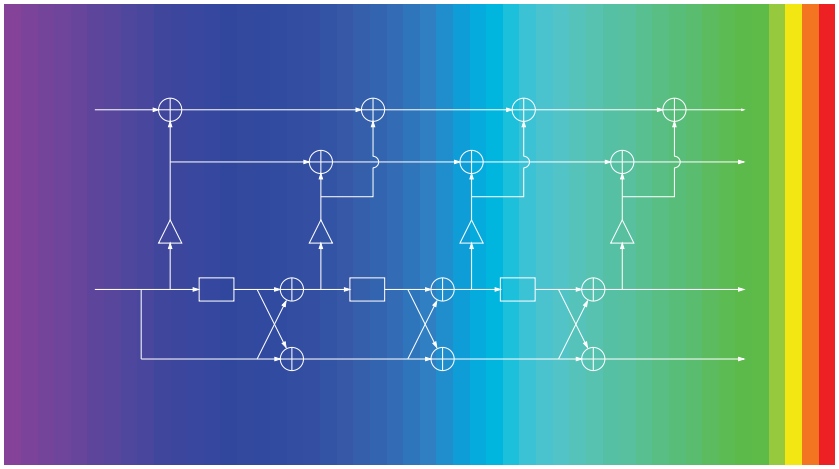

Applied Optimum Signal Processing

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http://www.ece.rutgers.edu/~orfanidi/aosp

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A MATLAB-based Introduction

Sophocles J. Orfanidis

Rutgers University

2018

`http://www.ece.rutgers.edu/~orfanidi/aosp`

To my parents
John and Clio Orfanidis

And To
Monica, John, Anna, and Owen

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CVX software by Michael Grant and Stephen Boyd, *CVX: Matlab software for disciplined convex programming*, version 2.0 beta, September 2013, `http://cvxr.com/cvx`.

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