332:345 – Linear Systems & Signals – Fall 2009
Sample Exam-1 Questions

1. The impulse response $h(t)$ of an LTI system and an input signal $f(t)$ are nonzero over the following time ranges:

$$h(t), \quad a \le t \le b$$
$$f(t), \quad c \le t \le d$$

The corresponding output is given by the convolutional equation:

$$y(t) = \int h(t-\tau) f(\tau)\, d\tau$$

Determine the time range for $y(t)$, and the precise limits of the above integral.

2. Sketch the two signals $h(t) = e^{-t}u(t)$ and $f(t) = u(t) - u(t-5)$. Then, determine their convolution $y(t) = \int h(t-\tau) f(\tau)\, d\tau$ using the following two methods:

   (a) By performing the indicated time integration.

   (b) By working with Laplace transforms.

3. Using Laplace transforms, solve the following differential equation,

$$\ddot{y}(t) + 5\dot{y}(t) + 4y(t) = 3f(t)$$

where $f(t) = e^{-3t}u(t)$ with arbitrary initial conditions: $y(0-)$ and $\dot{y}(0-)$.

Identify the parts of the solution that correspond to the decomposition of $y(t)$ into a "homogeneous solution" and a "particular solution". Similarly, identify the parts that correspond to the "zero-input solution" and the "zero-state solution".

4. Determine the impulse response $h(t)$ of the system of the previous problem using Laplace transform techniques.

5. The input signal $f(t) = u(t)$ to an unknown LTI system causes the following output signal: $y(t) = 2u(t) - e^{-2t}u(t)$.

   (a) What input signal $f(t)$ would cause the output $y(t) = e^{-2t}u(t)$?

   (b) What is the impulse response $h(t)$ of this system? What is its transfer function $H(s)$?

6. Using long division and partial-fraction expansions, determine the impulse response $h(t)$ of the system with the following transfer function:

$$H(s) = \frac{s^3 + 4s^2 + 2s + 2}{s^2 + 5s + 4}$$

7. Consider the signals:

$$f(t) = (2-t)\left[u(t) - u(t-2)\right], \quad h(t) = u(t) - u(t-1)$$

Sketch the two signals and then determine their convolution in two different ways:

   (a) Using the time-domain convolution formula $y(t) = \int h(t-\tau) f(\tau)\, d\tau$.

   (b) Using Laplace transform techniques, i.e., by inverting the Laplace transform of the output signal, $Y(s) = H(s)F(s)$.

Demonstrate that the answers in (a) and (b) are equivalent.

8. Consider the Laplace transform pair:

$$f(t) = t^2 u(t) \quad \Leftrightarrow \quad F(s) = \frac{2}{s^3}, \quad \mathrm{Re}(s) > 0$$

Demonstrate its correctness by the following three methods:

   (a) Using the definition of Laplace transforms.

   (b) Using the time-multiplication property and the fact that $u(t)$ has Laplace transform $1/s$.

   (c) Working with the doubly differentiated signal $\ddot{f}(t)$ and applying the differentiation property.

9. The general solution of the differential equation,

$$\dot{y}(t) + 2y(t) = \dot{f}(t) + 3f(t) \qquad \text{(i.e., from Lab-1)}$$

for arbitrary input $f(t)$ and with zero initial conditions is given by,

$$y(t) = f(t) + \int_0^t e^{-2(t-\tau)} f(\tau)\, d\tau, \quad t \ge 0$$

   (a) Verify that it is a solution.

   (b) Derive it by first finding the impulse response $h(t)$ of the above system and then using the convolutional formula.

   (c) Explain how to turn this solution into a difference equation using the zero-order hold technique discussed in class.

   (d) Then, write a MATLAB function that solves this difference equation. The function should have as inputs the sampling time T and a vector of input samples $f(t_n)$ corresponding to the sampling times $t_n = nT$. The function output should be the vector of corresponding output samples $y(t_n)$.

10. What Laplace transform properties would you use to prove the following Laplace transform pair?:

$$f(t) = \cos(\omega_0 t)\, u(t) \quad \Leftrightarrow \quad F(s) = \frac{s}{s^2 + \omega_0^2}$$