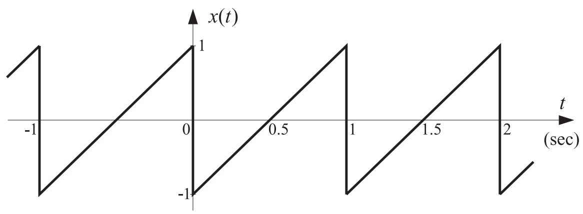
332:346 – Digital Signal Processing
Exam 1 – February 23, 2006

Please fully explain your reasoning and justify all steps.
Each subquestion is worth 10 points.

1. Consider a periodic sawtooth waveform $x(t)$ with period $T_0 = 1$ sec shown below:



For your reference, this periodic signal admits a Fourier series expansion containing only sine terms with harmonics at the frequencies $f_m = m/T_0$, $m = 1, 2, 3, 4, \ldots$ Hz, where t is in seconds:

$$x(t) = \sum_{m=1}^{\infty} b_m \sin(2\pi m t) = b_1 \sin(2\pi t) + b_2 \sin(4\pi t) + b_3 \sin(6\pi t) + \cdots$$

You don't need to know the b_m coefficients. The sawtooth waveform $x(t)$ is now sampled at the rate of $f_s = 5$ Hz and the resulting samples are reconstructed by an *ideal* reconstructor. Note that the value of the sawtooth at the discontinuities $(t = 0, 1, 2, \ldots)$ is equal to zero.

(a) Determine the harmonics within the Nyquist interval with which all the higher harmonics are aliased.

(b) Explain why the signal $x_a(t)$ at the output of the reconstructor will have the following form, with a_1, a_2 to be determined:

$$x_a(t) = a_1 \sin(2\pi t) + a_2 \sin(4\pi t)$$

(c) Set up but do not solve the equations from which the coefficients a_1, a_2 may be determined. (For your reference, the solutions are $a_1 = -0.5506$ and $a_2 = -0.1300$.)

2. Consider the following IIR filter $h(n) = a^n u(n)$, with $|a| < 1$. For each of the following three input signals $x(n)$, determine the corresponding output signal $y(n)$ as a function of n:

(a) $x(n) = \delta(n)$

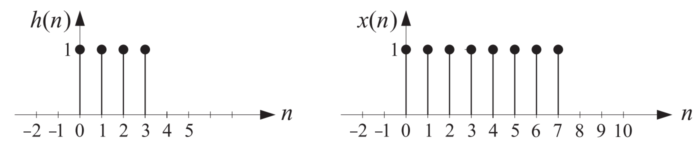
(b) $x(n) = 2\delta(n) + 3\delta(n-10)$

(c) $x(n) = u(n)$. In this case, show that the output has the form:

$$y(n) = Aa^n u(n) + Bu(n)$$

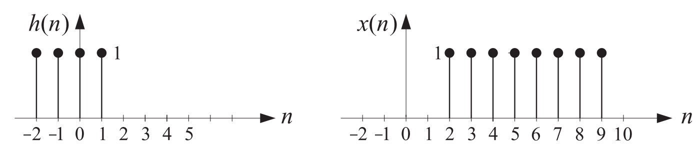
and determine the coefficients A, B in terms of a.

3. Consider a filter $h(n)$ and signal $x(n)$ defined in the following figure, where $h(n)$ is equal to 1 over $0 \le n \le 3$ and $x(n)$ is equal to 1 over $0 \le n \le 7$.



(a) Using any method you want, but showing all the computational steps, calculate the convolution $y(n)$ of these two sequences and determine the *range* of the output index n. Make a sketch of the signal $y(n)$ versus n. (There is no need to determine the summation limits in the convolution summation formula.)

(b) Repeat question (a), for the following case shown below, where $h(n)$ is equal to 1 for $-2 \le n \le 1$ and $x(n)$ is equal to 1 for $2 \le n \le 9$.



4. The ADSP-2181 Analog Devices 16-bit DSP chip uses the 1.15 fixed-point number format to represent real numbers internally. In this format, a 16-bit pattern $[b_1, b_2, \ldots, b_{16}]$ represents the following number x that lies in the interval $-1 \le x < 1$:

$$x = 2(\bar{b}_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + \cdots + b_{16} 2^{-16} - 0.5)$$

where $\bar{b}_1 = 1 - b_1$ is the complement of the b_1 bit.

(a) Show that the maximum number representable by this format is $x_{\max} = 1 - 2^{-15}$. What bit pattern $[b_1, b_2, \ldots, b_{16}]$ corresponds to this number?

(b) Show that the minimum representable number is $x_{\min} = -1$. What bit pattern $[b_1, b_2, \ldots, b_{16}]$ corresponds to this number?

[*Hint:*] Use the finite geometric series.

332:346 – Digital Signal Processing
Exam 2 – April 6, 2006

Please fully explain your reasoning and justify all steps.

Each subquestion is worth 10 points.

1. An oversimplified version of an allpass reverb filter has transfer function:

$$H(z) = \frac{-0.5 + z^{-3}}{1 - 0.5z^{-3}}$$

(a) Draw its *canonical* realization form and write down its sample processing algorithm using a *circular* delay-line buffer.

(b) Draw its *transposed* realization form and write down its sample processing algorithm also using a *circular* delay-line buffer.

(c) For the six input samples x shown in the following table, iterate the sample processing algorithm of part (a), and calculate the values of the circular-buffer entries w_0, w_1, w_2, w_3 as well as the values of the states s_0, s_3, and of the output y. Indicate the successive positions of the circular pointer p by circling the corresponding entries of the buffer $\mathbf{w}$. (Note that the states s_1, s_2 are not needed but may be inserted in the table if you so prefer.)

x	w_0	w_1	w_2	w_3	s_0	s_3	y
8	*	*	*	*	*	*	*
6	*	*	*	*	*	*	*
4	*	*	*	*	*	*	*
8	*	*	*	*	*	*	*
6	*	*	*	*	*	*	*
4	*	*	*	*	*	*	1

(check: the last value of y is one)

(d) The frequency response $H(\omega)$ is obtained by setting $z = e^{j\omega}$ into $H(z)$. Show that this is indeed an "allpass" filter by proving that

$$|H(\omega)| = 1, \quad \text{for all } \omega$$

2. Using pole/zero placement, design a second-order IIR notch filter operating at a rate of 8 kHz that has a notch frequency at 800 Hz and a 3-dB width of 80 Hz. (The dc-gain normalization factor may be omitted.)

(a) Write down the designed transfer function $H(z)$. Show all work.

(b) Calculate the 60-dB time constant of this filter in milliseconds.

3. Consider the following filter $H(z)$:

$$H(z) = \frac{0.75(1 + z^{-2})}{1 + 0.5z^{-2}}$$

(a) Draw a rough sketch of its magnitude response $|H(\omega)|$ versus frequency ω in the range $0 \le \omega \le \pi$.

(b) The following causal signal is sent to the input of the above filter:

$$x(n) = 2u(n) + 3\sin\left(\frac{\pi n}{2}\right)u(n) + 4\cos\left(\frac{\pi n}{2}\right)u(n) + 5\cos(\pi n)u(n)$$

Determine the exact form of the causal *steady-state* output signal, $y_{\text{steady}}(n)$, i.e., the output signal after steady state has been reached and the filter transients have died out. Express $y_{\text{steady}}(n)$ as a sum of sinusoids. Explain your approach (do not use z-transforms.)

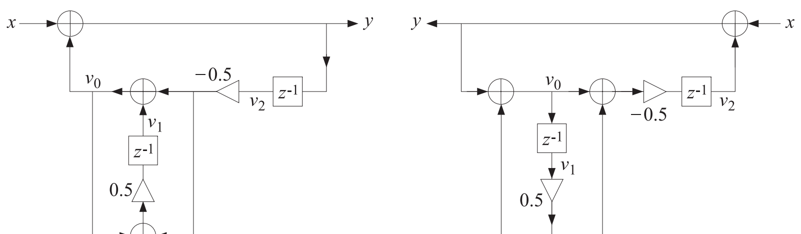
(c) Approximately, how many time samples does it take for the filter transients to die out?

(d) Based on the given $H(z)$, explain why some of the sinusoidal terms of the above input $x(n)$ can go through this filter, while some other terms are filtered out.

332:346 – Digital Signal Processing
Final Exam – May 8, 2006

Please fully explain your reasoning and justify all steps.

1. [30 pts]. Consider the filter structures (A) and (B) shown below:



   (a) Explain why they are the transposed of each other (and hence, they have the same transfer function.)

   (b) Working with either structure, determine their common transfer function $H(z)$ and express it as a ratio of two polynomials in z^{-1}. Show all work.

   (c) For both structures, write the corresponding sample processing algorithms. Only the indicated variables x, y, v_1, v_2, v_0 must be used in stating these algorithms. (No credit will be given if the computational steps are listed in the wrong order.)

   (d) Working with structure (A), iterate its sample processing algorithm on the input signal $\mathbf{x} = [8, 6, 4, 2]$ and compute the corresponding output samples y, as well as the quantities v_0, v_1, v_2, at each time instant, that is, fill the entries of the table:

x	v_1	v_2	v_0	y
8	∗	∗	∗	∗
6	∗	∗	∗	∗
4	∗	∗	∗	∗
2	∗	∗	∗	∗

2. [25 pts]. Determine the transfer function $H(z)$ of a second-order IIR notch filter operating at a rate of 10 kHz that knocks out the middle term of the following input signal within 9 milliseconds and allows the other terms to pass through unchanged:

$$x(n) = 3u(n) + 4\sin\left(\frac{\pi n}{2}\right)u(n) + 5\cos(\pi n)u(n)$$

thus, resulting into the following steady-state output signal (after the initial 9 msec have elapsed):

$$y_{\text{steady}}(n) = 3u(n) + 5\cos(\pi n)u(n)$$

[*Notes:* You may assume that the 40-dB time constant of the filter is 9 msec. An overall gain factor must also be used in order to get the above steady-state output. You may use the approximate design methods of Ch.6 or the exact methods of Ch.11. Show all design steps.]

3. [15 pts]. A signal is the sum of three sinusoidal signals of frequencies of 1 kHz, 3 kHz, and 4 kHz. The signal is sampled at a rate of 10 kHz and 128 samples are collected and a 128-point FFT is computed.

   (a) Based on the peaks observed in the computed 128-point FFT, what would be the estimated frequencies of the three sinusoids in kHz?

   (b) In a short sentence, explain why the error in estimating these frequencies is reduced by increasing the length of the FFT.

4. [10 pts]. The following signal is sampled at a rate of 8 kHz and 8 samples are collected:

$$x(t) = 3\cos(2\pi t) + 5\cos(8\pi t) + 4\sin(14\pi t)$$

where t is in msec. Without performing any DFT/FFT computations determine the 8-point DFT of the 8 collected time samples.

5. [10 pts]. Using the FFT algorithm, compute the 8-point DFT of the signal:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1+3\sqrt{2} \\ -1 \\ 1+3\sqrt{2} \\ 1 \\ 1-3\sqrt{2} \\ 3 \\ 1-3\sqrt{2} \end{bmatrix}$$

6. [10 pts]. Showing all work, calculate the inverse FFT of the following FFT using the formula $\text{IFFT}(\mathbf{X}) = \left[\text{FFT}(\mathbf{X}^*)\right]^*/N$:

$$\mathbf{X} = \begin{bmatrix} 4 \\ 0 \\ -4j \\ 0 \\ 4 \\ 0 \\ 4j \\ 0 \end{bmatrix}$$

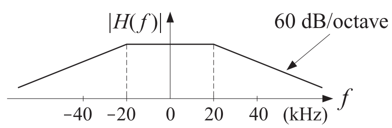
332:346 – Digital Signal Processing
Exam 1 – February 22, 2007

1. Consider the following sound signal, where t is in milliseconds:

$$x(t) = \sin(10\pi t) + \sin(30\pi t) + \sin(50\pi t) + \sin(70\pi t)$$

The first two terms are audible, the last two, inaudible. This signal is prefiltered by an analog antialiasing prefilter $H(f)$ and then sampled at a rate of 40 kHz, as shown below. The resulting samples are immediately reconstructed by an *ideal reconstructor*.

The specifications of the prefilter are such that it has a flat passband (i.e. zero-dB attenuation) up to the Nyquist frequency of 20 kHz, and for $f \geq 20$ kHz, it attenuates at a rate of 60 dB per octave, as shown below.



(a) Determine the outputs of the prefilter and reconstructor, that is, the signals $y(t)$ and $y_a(t)$, as sums of sinusoids, and compare them with the audible part of $x(t)$. Justify all steps.

(b) Repeat part (a) if the prefilter is replaced by an ideal prefilter with cutoff at Nyquist. Justify all steps.

(c) Repeat part (a) if the prefilter is removed completely. Justify all steps.

2. Consider a 5-bit, two's complement, bipolar, successive approximation, A/D converter with *rounding*. The full-scale range is 8 volts.

(a) By stepping through the successive approximation algorithm, carry out the conversion of the voltage level $x = 3.2$ volts, and determine the nearest quantization level in volts, and its 5-bit pattern. Show all steps.

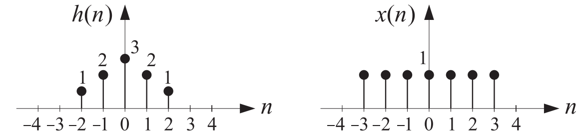
(b) Repeat for $x = -1.3$ volts.

3. A unit-step input $x(n) = u(n)$ is sent into a causal, linear, time-ivariant, system, and the following output signal is observed:

$$y(n) = 5u(n) - 4(0.8)^n u(n)$$

Working in the time domain and using linearity and time-invariance, determine the impulse response $h(n)$ of this system, for all n. Justify all steps. [*Hint:* $\delta(n) = u(n) - u(n-1)$.]

4. Consider a filter $h(n)$ and signal $x(n)$ defined in the following figure.



Using any method you want, but showing all the computational steps, calculate the convolution $y(n)$ of these two sequences and determine the *range* of the output index n. Make a sketch of the signal $y(n)$ versus n.

332:346 – Digital Signal Processing
Exam 2 – April 5, 2007

Please fully explain your reasoning and justify all steps.

1. [30 pts]. For each of the following signals, determine its z-transform (if it exists) and the corresponding ROC.

   (a) $x(n) = u(n)$

   (b) $x(n) = u(n) - u(n-1)$

   (c) $x(n) = u(n) + u(-n-1)$

   (d) $x(n) = (0.5)^n u(n) + 2^n u(-n-1)$

   (e) $x(n) = 2^n u(n) + (0.5)^n u(-n-1)$

   (f) $x(n) = (0.5)^n u(n) + 2^n u(-n)$

   You may leave the answers in their partial-fraction-expansion forms. If a z-transform does not exist, please explain why.

2. [30 pts]. A single-notch digital filter operating at a rate of 6 kHz was designed using the pole-zero placement method and its transfer function was found to be:

$$H(z) = G \cdot \frac{1 - z^{-1} + z^{-2}}{1 - 0.95z^{-1} + 0.9025z^{-2}}$$

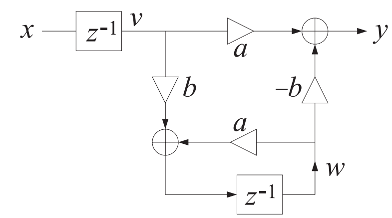
   (a) Determine the notch frequency f_0 in kHz. Explain.
   (b) Determine the 3-dB width of the notch in Hz. Show all work.
   (c) Determine the 60-dB time-constant of this filter in milliseconds.
   (d) Determine the value of the gain G in order for the filter to have unity gain at DC.

3. [25 pts]. Consider the filter: $H(z) = \dfrac{1 + z^{-2}}{1 - z^{-1} + 0.5z^{-3}}$.

   (a) Draw its canonical realization and write the corresponding sample processing algorithm using a circular-delay-line buffer.
   (b) For the input signal $\mathbf{x} = [8, 6, 4, 2, 1]$, iterate the sample processing algorithm, and compute the output signal $y(n)$ for $0 \le n \le 4$. In the process, fill in the table of values of the circular buffer $\mathbf{w}$ and the states s_0, s_1, s_2, s_3:

x	w_0	w_1	w_2	w_3	s_0	s_1	s_2	s_3	y
8	*	*	*	*	*	*	*	*	*
6	*	*	*	*	*	*	*	*	*
4	*	*	*	*	*	*	*	*	*
2	*	*	*	*	*	*	*	*	*
1	*	*	*	*	*	*	*	*	*

4. [15 pts]. Consider the filter defined by the following block diagram:



   (a) Using the internal-state variables v, w (that is, the contents of the two delays), write the sample processing algorithm for computing each output sample y from each input sample x. [No credit will be given if the computational steps are listed in a non-computable order.]
   (b) Assuming that a is real such that $|a| < 1$ and that $b = (1 - a^2)^{1/2}$, show that the transfer function $H(z)$ of this filter is:

$$H(z) = \frac{z^{-1}(a - z^{-1})}{1 - az^{-1}}$$

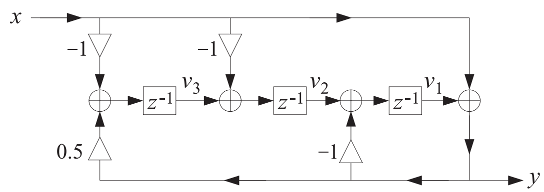
   (c) Show that the above transfer function represents an allpass filter, that is, show analytically that

$$|H(\omega)| = 1\,, \qquad \text{for all } \omega$$

332:346 – Digital Signal Processing
Final Exam – May 9, 2007

Please fully explain your reasoning and justify all steps.

1. [20 pts]. A digital filter is described by the following block diagram:



   (a) Let v_1, v_2, v_3 denote the contents of the three delays that appear in this realization. With the help of these variables, state the sample processing algorithm for computing each output sample y from each input sample x. Only the variables v_1, v_2, v_3, x, y must appear in the algorithm.

   (b) For the following input signal $\mathbf{x} = [8, 6, 4, 2, 1]$, iterate the above sample processing algorithm to compute the corresponding output samples, and in the process fill in the entries of the following table:

x	v_1	v_2	v_3	y
8	∗	∗	∗	∗
6	∗	∗	∗	∗
4	∗	∗	∗	∗
2	∗	∗	∗	∗
1	∗	∗	∗	−4
	∗	−6	∗	

   To help you check your answer, a couple of table entries are given.

2. [10 pts]. Determine the transfer function $H(z)$ of the filter of the previous problem. Show all work.

3. [20 pts]. Calculate the 8-point FFT of the 8-point signal:

$$\mathbf{x} = [1,\ 0,\ 1,\ \sqrt{2},\ -1,\ 0,\ -1,\ -\sqrt{2}]$$

4. [20 pts]. Without performing any DFT/FFT calculations determine the 8-point DFT of the 8-point signal:

$$x(n) = 2\sin\left(\frac{3\pi n}{4}\right) + 3\cos\left(\frac{3\pi n}{4}\right), \quad n = 0, 1, \dots, 7$$

   by casting $x(n)$ in the form of an inverse DFT.

5. [20 pts]. Consider the following analog signal consisting of three sinusoids:

$$x(t) = \cos(3\pi t) + 0.02\cos(5\pi t) + \cos(7\pi t)$$

   where t is in msec. The middle term represents a weak sinusoid whose presence we wish to detect by sampling $x(t)$ and computing its FFT. The signal is sampled at 10 kHz and 128 samples are collected.

   (a) The 128 samples are windowed by a Hamming window and the corresponding 128-point FFT is computed. Would the resulting FFT spectrum be able to detect the three sinusoidal components? Explain why or why not.

   (b) Could a rectangular window be used instead of a Hamming window for part (a)? Explain why or why not.

   (c) At what DFT integer indices in the range $0 \le k \le 127$ do you expect to see peaks in the 128-point FFT of this signal?

   (d) What would be the best estimates (in kHz) of the frequencies of the three sinusoids that one could guess on the basis of the 128-point FFT of the above signal?

6. [10 pts]. A real-valued 8-point signal $x(n)$ has the following 8-point DFT:

$$\mathbf{X} = [0,\ 12-16j,\ 8j,\ -4j,\ 0,\ *,\ *,\ *]$$

   (a) Determine the numerical values of the starred entries. Explain your reasoning.

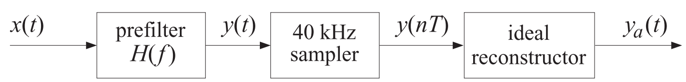
   (b) Express $x(n)$ as a linear combination of *real-valued* sinusoidal and/or cosinusoidal signals.

332:346 – Digital Signal Processing
Exam 1 – February 28, 2008

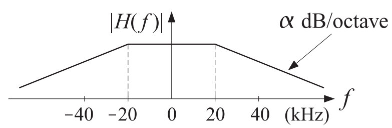
1. Consider the following sound signal, where t is in milliseconds:

$$x(t) = \cos(10\pi t) + \cos(90\pi t) + \cos(100\pi t)$$

This signal is prefiltered by an analog antialiasing prefilter $H(f)$ and then sampled at a rate of 40 kHz, as shown below. The resulting samples are immediately reconstructed by an *ideal reconstructor*.



The prefilter has a flat passband (that is, zero dB attenuation) up to the Nyquist frequency of 20 kHz, and for $f \geq 20$ kHz, it attenuates at a rate of α dB per octave, as shown below.



(a) It is required that all aliased components that are aliased into the Nyquist interval be suppressed by at least 40 dB. Determine the prefilter's attenuation rate α in dB/octave that will meet this requirement.

(b) What is the reconstructed output $y_a(t)$ in this case? (If you were unable to do part (a), then carry out this part with the assumed value of $\alpha = 42$ dB/octave.)

2. Consider a 5-bit, two's complement, bipolar, successive approximation, A/D converter with *rounding*. The full-scale range is 8 volts.

By stepping through the successive approximation algorithm, carry out the conversion of the two voltage levels $x = -1.6$ and $x = 1.3$ volts, and determine the nearest quantization levels in volts, and their 5-bit patterns. Show all steps.

3. Using the convolution table, calculate the convolution of the following filter and input signal:

$$\mathbf{h} = [1, 2, 2, 1] \quad \mathbf{x} = [1, 1, 2, 2, 1, 1]$$

4. A DSP chip with instruction time of T_{instr} nsec is used to process audio samples at a rate of $f_s = 1/T$ kHz.

Show that the maximum delay z^{-D} that can be implemented with this processor using a linear delay-line buffer is given in seconds by

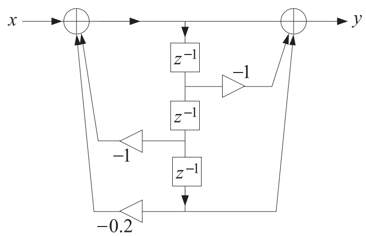
$$T_D = \frac{T^2}{2T_{\text{instr}}}$$

Explain your reasoning. What is the value of T_D in msec if f_s is 40 kHz and T_{instr} is 25 nsec? What is the value of D in this case?

(*Hint:* You may assume that each data shift requires two instructions, i.e., a read from memory and a write to memory.)

332:346 – Digital Signal Processing
Exam 2 – April 10, 2008

1. [40 pts]. Consider the filter shown in the following block diagram.



   (a) Determine the transfer function $H(z)$ of this filter.

   (b) Using a circular delay-line buffer implementation, state the sample processing algorithm for computing the output sample y from each input sample x.

   (c) For the input signal $\mathbf{x} = [1, 2, 3, 4, 5]$, iterate the sample processing algorithm, and compute the output signal $y(n)$ for $0 \le n \le 4$. In the process, fill in the table of values of the circular buffer $\mathbf{w}$ and the states s_0, s_1, s_2, s_3:

x	w_0	w_1	w_2	w_3	s_0	s_1	s_2	s_3	y
1	*	*	*	*	*	*	*	*	*
2	*	*	*	*	*	*	*	*	*
3	*	*	*	*	*	*	*	*	*
4	*	*	*	*	*	*	*	*	*
5	*	*	*	*	*	*	*	*	*

   (d) Draw the transposed realization of this filter.

   (e) Introducing appropriate internal states, state the sample processing algorithm for the transposed realization.

2. [40 pts]. A single-notch digital filter operating at a rate of 9 kHz was designed using the pole-zero placement method and its transfer function was found to be:

$$H(z) = G \cdot \frac{1 + z^{-1} + z^{-2}}{1 + 0.90z^{-1} + 0.81z^{-2}}$$

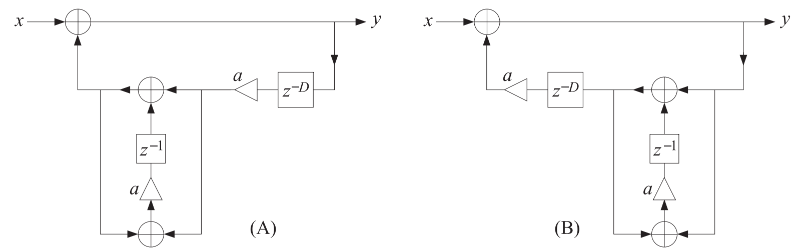
   (a) Determine the notch frequency in units of radians/sample and in kHz. Explain.

   (b) Determine the 3-dB width of the notch in units of radians/sample and in Hz. Show all work.

   (c) Determine the 60-dB time-constant of this filter in milliseconds.

   (d) Determine the value of the gain G in order for the filter to have unity gain at DC.

3. [20 pts]. A guitar-synthesis filter can be realized by either of the two filter structures (A) or (B) shown below.



   (a) Show that the two filter structures (A) and (B) have the same overall transfer function from x to y, and determine that transfer function.

   (b) For each of the two filter structures, introduce appropriate internal states, and using a circular buffer for the multiple delay z^{-D}, write the sample processing algorithm for computing the output sample y from the input sample x.

332:346 – Digital Signal Processing
Final Exam – May 14, 2008

1. Consider the following notch filter operating at a rate of 6 kHz:

$$H(z)=\frac{1+z^{-3}}{1+0.5\,z^{-3}}$$

(a) Determine the poles and zeros of this filter and place them on the complex z-plane. Then, determine the notch frequencies of the filter in kHz.

(b) The following signal is sent to the input of this filter:

$$x(n)=3+\sin\left(\frac{\pi n}{3}\right)+\cos(\pi n)+\sin\left(\frac{5\pi n}{3}\right),\quad n\geq 0$$

Determine the steady-state output $y_{\text{steady}}(n)$ for large n. [*Hint:* Do not use z-transforms.]

2. Draw the canonical and transposed realizations of the above filter.

(a) For the *transposed* realization, write the sample processing algorithm for computing each output sample y from each input sample x, using a circular buffer to implement the multiple delay z^{-3}.

(b) Iterate the sample processing algorithm on the following input signal $\mathbf{x}=[5,4,3,2,1]$ and compute the corresponding output samples y. In the process, fill in the entries of the following table:

x	w_0	w_1	w_2	w_3	s_0	s_3	y
5	*	*	*	*	*	*	*
4	*	*	*	*	*	*	*
3	*	*	*	*	*	*	*
2	*	*	*	*	*	2.5	*
1	*	−0.25	*	*	*	*	*

where $[w_0,w_1,w_2,w_3]$ are the circular-buffer entries, and s_0,s_3 are the input and output signals of the multiple delay z^{-3}, respectively. A couple of table entries have been given as checkpoints.

3. In pressing the key "7" on a touchtone phone keyboard, the following dual-tone signal is generated, where t is in milliseconds:

$$x(t)=\sin(1.704\,\pi t)+\sin(2.418\,\pi t)$$

To detect the depressed digit, the receiver samples the above signal at a rate of 8 kHz, collects N samples, windows them by a length-N Hamming window, and computes an N-point FFT.

(a) What is the minimum value of N that would enable the FFT to distinguish between the above two sinusoidal terms? Explain.

(b) The FFT can slightly miss the correct frequencies present in the above signal. Suppose, for example, that $N=64$. Determine the DFT indices k at which the FFT of the above signal will exhibit peaks, and then, determine the corresponding frequencies predicted by the FFT and compare them with the actual frequencies contained in the above signal. [*Hint:* Do not ignore the negative frequencies.]

4. Consider the following 8-point signal:

$$x(n)=5+3\cos\left(\frac{3\pi n}{4}\right)+5\sin\left(\frac{5\pi n}{4}\right)+\cos(\pi n),\quad n=0,1,\dots,7$$

(a) Without any DFT/FFT numerical calculations, determine the 8-point DFT of this signal.

(b) For a general N-point signal $x(n)$, show that if $x(n)$ is real-valued, then its N-point DFT must satisfy the Hermitian property:

$$X(k)^*=X(N-k),\quad k=0,1,\dots,N-1$$

5. Compute the 8-point FFT of the following 8-point signal:

$$\mathbf{x}=\begin{bmatrix}7\\4+\sqrt{2}\\7\\4+\sqrt{2}\\7\\4-\sqrt{2}\\3\\4-\sqrt{2}\end{bmatrix}$$

<u>General Hints</u>

$$x^3+1=(x+1)(x^2-x+1),\quad e^{j\pi/3}=\frac{1+j\sqrt{3}}{2}$$

$$x(n)=e^{j\omega n}u(n)\Rightarrow y(n)=H(\omega)e^{j\omega n},\quad \text{as } n\to\infty$$

$$x(n)=\frac{1}{N}\sum_{k=0}^{N}X(k)e^{j\omega_k n},\quad n=0,1,\dots,N-1$$

$$-\omega_k\equiv\omega_{N-k}\mod(2\pi)$$