RUTGERS UNIVERSITY
The State University of New Jersey
School of Engineering
Department of Electrical and Computer Engineering

332:223 Principles of Electrical Engineering I Laboratory

Experiment #3

Title: Proportionality, Superposition, Thévenin, and Maximum Power Transfer theorems

1- INTRODUCTION: The objective of this experiment is to study some simple resistive circuits in order to demonstrate Proportionality, Superposition, Thévenin, and Maximum Power Transfer theorems. This experiment is more involved than the previous two experiments and thus it will be weighed more in deciding your lab grade.

2- THEORETICAL CONSIDERATIONS:

2.1 Proportionality Theorem:

The **v-i** characteristics of a linear resistor, **v = Ri**, shows that if the current is doubled then the voltage is also doubled. However power is quadrupled, which is not a proportional change. Only the current and voltage satisfy the proportionality property. For linear resistive circuits the proportionality relationship can be written as

$$y = K\,x$$

where **x** is the input, **y** is the output and **K** is a constant. The input **x** is multiplied by the scalar constant **K** to produce the output **y**. In the following circuit the proportionality relationship can be represented as

$$V_{out} = K\,V_{in}$$

where **K** is a scalar constant whose value depends on the values of the resistors R1, R2, R3, and R4.

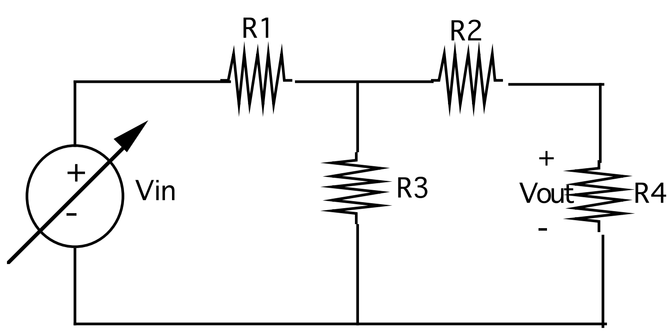

Fig. 1 Proportionality Theorem

2.2 Superposition Theorem:

The input-output relationships of linear circuits possess the additive property of linear functions. For linear resistance circuits this means that we can write any output y as

$$y = K_1\, x_1 + K_2\, x_2 + K_3\, x_3 + ...$$

where x_1, x_2, x_3, .. are circuit inputs and K_1, K_2, K_3, ... are constants that depend on the circuit. Briefly stated, the output of a linear resistance circuit is a linear combination of the outputs due to each input. That is, the output with all the inputs present is the same as the sum of the outputs with each input applied separately while the other inputs are set to zero. The following circuit explains the above principle of superposition. The output is taken across the resistor R_3; in the first case with both the inputs present and then with each input independently set while the other is set to zero. Thus,

$$V_{out} = V_{1out} + V_{2out}$$

where V_{out} is the output with both the inputs present, V_{1out} is the output with only V_1 as input and V_{2out} is the output with only V_2 as input.

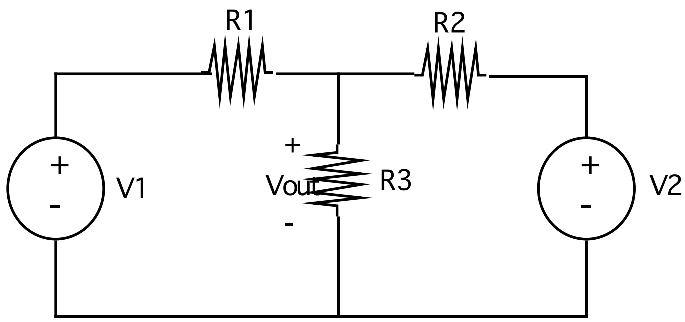

Fig. 2 Superposition theorem

2.3 Thévenin-Norton Theorem:

For simplicity, we will concentrate only on **Thévenin Theorem** as applied to linear resistive networks. Consider a given linear circuit that contains only resistances and sources which could be independent or dependent sources. Also, consider two of its terminals, say **e** and **g**, having a certain load across them. Let us also, consider another circuit called a Thévenin equivalent circuit containing a voltage source V_T in series with a resistance R_T connected to its terminals **e** and **g** and also having the same load across the terminals **e** and **g** as that of the given original circuit (see Fig. 5A given in a later page). Then, there exist the parameters V_T and R_T such that both the given circuit and the Thévenin equivalent circuit have the same voltage across **e** and **g** and the same current through **e** and **g** whatever might be the load that is connected across **e** and **g** in the given circuit provided the same load is across **e** and **g** in the Thévenin equivalent circuit. The voltage V_T is equal to the open circuit voltage across the terminals **e** and **g** (i.e. the voltage across **e** and **g** when the load across **e** and **g** draws no current through it). The resistance R_T is the resistance seen from the terminals **e** and **g** of the original given circuit when all its independent sources are set to zero.

Fig. 3 illustrates a physical resistance network that will be used for an application of the Thévenin Theorem and the Maximum Power Transfer theorem.

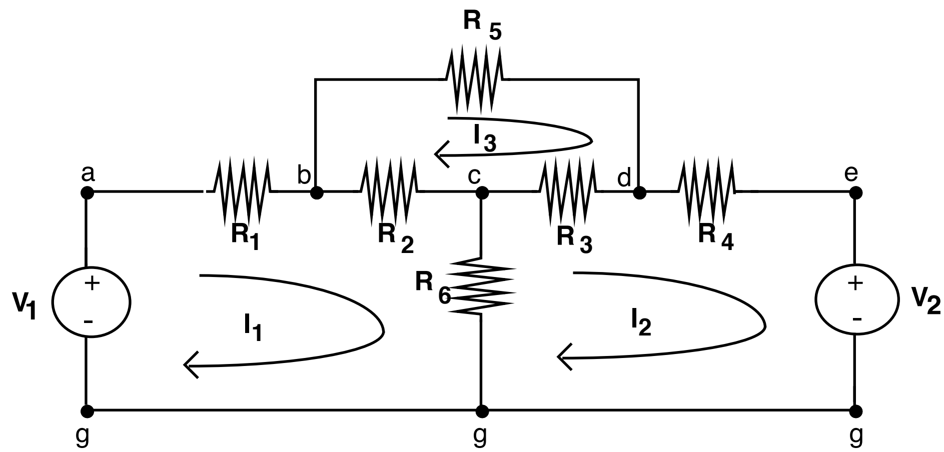

Fig. 3 A Resistive Network

If the ideal source V_2 is removed (i.e. **e-g** is an open circuit) but V_1 is retained, the network becomes a two-terminal network (a one-port) which has a Thévenin equivalent with respect to terminals **e-g**.

There are various ways by which the parameters V_T and R_T of the equivalent circuits can be determined. V_T, of course, is the open-circuit voltage at terminals **e-g**, but this is the same as saying that V_T is the value needed for V_2 to reduce I_2 to zero in the original network. This value can be determined by using mesh or node analysis in the original network, and obtaining a relationship between I_2 and V_2. Setting I_2 to zero solves for $V_2 = V_T$. Setting V_2 to zero implies that the terminals **e** and **g** are shorted; evaluating I_2 then yields the short circuit current I_{Sh} which also equals Norton current I_N. The ratio V_T/I_{Sh} equals R_T.

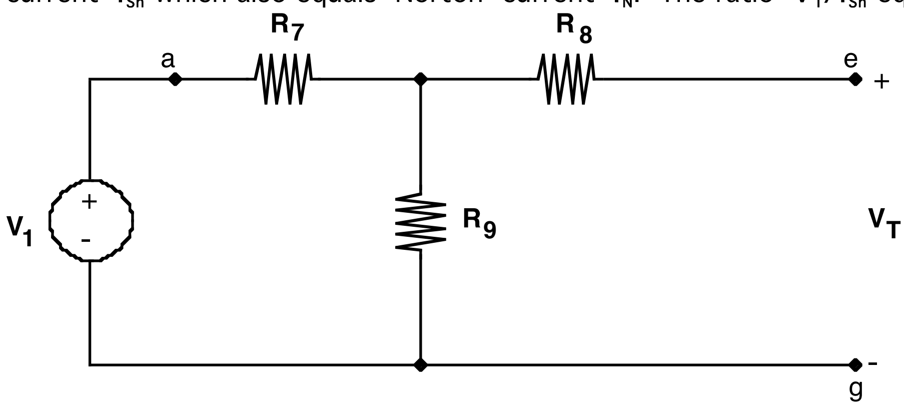

Fig. 4 Delta-Wye Conversion of the Circuit in Fig. 3

For the circuit of Fig. 3, an alternate method for obtaining the Thévenin Parameters is as follows: If we apply the DELTA-WYE conversion to terminals **b-c-d** , the two terminal network eventually reduces to the form shown in Fig. 4. The Thévenin parameters can then be calculated relatively easily by the same procedure outlined above.
In Fig. 4, the Thévenin voltage V_T is the open circuit voltage. It is the voltage at terminals **e-g** which is also the voltage across R_9 when no current flows in R_8. The Thévenin resistance R_T is the equivalent resistance of the network looking at **e-g** with $V_1 = 0$ V.

2.4 Maximum Power Transfer Theorem:

The ability of the network to transfer power to a load connected across **e** and **g** is determined by the Thévenin parameters.
Let V_L and I_L be the load variables connected to the original network as shown in Fig. 5 where the Thévenin equivalent has been used. The power to the load, P_L is maximum

when $V_L = V_T/2$ and $I_L = I_{sh}/2$ and therefore $R_L = R_T$ (to be derived later on for the report). The maximum power denoted by $P_{L(max)}$ that can be transferred to the load is given by

$$P_{L(max)} = V_T^2/4R_T = I_N^2/4G_N = I_N^2R_T/4 \qquad \text{(Eq. 1)}$$

where $G_N = 1/R_T$ is the conductance.
If the load is a pure resistance, R_L, the power function is then given in dimensionless form by

$$P = 4r/(1+r)^2 \qquad \text{(Eq. 2)}$$

where $P = P_L/P_{L(max)}$, and $r = R_L/R_T$.

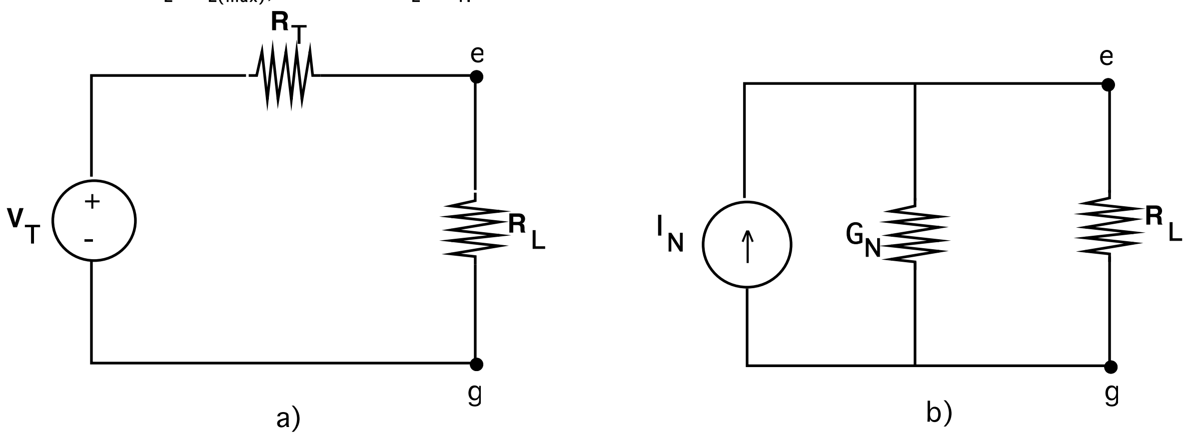

Fig. 5 R_L is connected to a) Thévenin Equivalent, b) Norton Equivalent

3- PRE-LAB EXERCISES:

3.1 For the circuit of Fig. 1 and for some appropriately chosen values for the resistances (all resistances must be greater than one Kilo ohm) and the source V_{in} = 10 V, determine first the output voltage and then determine the proportionality coefficient **K**.

3.2 Consider Fig. 2 for some appropriately chosen values for the resistances (all resistances must be greater than one Kilo ohm) with V_1 = 10 V and V_2 = 5 V. Determine the output voltage across the resistance R_3 and denote it by V_{out}. With V_2 set to zero and V_1 = 10 V, determine the output voltage across the resistance R_3 and denote it by V_{1out}. Similarly, with V_1 set to zero and V_2 = 5 V, determine the output voltage across the resistance R_3 and denote it by V_{2out}. Verify that
$V_{out} = V_{1out} + V_{2out}$.

3.3 Consider Fig. 3 with resistance values as given in Fig. 6 in the next page. Using Mesh Analysis, derive the expressions for the currents I_1, I_2, and I_3 in terms of V_1 and V_2.

3.4 Determine the theoretical values of the open circuit voltage V_T and the Thévenin resistance R_T as explained in Section 2.3, and with the use of the results of item 3.3 as given above. Use the value of V_1 = 10V.

3.5 Consider Fig. 3 with resistance values as given in Fig. 6 in the next page. Use the Delta-Wye transform to terminals **b-c-d** so that the network in Fig. 3 is replaced by the network shown in Fig. 4. Determine R_7, R_8, and R_9 of Fig. 4. Show all your work.

3.6 From the network obtained in item 3.5 as given above, determine V_T and R_T with V_1 = 10V. Show all your work.

4- EXPERIMENT:

Suggested Equipment:

TEKTRONIX PS 503 Power Supply
Keithly 179A TRMS Multimeter
0 - 1000 Simpson Milliammeter or a Suitable DMM as available
2:2.0KΩ, 2:3.3KΩ, 2:3.6KΩ and other resistances as needed
Variable Resistance Box

4.1 Proportionality Theorem:

Prepare a table with three columns, the first labeled V_{in}, the second one for V_{out} and the third for the calculated proportionality coefficient K. With the resistance values as chosen in pre-lab exercise 3.1, construct the circuit of Fig. 1. At four different input voltages of your choice, measure V_{in} and V_{out}. At each point calculate and record K.

Plot these four actual points on the graph (Response V_{out} vs V_{in}) and connect them with a best (in your judgement) fit line going through the origin. The slope of the line gives the coefficient K that fits the data best.

4.2 Superposition Theorem:

1. With the resistance values and source values as chosen in pre-lab exercise 3.2, construct the circuit of Fig. 2.

2. Measure the voltage across and the current through the resistor R_3. Record these values.

3. Keep V_1 and remove the power supply V_2, and in its place connect a wire. Measure the voltage across and current through the resistor R_3. Record these values.

4. Repeat the step 3 with V_2 present and V_1 removed.

5. Verify the superposition theorem.

4.3 Thévenin Equivalent To The Left Of Terminals e-g:

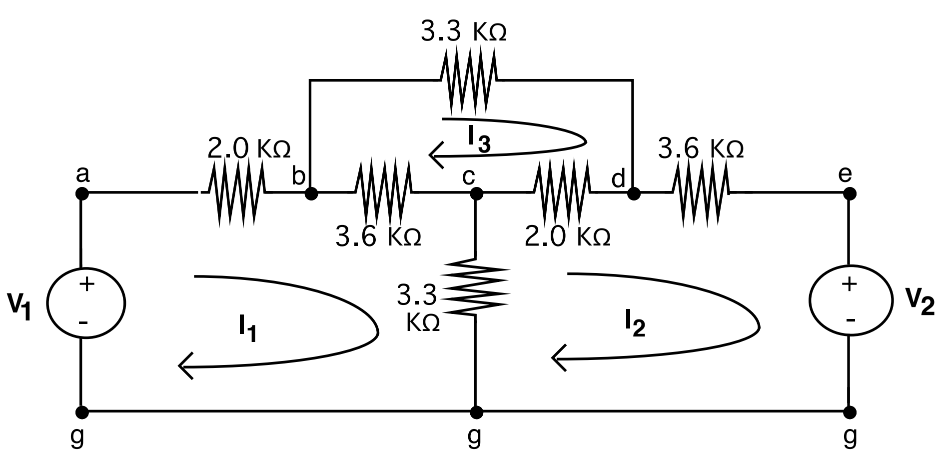

Fig. 6 Resistive Circuit

1. Construct the circuit of Fig. 6.

2. Remove V_2. With **e-g** as an open circuit and V_1 = 10V, measure first the output voltage across the terminals **e-g**. This is the Thévenin voltage V_T.

3. Remove V_2. With **e-g** as a short circuit and V_1 = 10V, measure the short circuit current I_{sh} through **e-g**. The ratio V_T/I_{sh} equals the Thévenin resistance R_T.

4. Remove V_1 and V_2. Connect a wire across the terminals **a-g**. Measure the equivalent resistance R_T looking from the terminals **e-g**.

5. Verify your results with those of in pre-lab exercise **3.6.**

By measuring V_T, I_{sh}, and R_T, the network to the left of **e-g** is easily replaced by Thévenin or Norton equivalent.

4.4 Power computations:

The experiment of Section 4.3 is continued to see the effect of the load on the given circuit. Prepare a data table with the column headings suggested below: The values of R_T and $P_{L(max)}$ should be based on the measurements just taken in Section 4.3.

Load Resistance	Load Voltage	Load Power	Dimensionless Values	
			Resistance	Power
R_L	V_L	P_L	$r= R_L/R_T$	$P= P_L/P_{L(max)}$
KΩ	Volts	mW	....	....

In Fig. 6, remove V_2 and set V_1 = 10V. Connect R_L across the terminals **e-g**. The load R_L is to be taken from a 10KΩ potentiometer. Increase R_L in the following way:

0KΩ 5KΩ increment by 0.5KΩ
5KΩ 7KΩ increment by 0.2KΩ
7KΩ 10KΩ increment by 0.5KΩ

and measure the voltage across the load $V_L = V_{eg}$. Record the data in the table given above. The power P_L can be computed by knowing V_L and R_L.

From the data collected, determine the value of R_L that causes **the maximum power P_L**. Also, determine the value of R_L **that causes the maximum current** to be delivered to R_L and the value of the maximum current. Similarly, determine the value of R_L **that causes the maximum voltage** to be delivered to R_L and the maximum voltage.

5- REPORT:

5.1 For the circuit of Fig.1 how did the measurements made in the lab compare with the predicted output as calculated in pre-lab exercise 3.1. Explain any differences.

5.2 For each of the three circuits you built for the superposition portion of the exercise, how well did the measured outputs compare with those in pre-lab exercise 3.2? Explain any differences.

5.3 Tabulate the measured Thévenin parameters and the theoretical ones determined in pre-lab exercises. Compare the two and explain any differences.

5.4 Draw the Thévenin equivalent circuit of f Fig. 6 at the terminals e-g using the measured parameters.

5.5 Show mathematically that the power P_L to the load shown in Fig. 5 is maximum when $V_L = V_T/2$, $I_L = I_{Sh}/2$ and therefore, $R_L = R_T$. Use these results to derive Eq. 1.

5.6 As usual, assuming that the load is a pure resistance R_L, derive Eq. 2.

5.7 Plot the dimensionless power function given by Eq. 2. on a graph paper with rectangular coordinates for the range $0 \leq r \leq 2$. Notice that this is a "universal" curve, i.e. it is independent of R_T and R_L and hence is valid for any Thévenin source feeding any pure resistance load.

5.8 Complete the Table in Section 4.4. Plot the dimensionless power vs the dimensionless resistance on the same graph paper of item 5.7 using the values measured. Compare the two graphs.

5.9 Looking at the graphs of the dimensionless power, determine the load resistance where the power is maximum. Is it equal to the Thévenin resistance R_T? If not, why?

5.10 Simulate the network in Fig. 6 in PSPICE or MULTISIM to obtain the currents I_1, I_2, and I_3 for:

a) V_1 = 10V, V_2 = 0V
b) V_1 = 0V, V_2 = 5V
c) V_1 = 10V, V_2 = 5V

5.11 With V_2 disconnected, simulate the circuit of Fig. 6 in PSPICE or MULTISIM to obtain the Thévenin equivalent parameters V_T and R_T.

5.12 Prepare a summary.