1- INTRODUCTION: The objective of this experiment is to study some simple resistive circuits in order to demonstrate Proportionality, Superposition, Thévenin, and Maximum Power Transfer theorems. This experiment is more involved than the previous two experiments and thus it will be weighed more in deciding your lab grade.

2- THEORETICAL CONSIDERATIONS:

2.1 Proportionality Theorem:

The v-i characteristics of a linear resistor, \( V = R I \), shows that if the current is doubled then the voltage is also doubled. However power is quadrupled, which is not a proportional change. Only the current and voltage satisfy the proportionality property. For linear resistive circuits the proportionality relationship can be written as

\[ y = K x \]

where \( x \) is the input, \( y \) is the output and \( K \) is a constant. The input \( x \) is multiplied by the scalar constant \( K \) to produce the output \( y \). In the following circuit the proportionality relationship can be represented as

\[ V_{out} = K V_{in} \]

where \( K \) is a scalar constant whose value depends on the values of the resistors R1, R2, R3, and R4.

![Fig. 1 Proportionality Theorem](image)
2.2 Superposition Theorem:

The input-output relationships of linear circuits possess the additive property of linear functions. For linear resistance circuits this means that we can write any output $y$ as

$$y = K_1 x_1 + K_2 x_2 + K_3 x_3 + \ldots$$

where $x_1, x_2, x_3, \ldots$ are circuit inputs and $K_1, K_2, K_3, \ldots$ are constants that depend on the circuit. Briefly stated, the output of a linear resistance circuit is a linear combination of the outputs due to each input. That is, the output with all the inputs present is the same as the sum of the outputs with each input applied separately while the other inputs are set to zero. The following circuit explains the above principle of superposition. The output is taken across the resistor $R_3$; in the first case with both the inputs present and then with each input independently set while the other is set to zero. Thus,

$$V_{out} = V_{1out} + V_{2out}$$

where $V_{out}$ is the output with both the inputs present, $V_{1out}$ is the output with only $V_1$ as input and $V_{2out}$ is the output with only $V_2$ as input.

![Fig. 2 Superposition theorem](image)

2.3 Thévenin-Norton Theorem:

For simplicity, we will concentrate only on Thévenin Theorem as applied to linear resistive networks. Consider a given linear circuit that contains only resistances and sources which could be independent or dependent sources. Also, consider two of its terminals, say $e$ and $g$, having a certain load across them. Let us also, consider another circuit called a Thévenin equivalent circuit containing a voltage source $V_T$ in series with a resistance $R_T$ connected to its terminals $e$ and $g$ and also having the same load across the terminals $e$ and $g$ as that of the given original circuit (see Fig. 5A given in a later page). Then, there exist the parameters $V_T$ and $R_T$ such that both the given circuit and the Thévenin equivalent circuit have the same voltage across $e$ and $g$ and the same current through $e$ and $g$ whatever might be the load that is connected across $e$ and $g$ in the given circuit provided the same load is across $e$ and $g$ in the Thévenin equivalent circuit. The voltage $V_T$ is equal to the open circuit voltage across the terminals $e$ and $g$ (i.e. the voltage across $e$ and $g$ when the load across $e$ and $g$ draws no current through it). The resistance $R_T$ is the resistance seen from the terminals $e$ and $g$ of the original given circuit when all its independent sources are set to zero.

Fig. 3 illustrates a physical resistance network that will be used for an application of the Thévenin Theorem and the Maximum Power Transfer theorem.
If the ideal source $V_2$ is removed (i.e. e-g is an open circuit) but $V_1$ is retained, the network becomes a two-terminal network (a one-port) which has a Thévenin equivalent with respect to terminals e-g.

There are various ways by which the parameters $V_T$ and $R_T$ of the equivalent circuits can be determined. $V_T$, of course, is the open-circuit voltage at terminals e-g, but this is the same as saying that $V_T$ is the value needed for $V_2$ to reduce $I_2$ to zero in the original network. This value can be determined by using mesh or node analysis in the original network, and obtaining a relationship between $I_2$ and $V_2$. Setting $I_2$ to zero solves for $V_2 = V_T$. Setting $V_2$ to zero implies that the terminals e and g are shorted; evaluating $I_2$ then yields the short circuit current $I_{sh}$ which also equals Norton current $I_N$. The ratio $V_T/I_{sh}$ equals $R_T$.

For the circuit of Fig. 3, an alternate method for obtaining the Thévenin Parameters is as follows: If we apply the DELTA-WYE conversion to terminals b-c-d, the two terminal network eventually reduces to the form shown in Fig. 4. The Thévenin parameters can then be calculated relatively easily by the same procedure outlined above.

In Fig. 4, the Thévenin voltage $V_T$ is the open circuit voltage. It is the voltage at terminals e-g which is also the voltage across $R_9$ when no current flows in $R_8$. The Thévenin resistance $R_T$ is the equivalent resistance of the network looking at e-g with $V_1 = 0$ V.

2.4 Maximum Power Transfer Theorem:

The ability of the network to transfer power to a load connected across e and g is determined by the Thévenin parameters.

Let $V_L$ and $I_L$ be the load variables connected to the original network as shown in Fig. 5 where the Thévenin equivalent has been used. The power to the load, $P_L$ is maximum
when \( V_L = V_T/2 \) and \( I_L = I_{sn}/2 \) and therefore \( R_L = R_T \) (to be derived later on for the report). The maximum power denoted by \( P_{L(max)} \) that can be transferred to the load is given by

\[
P_{L(max)} = V_T^2/4R_T = I_n^2/4G_N = I_n^2R_T/4
\]

(Eq. 1)

where \( G_N = 1/R_T \) is the conductance.

If the load is a pure resistance, \( R_L \), the power function is then given in dimensionless form by

\[
P = 4r/(1+r)^2 \quad \text{(Eq. 2)}
\]

where \( P = P_L/P_{L(max)} \), and \( r = R_L/R_T \).

![Fig. 5](image)

**Fig. 5** \( R_L \) is connected to a) Thévenin Equivalent, b) Norton Equivalent

### 3- PRE-LAB EXERCISES:

3.1 For the circuit of Fig. 1 and for some appropriately chosen values for the resistances (all resistances must be greater than one Kilo ohm) and the source \( V_{in} = 10 \) V, determine first the output voltage and then determine the proportionality coefficient \( K \).

3.2 Consider Fig. 2 for some appropriately chosen values for the resistances (all resistances must be greater than one Kilo ohm) with \( V_1 = 10 \) V and \( V_2 = 5 \) V. Determine the output voltage across the resistance \( R_3 \) and denote it by \( V_{out} \). With \( V_2 \) set to zero and \( V_1 = 10 \) V, determine the output voltage across the resistance \( R_3 \) and denote it by \( V_{1out} \). Similarly, with \( V_1 \) set to zero and \( V_2 = 5 \) V, determine the output voltage across the resistance \( R_3 \) and denote it by \( V_{2out} \). Verify that \( V_{out} = V_{1out} + V_{2out} \).

3.3 Consider Fig. 3 with resistance values as given in Fig. 6 in the next page. Using Mesh Analysis, derive the expressions for the currents \( I_1, I_2, \) and \( I_3 \) in terms of \( V_1 \) and \( V_2 \).

3.4 Determine the theoretical values of the open circuit voltage \( V_T \) and the Thévenin resistance \( R_T \) as explained in Section 2.3, and with the use of the results of item 3.3 as given above. Use the value of \( V_1 = 10V \).

3.5 Consider Fig. 3 with resistance values as given in Fig. 6 in the next page. Use the Delta-Wye transform to terminals b-c-d so that the network in Fig. 3 is replaced by the network shown in Fig. 4. Determine \( R_7, R_8, \) and \( R_9 \) of Fig. 4. Show all your work.
3.6 From the network obtained in item 3.5 as given above, determine $V_T$ and $R_T$ with $V_1 = 10V$. Show all your work.

4- EXPERIMENT:
Suggested Equipment:
- TEKTRONIX PS 503 Power Supply
- Keithly 179A TRMS Multimeter
- 0 - 1000 Simpson Milliammeter or a Suitable DMM as available
- 2:2.0kΩ, 2:3.3kΩ, 2:3.6kΩ and other resistances as needed
- Variable Resistance Box

4.1 Proportionality Theorem:
Prepare a table with three columns, the first labeled $V_{in}$, the second one for $V_{out}$ and the third for the calculated proportionality coefficient $K$. With the resistance values as chosen in pre-lab exercise 3.1, construct the circuit of Fig. 1. At four different input voltages of your choice, measure $V_{in}$ and $V_{out}$. At each point calculate and record $K$. Plot these four actual points on the graph (Response $V_{out}$ vs $V_{in}$) and connect them with a best (in your judgement) fit line going through the origin. The slope of the line gives the coefficient $K$ that fits the data best.

4.2 Superposition Theorem:
1. With the resistance values and source values as chosen in pre-lab exercise 3.2, construct the circuit of Fig. 2.
2. Measure the voltage across and the current through the resistor $R_3$. Record these values.
3. Keep $V_1$ and remove the power supply $V_2$, and in its place connect a wire. Measure the voltage across and current through the resistor $R_3$. Record these values.
4. Repeat the step 3 with $V_2$ present and $V_1$ removed.
5. Verify the superposition theorem.

4.3 Thévenin Equivalent To The Left Of Terminals e-g:

![Fig. 6 Resistive Circuit]
1. Construct the circuit of Fig. 6.

2. Remove $V_2$. With $e-g$ as an open circuit and $V_1 = 10V$, measure first the output voltage across the terminals $e-g$. This is the Thévenin voltage $V_T$.

3. Remove $V_2$. With $e-g$ as a short circuit and $V_1 = 10V$, measure the short circuit current $I_{sh}$ through $e-g$. The ratio $V_T/I_{sh}$ equals the Thévenin resistance $R_T$.

4. Remove $V_1$ and $V_2$. Connect a wire across the terminals $a-g$. Measure the equivalent resistance $R_T$ looking from the terminals $e-g$.

5. Verify your results with those of in pre-lab exercise 3.6.

By measuring $V_T$, $I_{sh}$, and $R_T$, the network to the left of $e-g$ is easily replaced by Thévenin or Norton equivalent.

**4.4 Power computations:**

The experiment of Section 4.3 is continued to see the effect of the load on the given circuit. Prepare a data table with the column headings suggested below: The values of $R_T$ and $P_L(\text{max})$ should be based on the measurements just taken in Section 4.3.

<table>
<thead>
<tr>
<th>Load Resistance</th>
<th>Load Voltage</th>
<th>Load Power</th>
<th>Dimensionless Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_L$</td>
<td>$V_L$</td>
<td>$P_L$</td>
<td>$r = R_L / R_T$</td>
</tr>
<tr>
<td>$K\Omega$</td>
<td>Volts</td>
<td>mW</td>
<td>$P = P_L / P_L(\text{max})$</td>
</tr>
</tbody>
</table>

In Fig. 6, remove $V_2$ and set $V_1 = 10V$. Connect $R_L$ across the terminals $e-g$. The load $R_L$ is to be taken from a $10K\Omega$ potentiometer. Increase $R_L$ in the following way:

- $0K\Omega$ to $5K\Omega$ increment by $0.5K\Omega$
- $5K\Omega$ to $7K\Omega$ increment by $0.2K\Omega$
- $7K\Omega$ to $10K\Omega$ increment by $0.5K\Omega$

and measure the voltage across the load $V_L = V_{eg}$. Record the data in the table given above. The power $P_L$ can be computed by knowing $V_L$ and $R_L$.

From the data collected, determine the value of $R_L$ that causes the maximum power $P_L$. Also, determine the value of $R_L$ that causes the maximum current to be delivered to $R_L$ and the value of the maximum current. Similarly, determine the value of $R_L$ that causes the maximum voltage to be delivered to $R_L$ and the maximum voltage.
5- REPORT:

5.1 For the circuit of Fig.1 how did the measurements made in the lab compare with the predicted output as calculated in pre-lab exercise 3.1. Explain any differences.

5.2 For each of the three circuits you built for the superposition portion of the exercise, how well did the measured outputs compare with those in pre-lab exercise 3.2? Explain any differences.

5.3 Tabulate the measured Thévenin parameters and the theoretical ones determined in pre-lab exercises. Compare the two and explain any differences.

5.4 Draw the Thévenin equivalent circuit of Fg. 6 at the terminals e-g using the measured parameters.

5.5 Show mathematically that the power $P_L$ to the load shown in Fig. 5 is maximum when $V_L = V_T/2$, $I_L = I_{Sh}/2$ and therefore, $R_L = R_T$. Use these results to derive Eq. 1.

5.6 As usual, assuming that the load is a pure resistance $R_L$, derive Eq. 2.

5.7 Plot the dimensionless power function given by Eq. 2. on a graph paper with rectangular coordinates for the range $0 \leq r \leq 2$. Notice that this is a “universal” curve, i.e. it is independent of $R_T$ and $R_L$ and hence is valid for any Thévenin source feeding any pure resistance load.

5.8 Complete the Table in Section 4.4. Plot the dimensionless power vs the dimensionless resistance on the same graph paper of item 5.7 using the values measured. Compare the two graphs.

5.9 Looking at the graphs of the dimensionless power, determine the load resistance where the power is maximum. Is it equal to the Thévenin resistance $R_T$? If not, why?

5.10 Simulate the network in Fig. 6 in PSPICE or MULTISIM to obtain the currents $I_1$, $I_2$, and $I_3$ for:
   a) $V_1 = 10V$, $V_2 = 0V$
   b) $V_1 = 0V$, $V_2 = 5V$
   c) $V_1 = 10V$, $V_2 = 5V$

5.11 With $V_2$ disconnected, simulate the circuit of Fig. 6 in PSPICE or MULTISIM to obtain the Thévenin equivalent parameters $V_T$ and $R_T$.

5.12 Prepare a summary.