332:525 – Optimum Signal Processing
Computer Experiment 4 – Due February 24, 2011

Consider the noisy random walk signal model:

$$\begin{aligned} x_{n+1} &= x_n + w_n \\ y_n &= x_n + v_n \end{aligned} \tag{1}$$

where w_n, v_n are mutually uncorrelated, zero-mean, white noise signals of variances $Q = \sigma_w^2$ and $R = \sigma_v^2$. We showed in class that the optimum Wiener/Kalman filter for predicting x_n is equivalent to the exponential smoother,

$$\hat{x}_{n+1/n} = \lambda \hat{x}_{n/n-1} + (1-\lambda) y_n$$

provided we identify the forgetting factor λ (and the predictor's Kalman gain K) by

$$1 - \lambda = K = \frac{\sqrt{q^2 + 4q} - q}{2}, \quad q = \frac{Q}{R}$$

a. For the following values $\sigma_w = 0.1$ and $\sigma_v = 1$, generate $N = 300$ samples of x_n, y_n from Eq. (1) and run y_n through the equivalent Kalman exponential smoother to compute $\hat{x}_{n/n-1}$. On the same graph, plot all three signals $y_n, x_n, \hat{x}_{n/n-1}$ versus $0 \le n \le N-1$.

b. A possible way to determine λ from the data y_n is as follows. Assume a tentative value for λ, compute $\hat{x}_{n/n-1}$, then the error $e_{n/n-1} = x_n - \hat{x}_{n/n-1}$, and the mean-square error:

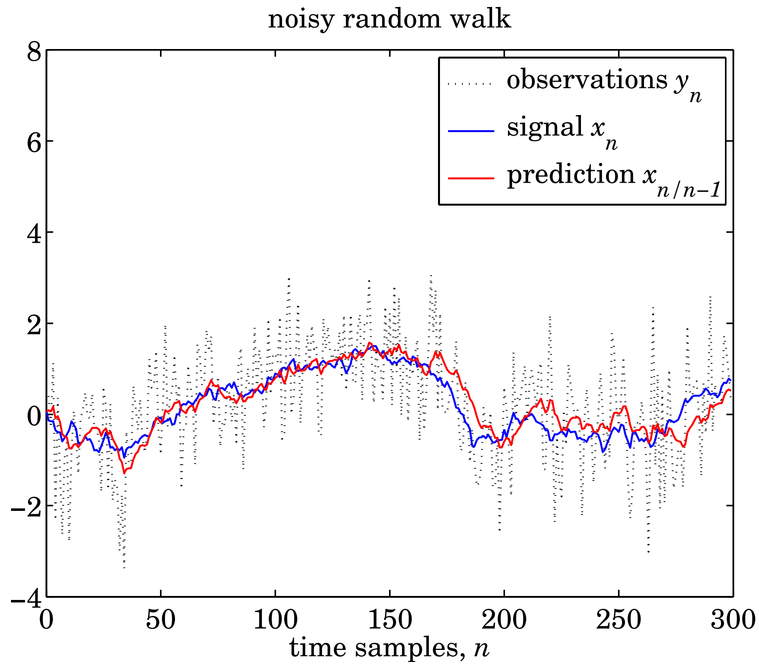
$$\text{MSE}(\lambda) = \sum_n e_{n/n-1}^2$$

Repeat the calculation of $\text{MSE}(\lambda)$ over a range of λs, for example, $\lambda_1 \le \lambda \le \lambda_2$, chosen such that the interval $[\lambda_1, \lambda_2]$ contain the true λ. Then find that λ that minimizes $\text{MSE}(\lambda)$, which should be close to the true value.

Because the estimated λ depends on the particular realization of the model (1), generate 20 different realizations of the pair x_n, y_n with the same Q, R, and for each realization carry out the estimate of λ as described above, and finally form the average of the 20 estimated λs.

c. Repeat part (b), by replacing the MSE by the mean-absolute-error:

$$\text{MAE}(\lambda) = \sum_n |e_{n/n-1}|$$


noisy random walk
observations y_n
signal x_n
prediction $x_{n/n-1}$
8
6
4
2
0
−2
−4
0
50
100
150
200
250
300
time samples, n