP} and the frequency of the wave using the frequency counter. Study the images and download them for your report.

4.3.5 Differentiation of a square wave:
Change to a square wave and decrease Channel 2 sensitivity until the derivative becomes visible. Study and copy the images. With 1 volt/div on Channel 1 and 2 volts/div on Channel 2, notice that the peak V_R is twice the peak input.

4.3.6 Differentiation of a sine wave:
Change to sine wave. Adjust input to V_{PP} = 6.0 and make the usual positioning adjustments. Tweak the frequency to make the phase difference stand out. Study the images and download them.

5 Report

5.1 Determine the time constant τ in the following two ways:

(a). Plot f(t) data from table 1 on a graph paper with rectangular coordinates. The value of τ is the time at which

$$f(t) = f(0)e^{-1} = 0.368f(0)$$

(b). $f(t) = A\, e^{-t/\tau}\, u(t)$

- $\Rightarrow \ln(f(t)) = (-1/\tau)t + \ln(A)$ for $t \geq 0$ which is in the form: $Y = A_1\, t + A_0$ and $\ln(f(t))$ vs t should plot as a straight line.
- Plot $\ln(f(t))$ vs t on a graph paper with rectangular coordinates and find the best straight line fit to the data.
- The slope of the straight line must be $(-1/\tau)$, hence the value of τ can be computed from the slope.

5.2 Determine the time constant by integration using the following method:

(a) From the equation $f(t) = Ae^{-t/\tau}$, it is easy to show that the area under the complete f(t) curve is equal to $A\tau$.

(b) The trapezoidal rule is used to find the area. For a set of observations $f_1 \ldots f_N$ spaced at a common interval Δt, the area τ in the interval $t_1 \leq t_i \leq t_N$ is given approximately by:

$$Area = \tau = \Delta t\left[\frac{f_1 + f_N}{2} + \sum_{i=2}^{N-1} f_i\right]$$

5.3 Determine the time constant by differentiation using the following method:

(a) At any point on the graph of f(t) vs t, the slope line (tangent) will always reach zero in a time $t = \tau$. Given

$$f(t) = f(t_o)e^{-\frac{t-t_o}{\tau}}$$

it can be shown that (do not prove)

$$\tau = f(t_o)\frac{\Delta t}{f(t_o) - f(t_o + \Delta t)}$$

(b) Apply the above equation for τ to each of the data points to the first 3 minutes and average the values of τ to obtain the time constant.

5.4 From the value of V_{CPP} measured in Section 4.2f, determine V_{PP}/V_{CPP}. Compare with the theoretical value obtained from Eq. 2.

5.5 Submit all images copied from the scope, using the appropriate scales and label with the appropriate descriptive labels.

5.6 From Section 4.3.1, determine the minimum τ/T for good integration of a square wave. Determine the minimum V_{PP}/V_{CPP}. Compare with the theoretical value obtained from Eq. 2.

5.7 From Section 4.3.2, Show that the integration of the triangular wave is a parabolic wave, i.e, show that the amplitude of the wave at 1/8 period is 3/4 of peak.

5.8 From Section 4.3.4, determine the minimum T/τ for good differentiation of the triangular wave. Determine the minimum V_{PP}/V_{RPP}. What does the output waveform look like?

5.9 Simulate in PSpice all parts of section 4.3. Use the frequencies obtained in the Lab. Plot the output waveforms. Compare with the experimental ones.

5.10 Design a first order RC circuit that produces the following response:

$$v_c(t) = 10 - 5\, e^{-3000t} \text{ V for } t \geq 0.$$

5.11 Prepare a summary.