

Fall 2009 – 332:347 – Linear Systems Lab – Lab 5

1. In this part, you will study the steady-state and transient response of a filter. Consider the following sinusoidal input signal and filter:

$$f(t) = \sin(\omega_0 t)u(t), \quad H(s) = \frac{s+3}{s^2+s+1.25}$$

where $\omega_0 = 4$ rad/sec and t is in seconds.

- (a) Determine the poles of the filter and calculate its 40-dB time constant in sec. Calculate the frequency response $H(\omega_0)$, as well as its magnitude $|H(\omega_0)|$, and phase $\phi_0 = \arg H(\omega_0)$.
- (b) Define a vector of 1001 equally-spaced time points t spanning the interval $0 \leq t \leq 10$, and evaluate the corresponding steady-state output signal:

$$y_{st}(t) = |H(\omega_0)| \sin(\omega_0 t + \phi_0) \quad (1)$$

- (c) The exact response to a complex sinusoidal input $e^{i\omega_0 t}u(t)$ was worked out in class for any filter:

$$y_c(t) = H(\omega_0) e^{i\omega_0 t} u(t) + \sum_{i=1}^M B_i e^{p_i t} u(t) \quad (2)$$

where $p_i, i = 1, 2, \dots, M$, are the filter poles, M is the order of the denominator polynomial of $H(s)$, assuming that the order of the numerator polynomial was at most M . The exact response due to a real-valued sinusoidal input $\sin(\omega_0 t)u(t)$ is obtained by calculating the imaginary part of Eq. (2).

Using the function `residue`, compute the quantities B_i, p_i that appear in Eq. (2), and then evaluate $y_c(t)$ at the same vector of t 's defined above, and extract its imaginary part $y(t) = \text{Im}[y_c(t)]$.

On the same graph, plot $y(t), y_{st}(t)$, and the input $f(t)$, and observe how $y(t)$ tends to $y_{st}(t)$ as t increases.

Calculate the phase delay $t_{ph} = -\phi_0/\omega_0$ and verify that it is consistent with the observed phase shift of the steady-state output.

On a separate graph, plot only the transient part $y_{tr}(t) = y(t) - y_{st}(t)$. Is its rate of decrease consistent with the 40-dB time constant that you calculated above?

- (d) Repeat parts [a-c], for $\omega_0 = 8$.

2. In this part, you will study the effect of a bandlimited channel on an input pulse and observe the distortions introduced when the channel's bandwidth is narrower than the pulse's bandwidth. The main tradeoff to observe is that the distortions diminish as the time-bandwidth product $T\omega_B$ increases. Consider first a rectangular pulse of duration T :

$$f(t) = \text{rect}_T(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases} \Rightarrow F(\omega) = \frac{\sin(\omega T/2)}{\omega/2}$$

The channel has bandwidth ω_B and is defined by the ideal response:

$$H(\omega) = \begin{cases} 1, & |\omega| \leq \omega_B \\ 0, & |\omega| > \omega_B \end{cases}$$

The corresponding output signal was worked out in class:

$$y(t) = \frac{1}{2\pi} \int_{-\omega_B}^{\omega_B} \frac{\sin(\omega T/2)}{\omega/2} \cos(\omega t) d\omega \quad (3)$$

$$= \frac{1}{\pi} \left[\text{Si}(\omega_B(t + T/2)) - \text{Si}(\omega_B(t - T/2)) \right]$$

where $\text{Si}(x)$ is the standard Si (sine-integral) function defined by

$$\text{Si}(x) = \int_0^x \frac{\sin u}{u} du$$

The MATLAB function `Si.m` is part of this lab.

- (a) To gain an understanding of the function `Si`, plot $\text{Si}(x)$ versus x over $-30 \leq x \leq 30$, and also plot $\text{Si}(5x)$, which is a compressed version of $\text{Si}(x)$. Determine the maximum ripple and the value of x at which it occurs. Notice that the compressed version, does not eliminate the ripples, but squeezes them more tightly towards $x = 0$. Note also, that $\text{Si}(5x)$ resembles the sign-function scaled by $\pi/2$, that is, $\text{Si}(5x) \approx \text{sign}(x) \pi/2$.

The shifted/compressed copies of `Si` in Eq. (3) will resemble the square pulse. In fact, the original square pulse can be represented in terms of similarly shifted copies of the sign-function, for any value of ω_B :

$$f(t) = \frac{1}{2} \left[\text{sign}(\omega_B(t + T/2)) - \text{sign}(\omega_B(t - T/2)) \right] \quad (4)$$

- (b) Generate a vector t of equally-spaced time samples spanning the interval $-4 \leq t \leq 4$. For the numerical values $T = 1$ and $\omega_B = 1$, compute the signals $y(t)$ and $f(t)$ from Eqs. (3) and (4) and plot them on the same graph. Repeat for the values $\omega_B = 5, 10, 25, 50$.

(c) Consider next the following Lorentzian/Cauchy pulse and its Fourier transform:

$$f(t) = \frac{T^2}{T^2 + t^2} \Leftrightarrow F(\omega) = \pi T e^{-T|\omega|}$$

The output of the ideal band-limited filter was obtained in Exam-2:

$$y(t) = \frac{1}{2\pi} \int_{-\omega_B}^{\omega_B} \pi T e^{-T|\omega|} \cos(\omega t) d\omega$$

$$= \left[\frac{T^2}{T^2 + t^2} \right] \cdot \left[1 + \frac{1}{T} e^{-T\omega_B} [t \sin(\omega_B t) - T \cos(\omega_B t)] \right] \quad (5)$$

For the values $T = 1$ and $\omega_B = 1$, plot $f(t)$ and $y(t)$ on the same graph. Repeat for $\omega_B = 2, 10$.

(d) The above examples were chosen because the inverse Fourier transform could be expressed in terms of known functions. In general, numerical integration is required. For example, the exact output:

$$y(t) = \frac{1}{2\pi} \int_{-\omega_B}^{\omega_B} F(\omega) e^{j\omega t} d\omega$$

may be approximated by the sum:

$$y_{\text{app}}(t) = \frac{1}{2\pi} \sum_{k=0}^{N-1} F(\omega_k) e^{j\omega_k t} \Delta\omega \quad (6)$$

where

$$\Delta\omega = \frac{2\omega_B}{N}, \quad \omega_k = -\omega_B + k \frac{2\omega_B}{N}$$

This is effectively the so-called inverse discrete Fourier transform (IDFT), and we will encounter it again in the DSP course. For example, applying Eq. (6) to the Lorentzian case, we have:

$$y_{\text{app}}(t) = \frac{1}{2\pi} \sum_{k=0}^{N-1} \pi T e^{-T|\omega_k|} e^{j\omega_k t} \Delta\omega \quad (7)$$

For $T = 1$, $\omega_B = 2$, and for the values $N = 5, 10, 50$, compute $y_{\text{app}}(t)$ and plot its real part on the same graph with $y(t)$ of Eq. (5). On separate graphs but using the same vertical scale as for the real part, plot the imaginary part of $y_{\text{app}}(t)$ and verify that it is essentially zero (it would be exactly zero if the sum over k was extended to be over the interval $0 \leq k \leq N$, instead of $0 \leq k \leq N - 1$.)

Typical Outputs



