7. Consider the signals:
\[ f(t) = (2 - t) \left[ u(t) - u(t - 2) \right], \quad h(t) = u(t) - u(t - 1) \]
Sketch the two signals and then determine their convolution in two different ways:
(a) Using the time-domain convolution formula
\[ y(t) = \int h(t - \tau) f(\tau) d\tau. \]
(b) Using Laplace transform techniques, i.e., by inverting the Laplace transform of the output signal, \( Y(s) = H(s)F(s) \).
Demonstrate that the answers in (a) and (b) are equivalent.

8. Consider the Laplace transform pair:
\[ f(t) = t^2u(t) \quad \Leftrightarrow \quad F(s) = \frac{2}{s^3}, \quad \text{Re}(s) > 0 \]
Demonstrate its correctness by the following three methods:
(a) Using the definition of Laplace transforms.
(b) Using the time-multiplication property and the fact that \( u(t) \) has Laplace transform \( 1/s \).
(c) Working with the doubly differentiated signal \( f''(t) \) and applying the differentiation property.

9. The general solution of the differential equation,
\[ y''(t) + 5y'(t) + 4y(t) = 3f(t) \quad (\text{i.e., from Lab-1}) \]
for arbitrary input \( f(t) \) and with zero initial conditions is given by,
\[ y(t) = f(t) + \int_{0}^{t} e^{-2(t-\tau)} f(\tau) d\tau, \quad t \geq 0 \]
(a) Verify that it is a solution.
(b) Derive it by first finding the impulse response \( h(t) \) of the above system and then using the convolutional formula.
(c) Explain how to turn this solution into a difference equation using the zero-order hold technique discussed in class.
(d) Then, write a MATLAB function that solves this difference equation. The function should have as inputs the sampling time \( T \) and a vector of input samples \( f(t_n) \) corresponding to the sampling times \( t_n = nT \). The function output should be the vector of corresponding output samples \( y(t_n) \).

10. What Laplace transform properties would you use to prove the following Laplace transform pair?:
\[ f(t) = \cos(\omega_0 t)u(t) \quad \Leftrightarrow \quad F(s) = \frac{s}{s^2 + \omega_0^2} \]