Fall 2009 – 332:347 – Linear Systems Lab – Lab 2

1. This problem demonstrates the time-invariance property of LTI systems and also studies the the numerical approximation to convolution achieved by the built-in function `conv`. Consider a system described by the following differential equation and corresponding impulse response:

$$\dot{y}(t)+ay(t)=af(t) \quad \Rightarrow \quad h(t)=ae^{-at}u(t)$$

Let the input signal $f(t)$ be a square pulse of duration of t_p seconds, starting at $t=0$, that is,

$$f(t)=\text{upulse}(t,t_p)=\begin{cases}1, & \text{for } 0\le t<t_p\\ 0, & \text{for other } t\end{cases}$$

The corresponding exact output, obtained by convolving $h(t)$ and $f(t)$, was found in class:

$$y_{\text{exact}}(t)=\int h(t-\tau)f(\tau)\,d\tau = e^{-at}\big[e^{a\min(t,t_p)}-1\big]u(t) \qquad (1)$$

A numerical approximation to the convolution integral is obtained by considering the discrete time instants $t_n=nT$, where T is a small sampling interval, and approximating:

$$y(t_n)=\int h(t_n-\tau)f(\tau)\,d\tau \approx T\sum_m h(t_n-t_m)f(t_m) \qquad (2)$$

It can be implemented by the MATLAB code:

```
y = T * conv(h,f);
```

(a) Define the signals $h(t), f(t)$ for the following choice of parameters over a maximum time interval of $T_{\max}$:

```
Tmax = 25; a = 0.5; tp = 10; T = Tmax/100;
t = 0:T:Tmax;
h = a*exp(-a*t);
f = ustep(t,tp);
```

Calculate the exact and approximate convolution outputs of Eqs. (1) and (2) and plot all three signals $f(t), y_{\text{exact}}, y(t)$ versus t on the same graph. Repeat when $T = T_{\max}/1000$ and discuss the improvement in the approximation.

For plotting purposes you may wish to keep only first $N = \text{length}(t)$ convolutional outputs. This can be accomplished by redefining the computed output vector by:

```
y = y(1:length(t));
```

(b) Repeat the previous question when the input is the delayed unit pulse:

$$f(t)=\text{upulse}(t-t_d,t_p)$$

with the choice of the delay $t_d=5$. Observe the corresponding delay in the computed output.

2. This problem illustrates transient and steady-state sinusoidal responses. Consider a signal consisting of three sinusoidal bursts (shown at end):

$$f(t)=\begin{cases}\sin(3t), & 0\le t<30\\ \sin(2t), & 30\le t<70\\ \sin(3t), & 70\le t<100\end{cases}$$

It can be generated over a period $0\le t\le 100$ by the code:

```
Tmax = 100; T = Tmax/1000; t = 0:T:Tmax;
f = sin(3*t) .* upulse(t,30) + ...
    sin(2*t) .* upulse(t-30,40) + ...
    sin(3*t) .* upulse(t-70,30);
```

It is desired to eliminate the middle burst by means of a notch filter:

$$H(s)=\frac{s^2+\omega_0^2}{s^2+\alpha s+\omega_0^2}$$

where $\omega_0=2$ is the notch frequency coinciding with the frequency of the middle burst, and $\alpha=0.3$ is a parameter that represents the 3-dB width of the notch (see graph on last page). As discussed in class, the impulse response of this filter is:

$$h(t)=\delta(t)-g(t)\,, \quad g(t)=\alpha e^{-\alpha t/2}\Big[\cos(\omega_r t)-\frac{\alpha}{2\omega_r}\sin(\omega_r t)\Big]u(t)$$

where $\omega_r=\sqrt{\omega_0^2-\alpha^2/4}$. It follows that the output signal will be:

$$y(t)=\int h(t-\tau)f(\tau)\,d\tau = f(t)-\int g(t-\tau)f(\tau)\,d\tau$$

which can be implemented by the MATLAB code:

```
y = f - T * conv(g,f);
```

(a) Compute the above output signal $y(t)$ and plot it versus t. On a separate graph, but using the same vertical and horizontal scales, plot the input signal $f(t)$. Note the removal of the middle burst after the transients have decayed. Explain quantitatively the slight attenuation of the first and third bursts.

Repeat with $\alpha=0.1$, which corresponds to a narrower notch, but with a longer time constant.

(b) For the case $\alpha=0.3$, plot $h(t)$ versus $0<t<T_{\max}$. The time constant of the filter is the effective duration of $h(t)$. On another graph, plot the magnitude response $|H(\omega)|^2$ over $0\le\omega\le 5$.

Typical Outputs