# 14:332:231
# DIGITAL LOGIC DESIGN

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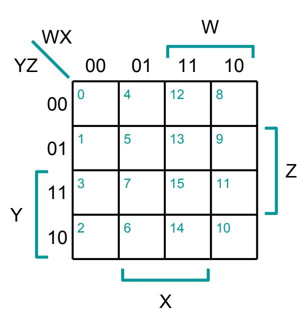
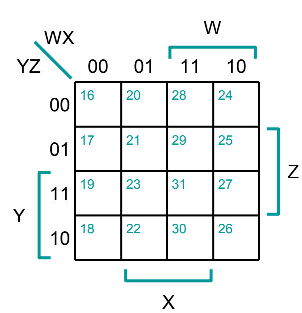
Lecture #7: Combinational Circuit Synthesis **II**

## What if we have 5 input variables?



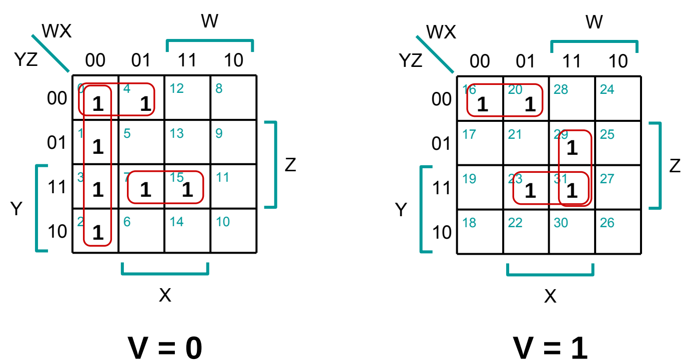

**V = 0**




**V = 1**

# Example with 5 variables

$F = \sum_{V,W,X,Y,Z}(0,1,2,3,4,7,15,16,20,23,29,31)$




$F = V'\cdot W'\cdot X' + W'\cdot Y'\cdot Z' + X\cdot Y\cdot Z + V\cdot W\cdot X\cdot Z$

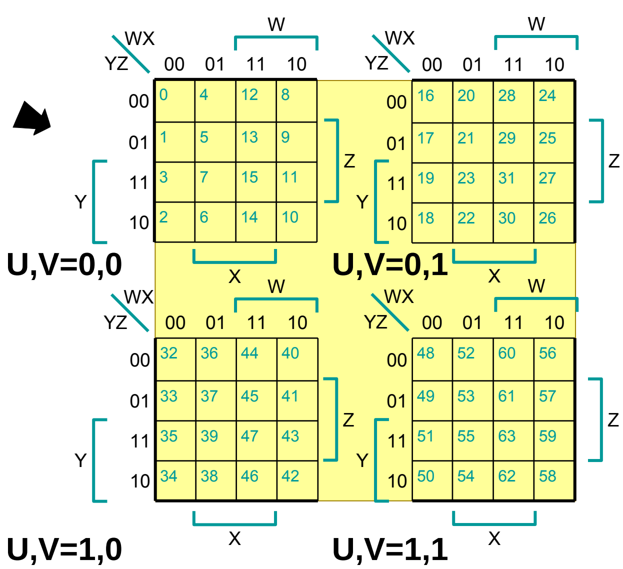
$W'\cdot Y'\cdot Z'$: $V'\cdot W'\cdot Y'\cdot Z' + V\cdot W'\cdot Y'\cdot Z'$ (same minterm in left & right tables)

$X\cdot Y\cdot Z$: $V'\cdot X\cdot Y\cdot Z + V\cdot X\cdot Y\cdot Z$ (same minterm in left & right tables)



# What if we have 6 input variables?

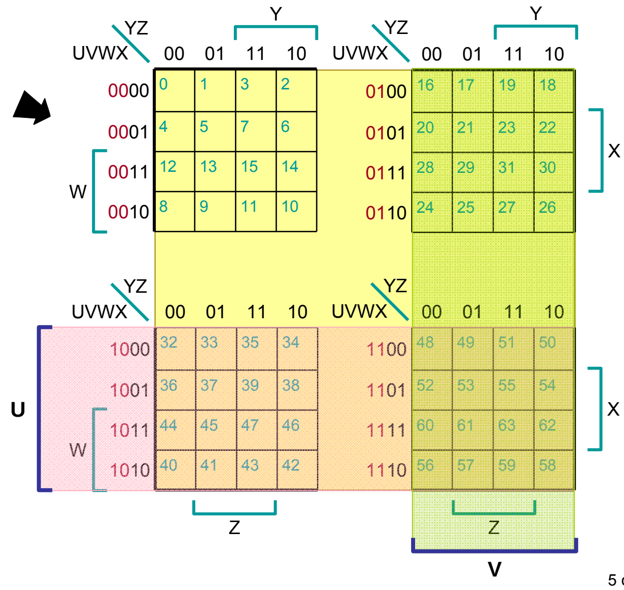
| Row | U | V | W | X | Y | Z | F |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 | |
| 6 | 0 | 0 | 0 | 1 | 1 | 0 | |
| 7 | 0 | 0 | 0 | 1 | 1 | 1 | |
| ... | | | | | | | ... |




Two most-significant bits (UV) are used to select which of the 4-variable maps is being used and the WXYZ bits are used to select the entry in the 4-variable Karnaugh map

# Six input variables — another view

| Row | U | V | W | X | Y | Z | F |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 | |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 | |
| 6 | 0 | 0 | 0 | 1 | 1 | 0 | |
| 7 | 0 | 0 | 0 | 1 | 1 | 1 | |
| … | | | | | | | … |



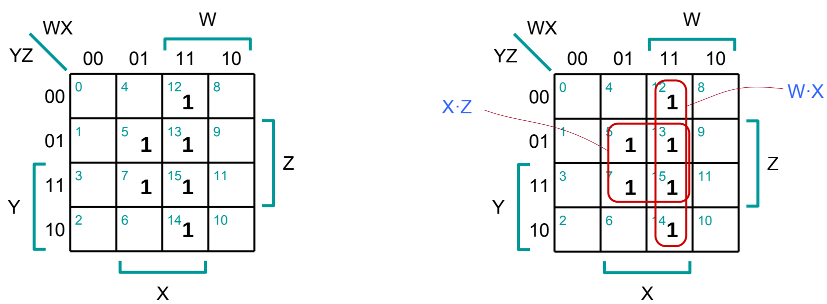

Two most-significant bits (UV) are used to select which of the 4-variable maps is being used and the WXYZ bits are used to select the entry in the 4-variable Karnaugh map



# Definitions

- **Minimal sum** of *F* — a sum-of-products such that
  - No sum-of-products for *F* has fewer product terms
  - Any sum-of-products with the same # of product terms has ≥literals
- **Prime Implicant** of *F* — a normal product term *P* that implies *F*, s.t. if any variable removed from *P* then *P** doesn't imply *F*
- **Complete sum** — the sum of all prime implicants of *F*
- **Distinguished 1-cell** — an input combination covered by only one prime implicant
- **Essential prime implicant** — a prime implicant that covers ≥1 distinguished 1-cells → it must be included in the minimal sum!

# Example of Prime Implicants

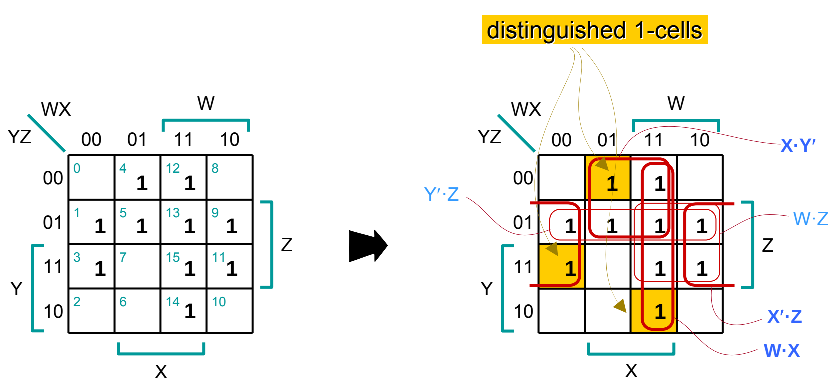



F = $\Sigma_{W,X,Y,Z}(5,7,12,13,14,15)$        F = X·Z + W·X

As seen, both prime implicants must be included in the minimal sum to cover all of the 1-cells



# Another example




F = $\Sigma_{W,X,Y,Z}(1,3,4,5,9,11,12,13,14,15)$

F = X·Y′ + X′·Z + W·X ~~+ Y′·Z + W·Z~~

*Essential* prime implicants
(circled with thicker lines)

Minimal sum

## Algorithm for minimum *sum-of-product* from K-maps
SoP

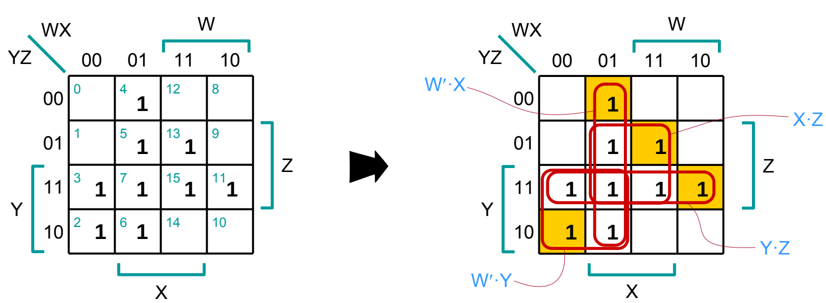
1. Circle all prime implicants.
2. Identify and select the essential prime implicants for cover.
3. Cover the 1-cells not covered by essential prime implicants.

## Minimum *product-of-sum*
PoS

1. Represent the ones of F′ in the K-map.
2. Find the minimum SoP of F′.
3. Complement the obtained expression by applying DeMorgan theorem.



## Example: All Prime Implicants Essential




F = $\Sigma_{W,X,Y,Z}(2,3,4,5,6,7,11,13,15)$

F = W′·X + W′·Y + X·Z + Y·Z

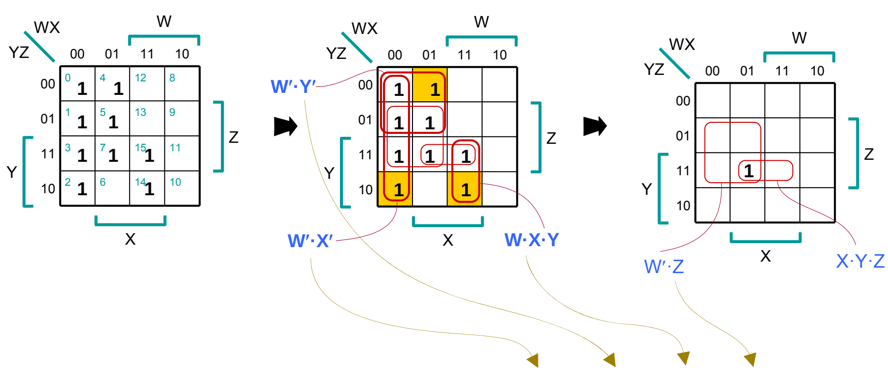
➔ all prime implicants are included in the minimal sum

## Few or None Essential Prime Implicants

- IF there are no essential prime implicants, or essential prime implicants do not cover all 1-cells,
- THEN select nonessential prime implicants to form a complete minimum-cost cover
- Selection Heuristics explained next …



## Too Few Essential Prime Implicants (1)




F = $\Sigma_{W,X,Y,Z}$ (0,1,2,3,4,5, 7,14,15)

F = W′·X′ + W′·Y′ + W·X·Y + W′·Z

Select nonessential prime implicant W′·Z over X·Y·Z because it has fewer inputs → lower cost

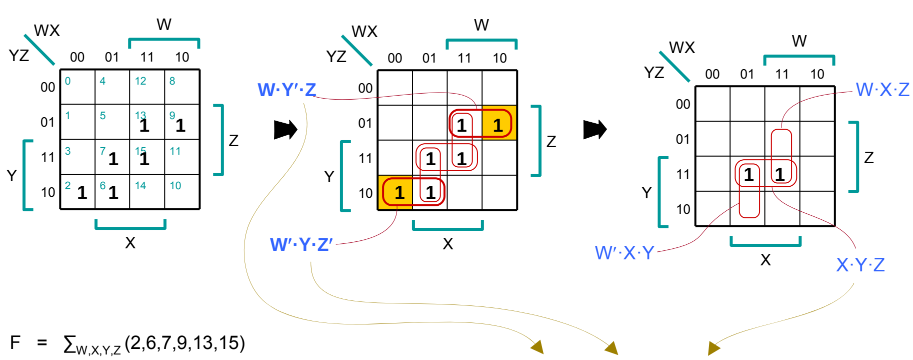
# More Definitions

(for more complex cases of too few essential prime implicants)

- **Eclipse**: Given two prime implicants P,Q
  P *eclipses* Q if P covers ≥ 1-cells covered by Q
- **Secondary essential prime implicant**: eclipses other prime implicants



# Too Few Essential Prime Implicants (2)

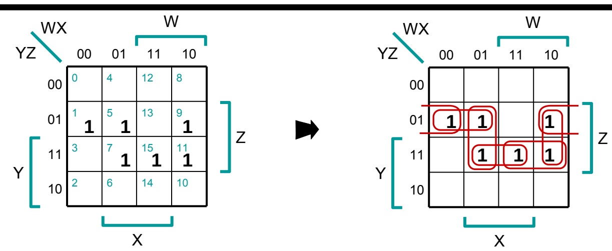



F = $\Sigma_{W,X,Y,Z}$(2,6,7,9,13,15)

F = W·Y′·Z + W′·Y·Z′ + X·Y·Z

X·Y·Z *eclipses* the other two prime implicants,
therefore it is a *secondary essential prime implicant*
→ must be included in the minimal sum

# No Essential Prime Implicants



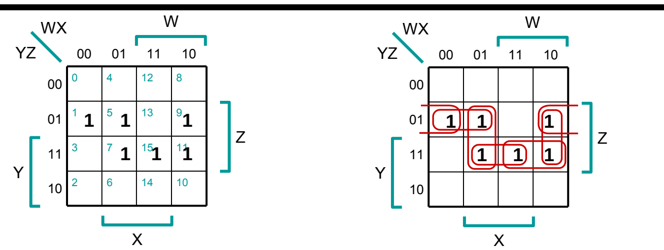

No distinguished 1-cells ➔ No essential prime implicants!



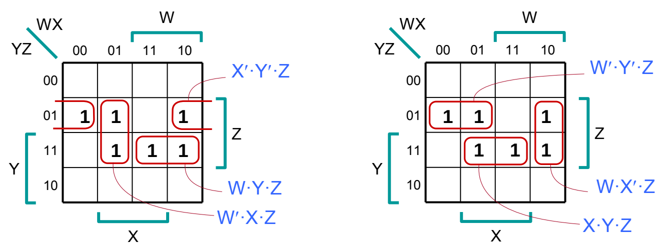
# Branching Heuristic

1. Starting w/ any 1-cell, arbitrarily select one prime implicant covering it
2. Include this p.i. as if it were *essential* and find a *tentative minimal sum-of-products*
3. Repeat the process, for all other prime implicants covering the starting 1-cell
   - Generates a different tentative minimal sum for each starting point
4. Finally, compare all tentative minimal sums and select the truly minimal

# Branching Heuristic Example




Two minimal sums:




F = W′·X·Z + W·Y·Z + X′·Y′·Z F = X·Y·Z + W·X′·Z + W′·Y′·Z

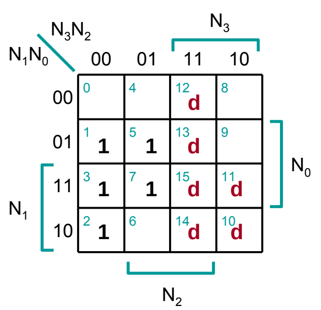


# Incompletely Specified Functions

## ("Don't-Care" Input Combinations)

*Don't-cares*: output doesn't matter for such input combinations (never occur in normal operation).

Example: Detect the prime numbers to ten, input is always a BCD digit.




F = $\Sigma_{N_3,N_2,N_1,N_0}$(1,2,3,5,7) + d(10,11,12,13,14,15)

(don't cares, a.k.a. *d-set*)

http://www.ddpp.com/DDPP4student/Supplementary_sections/Min.pdf

# Modified procedure for circling sets of 1's
(prime implicants)

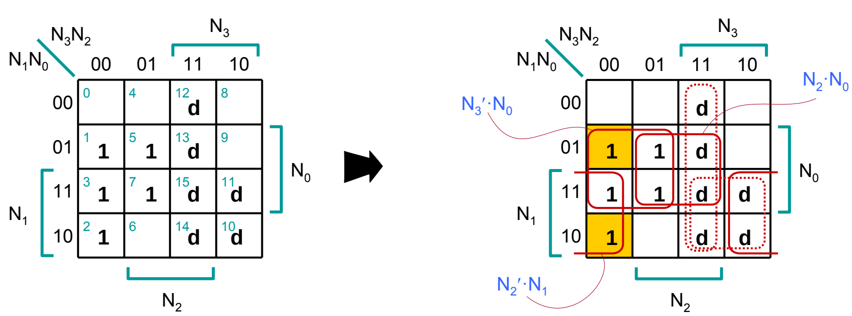
- Allow **d**'s to be included when circling sets of 1's, to make the sets as large as possible.
  - This reduces the number of variables in the corresponding prime implicants.
  - Two such prime implicants ($N_2 \cdot N_0$ and $N_2' \cdot N_1$) appear in the example.
- Do not circle any sets that contain only **d**'s.
  - Including the corresponding product term in the function would unnecessarily increase its cost.
  - Two such product terms ($N_3 \cdot N_2$ and $N_3 \cdot N_1$) are circled in the example.
- Reminder: As usual, do not circle any 0's



# Incompletely Specified Functions
("Don't-Care" Input Combinations)

*Don't-cares*: output doesn't matter for such input combinations (never occur in normal operation).

Example: Detect the prime numbers to ten, input is always a BCD digit.

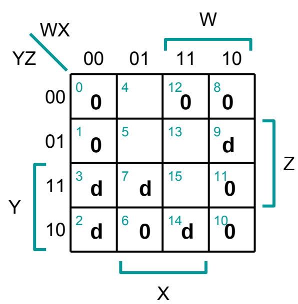


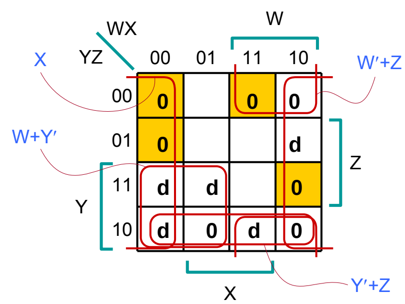

$F = \sum_{N_3,N_2,N_1,N_0}(1,2,3,5,7) + d(10,11,12,13,14,15)$
(don't cares, a.k.a. *d-set*)

$F = N_3' \cdot N_0 + N_2' \cdot N_1$

# Don’t Cares and Product-of-Sums Minimization

For Product-of-Sums (PoS) minimization,
apply the same technique as for Sum-of-Products (SoP) and the principle of duality




F = $\sum_{W,X,Y,Z}(4,5,13,15) + d(2,3,7,9,14)$

F(W,X,Y,Z) = X · (W′+Z) · (Y′+Z)

or X · (W′+Z) · (W+Y′)

RECALL:

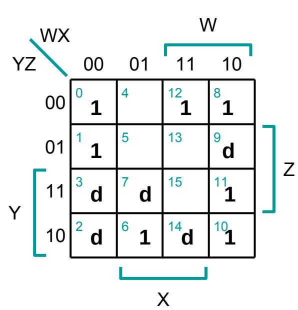
Minimum *product-of-sum* (PoS)

1. Represent the ones of F′ in the K-map.
2. Find the minimum SoP of F′.
3. Complement the obtained expression by applying DeMorgan theorem.



# SoP Minimization and Inverting the Result

F′ = $\sum_{W,X,Y,Z}(0,1,6,8,10,11,12) + d(2,3,7,9,14)$




F′ = X′ + W·Z′ + Y·Z′

F = (X′)′ · (W·Z′)′ · (Y·Z′)′ = X · (W′+Z) · (W+Y′)

(using DeMorgan’s theorem)

## Lots of Possibilities

- Can follow a “dual” procedure to find minimal products-of-sums (OR-AND realization)
- Can modify procedure to handle don’t-care input combinations.
- Can draw Karnaugh maps with up to six variables.



## Real-World Logic Design

- Lots more than 6 inputs --can’t use Karnaugh maps
- Design correctness more important than gate minimization
  - Use “higher-level language” to specify logic operations
- Use programs to manipulate logic expressions and minimize logic
- ABEL — developed for PLDs
- VHDL, Verilog — developed for ASICs