# 14:332:231 <br> DIGITAL LOGIC DESIGN 

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## Gate Delays

- When the input to a logic gate is changed, the output will not change immediately.
- The switching elements within a gate take a finite time to react to a change (transition) in input.
- As a result the change in the gate output is delayed w.r.t. to the input change.
- Such delay is called the propagation delay of the logic gate ( $t_{p}$ )
- The propagation delay for a 0-to-1 output change $\left(t_{p L H}\right)$ may be different than the delay for a 1-to-0 change ( $\mathrm{t}_{\mathrm{pHL}}$ ).


## [Recall from Lectre \#t] Transition Delays

(a) Ideal case of zero-time switching:

(b) A more realistic approximation:

(c) Actual timing for rise ( $\mathrm{t}_{\mathrm{r}}$, low-to-high $)$ and fall ( $\mathrm{t}_{\mathrm{f}}$, high-to-low) times: High
undefined Low
 $\mathrm{V}_{\text {IHmin }}$
$\mathrm{V}_{\text {ILmax }}$
$\quad 3$ of 2 L

## Propagation Delays for a CMOS Inverter

(a) Ignoring rise and fall times:

(b) Measured at midpoints of transitions:


## Effect of Gate Delays

- The analysis of a combinational circuit ignoring delays can predict only its steady-state behavior
- Predicts a circuit's output as a function of its inputs assuming that the inputs have been stable for a long time, relative to the delays in the circuit's electronics.
- Because of circuit delays, the transient behavior of a combinational logic circuit may differ from what is predicted by steady-state analysis.
- Timing hazard: a circuit's output may produce a short pulse ("glitch") at a time when steady state analysis predicts that the output should not change.


## Timing Hazard

- A gate has measurable response time $\mathrm{t}_{\mathrm{pLH}}$ and $\mathrm{t}_{\mathrm{pHL}}$. Around 10ns per gate.
- Delays through transmission gates can add up and introduce timing hazards.
- $\mathrm{t}_{\mathrm{pLH}}=$ low-to-high, $\mathrm{t}_{\mathrm{pHL}}=$ high-to-low propagation times
- static-1 hazard is a short "0" glitch when for a changed input, we expect (by logic theorems) the output to remain constant " 1 ".
- static-0 hazard is a short " 1 " glitch when we expect the output to remain constant " 0 ".



## Circuit with a Static-1 Hazard



- Assume: $X, Y, Z=111$
- Consider a transition to $X, Y, Z=110$
- What we logically expect:

$$
F=X \cdot Z^{\prime}+Y \cdot Z
$$

- Before: $\mathrm{F}=1 \cdot(1)^{\prime}+1 \cdot 1=1 \cdot 0+1=1$
- After: $F=1 \cdot(0)^{\prime}+1 \cdot 0=1 \cdot 1+0=1$
$\rightarrow$ No change in the output !!



## Real World Gates Introduce Delays



- Input signal of each gate is shifted in the output by a constant delay


## Different Paths Introduce Different Delays



Change occurs at input $Z$ and propagates to output $F$ along two paths with different delays

## Circuit with a Static-1 Hazard



## Timing Hazards and Karnaugh Maps

- Recall that in Sum-of-Products (AND-OR) circuits, AND gates correspond to prime implicants of the Karnaugh map
- A potential hazard exists wherever two adjacent 1-cells in a Karnaugh map are not covered by a single product term (prime implicant)
- A hazard occurs when there is a transition between adjacent prime implicants


## Eliminating the Timing Hazard

- To eliminate hazards, find a cover in which all adjacent 1-cells are covered by a prime implicant
- Define "consensus" prime implicant, as a product term not covered in $F$
- Therefore, include an extra product term to cover the hazardous input combination


## Eliminating the Timing Hazard



Minimal cost $F=X \cdot Z^{\prime}+Y \cdot Z$
(these adjacent 1-cells are NOT covered $\boldsymbol{\rightarrow}$ 1-hazard)

$F=X \cdot Z^{\prime}+Y \cdot Z+\underbrace{X \cdot Y}_{\text {(consensus term) }}$


A "redundant" term was introduced.


## Why This Works

Because the consensus term X•Y fills in ("covers") the temporary gaps during switching from one active AND gate (e.g., $X \cdot Z^{\prime}$ ) to another (e.g.,Y•Z)


## Another Example (four variables)

W' Z

$F=X \cdot Y^{\prime} \cdot Z^{\prime}+W^{\prime} \cdot Z+W \cdot X$

$F=X \cdot Y^{\prime} \cdot Z^{\prime}+W^{\prime} \cdot Z+W \cdot X$ $+W^{\prime} \cdot X \cdot Y^{\prime}+Y \cdot Z+W \cdot X \cdot Z^{\prime}$

## Circuit with a Static-0 Hazard



Product of sums: static-0 hazard

## Circuit with a Static-0 Hazard



## Dynamic Hazards

- Dynamic hazard: Output signal is supposed to change $1 \rightarrow 0$ or $0 \rightarrow 1$ but a short oscillation occurs before the output settles to its new logic value.
- Occurs if there are multiple paths with different delays from the changing input to the changing output.
- In practice many input variables, multiple outputs;
solved by computer programs


## Example of Dynamic Hazard



Change occurs at input $X$ and propagates to output $F$ along three paths with different delays

## Example of Dynamic Hazard



Change occurs at input $X$ and propagates to output $F$ along three paths with different delays

