Location Management Handoff Overhead in Hierarchically Organized Mobile Ad hoc Networks

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Abstract

Control overhead in a mobile ad hoc network may be reduced through hierarchical routing. However, to facilitate packet forwarding in a hierarchically organized network, each datagram must specify the hierarchical address of the destination. Maintaining and acquiring hierarchical addresses represents a location management (LM) problem and incurs control overhead in addition to that of a routing protocol. This paper considers the LM overhead due to handoff. That is, the transfer of LM data due to node mobility and volatility of the clustered hierarchy. It is shown that handoff overhead is only polylogarithmic in the node count.

1. Introduction

Hierarchical clustering represents a means to support scalable routing in mobile ad hoc networks (MANETs). However, hierarchical routing requires address management, or equivalently *location management* (LM).

This paper assesses control overhead that is due to *handoff* of LM data. The cost of handoff in a clustered hierarchical MANET is difficult to analyze because handoff may not only be triggered by node migration, but also by cluster reorganization. Cluster reorganization in a hierarchical network may result from the following:

- Clusterhead birth/death due to the actual birth/death of a node
- Clusterhead status change due to the clustering algorithm's reaction to a link state change

The occurrence of node birth/death is assumed here to be extremely rare and, therefore, its effect is not evaluated. For the purposes here, cluster reorganization is assumed to be solely due to the reaction of the clustering algorithm to link states.

Handoff occurs when a node migrates from one level-k cluster to another. This is referred to here as handoff due to node migration. In this case, the node in question must transfer $\Theta(\log|V|)$ LM entries to the appropriate members of its previous level-k cluster and acquire $\Theta(\log|V|)$ entries from its new cluster. The level-k topology remains intact, but handoff is required due to the distributed nature of the LM database considered here.

Handoff also occurs when a level-k cluster link state change impacts level-(k+1) cluster membership. This is referred to here as handoff due to clustered hierarchy reorganization. In this case, all nodes of the affected level-(k+1) clusters undergo a handoff process with the level-k cluster whose level-k cluster link state has changed.

This paper is organized as follows. The remainder of this section describes notation and network assumptions. Section 2 discusses hierarchical clustering principles. Section 3 provides an overview of hierarchical location management. Sections 4 and 5 assess LM overhead due to handoff and show that overhead is only $\Theta\left(\log^2|V|\right)$ packet transmissions per node. Lastly, conclusions are provided in Section 6.

1.1. Notation

$G = (V,E)$ represents the underlying network graph (G). V is the set of vertices (i.e., nodes) and E is the set of undirected edges (i.e., bi-directional links). The following definitions are useful:

– $V_k \equiv$ Set of level-k nodes at level-k of the clustered hierarchy ($V_0 = V$).
– $E_k \equiv$ Set of level-k links at level-k of the clustered hierarchy ($E_0 = E$).
– $c_k \equiv |V|/|V_k|$, $k \in \{1,2,\ldots,L\}$.
– $A_k \equiv$ Average area covered by a level-k cluster.
– $f_k \equiv$ Frequency of level-k node migration events (per node).
– $g_k \equiv$ Frequency of level-k link state change events (per node).
– $\alpha_k \equiv |V_{k-1}|/|V_k|$, $k \in \{1,2,\ldots,L\}$.
– $d_k \equiv$ Average degree of a level-k node.
– $h_k \equiv$ Average hop count, in terms of level-0 nodes, across a level-k cluster.

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- $\phi_k \equiv$ Average number of handoff packet transmissions per node due to node mobility to/from level-k clusters.
- $\gamma_k \equiv$ Average number of handoff packet transmissions per node due to level-k cluster link state change events.
- $N_k(v_k) \equiv$ Set of level-k neighbors of a level-k node.
- $\mu \equiv$ Node speed.

Some elementary consequences of this notation are as follows:

$$|E| = \frac{d \cdot |V|}{2} \Rightarrow |E_k| = \frac{d_k \cdot |V_k|}{2} \quad (1a)$$

$$|V_k| = \frac{|V_{k-1}|}{\alpha_k} \Rightarrow |E_k| = \frac{d_k \cdot |V_{k-1}|}{2 \cdot \alpha_k} \quad (1b)$$

$$c_k = \frac{\alpha_k \cdot |V|}{|V_{k-1}|} = \prod_{j=1}^{k} \alpha_j \quad (2a)$$

$$\frac{|V|}{|V_k|} = \prod_{j=1}^{k} \alpha_j \Leftrightarrow |V_k| = \frac{|V|}{\prod_{j=1}^{k} \alpha_j} = \frac{|V|}{c_k} \quad (2b)$$

$$h_k = \Theta\left(\sqrt{c_k}\right) = \Theta\left(\prod_{j=1}^{k} \sqrt{\alpha_j}\right) \quad (3)$$

1.2. Assumptions

It is assumed that nodes are situated in accordance with a two-dimensional uniform random distribution throughout a circularly shaped area. For the purposes of assessing scalability with increasing node count, it is assumed that the circular area increases proportionally with the node count so that the average node density is fixed with increasing $|V|$. This also implies the *sparse graph* phenomenon of $\Theta(|E|) = \Theta(|V|)$. Further, it is assumed that the underlying network graph $G = (V,E)$ is connected. An undirected edge (i.e., bi-directional link) $e = (u,v)$ is assumed to exist between a pair of nodes u and v if the two nodes are situated within R_{TX} m of one another where R_{TX} is the transmission radius for the omni-directional transmitters operating at each node. This bi-directional link model is referred to here as a unit-disk transmission model.

It is assumed that clustering is performed via recursive application of an asynchronous version of the link cluster algorithm of [1]. Further discussion of the clustering is provided in Section 2.2.

The fact that the average density of the network remains fixed with increasing node count implies that $\Theta(A_k) = \Theta(c_k)$. That is, the average area of coverage of a level-k cluster is proportional to the factor (c_k) by which k levels of clustering have aggregated the network topology.

It is shown in [2] that the average hop count (h) on the shortest path between an arbitrary pair of nodes in a two-dimensional network is $\Theta\left(\sqrt{|V|}\right)$. As noted in [3], to maintain connectivity in random graphs, R_{TX} must be $\Theta\left(\sqrt{\log|V|}\right)$. Thus, for random graphs h is actually $\Theta\left(\sqrt{|V|/\log|V|}\right)$. However, the $\Theta(\log|V|)$ term that appears in the expression for h will be ignored here for the sake of simplicity and compactness of notation and the $\Theta\left(\sqrt{|V|}\right)$ result given in [2] is employed, instead.

The mobility scenario under consideration here is the random waypoint model investigated in [4]. In this model, each node picks a random destination within the network area and proceeds to the waypoint coordinates with speed μ m/s. Further, it is assumed that the pause time at each waypoint is zero seconds. Combining the random waypoint model with the unit-disk transmission model implies that the average duration for which a level-0 cluster link $e = (u,v)$ is maintained is $\Theta(R_{TX}/\mu)$. Further, this implies that the frequency at which level-0 cluster link state change events occur *per node* is:

$$f_0 = \Theta\left(\frac{|E| \cdot \mu}{|V| \cdot R_{TX}}\right) = \Theta\left(\frac{|V| \cdot \mu}{|V| \cdot R_{TX}}\right) = \Theta(1) \quad (4)$$

Where, the second equality in (4) is due to the sparse graph feature of $\Theta(|E|) = \Theta(|V|)$.

2. Hierarchical Routing Overview

2.1. Hierarchical Principles

Fig. 1 illustrates the fundamental concept of a clustered hierarchy. All network nodes (i.e., V) are level-0 clusters. Level-0 clusters organize themselves into level-1 clusters, via some clusterhead election process such as one of the methods described in [8]. The level-1 clusters, in turn, organize themselves into level-2 clusters. That is, a level-k node that is elected as the clusterhead becomes a level-(k+1) node. This clustering procedure is performed recursively until the desired number of cluster levels has been constructed.

Hierarchical routing has long been known to afford scalability in computer networks. Reference [7] shows the possible theoretical reduction in the average size of the routing table maintained at each node. This also results in reduced control packet overhead.

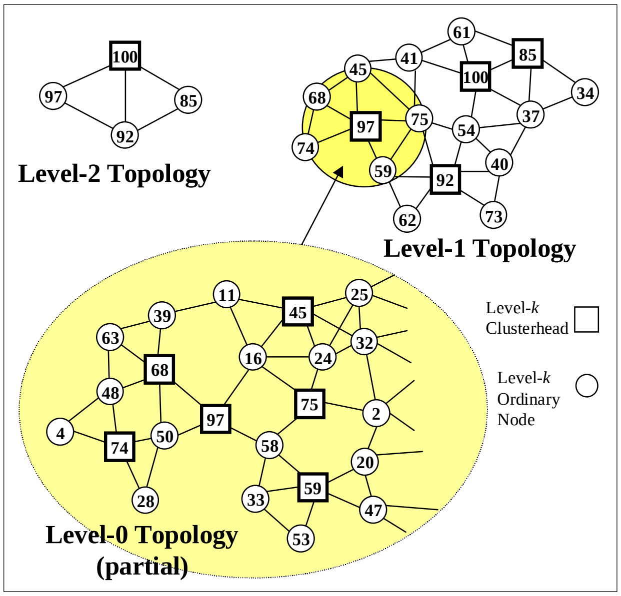

Fig. 1: Example of 3-level hierarchy.

The principles of hierarchical routing have seen application in military-based packet radio networks, such as the Survivable Packet Radio Network (SURAN) described in [9] and [10]. More recently, the Hierarchical State Routing (HSR) protocol proposed in [11,12] and multimedia support for mobile wireless networks (MMWN) proposed in [13] represent hierarchical approaches designed to support group mobility and multimedia, respectively, in the MANET environment.

The analysis of this paper assumes *strict hierarchical routing*, based on the description provided in [14], to be in effect. An important concept concerning packet forwarding in hierarchical networks is that packet forwarding decisions are made solely on the hierarchical address of the destination node and every node has a $O(\log|V|)$ hierarchical map for the clusters of the network hierarchy to which it belongs. This means that forwarding of user packets need *not* be directed through clusterheads and are forwarded via clusterhead and/or non-clusterhead nodes along the shortest hierarchical path to the destination.

2.2. Clustering Techniques

A number of clustering schemes have been proposed in previous literature (e.g., [1], [6], [8] and [15]). Of particular interest here are the max-min h-hop clustering strategy of [8] and the linked cluster algorithm (LCA) of [1]. Each of these approaches is an ID-based clustering technique. The max-min h-hop strategy is shown to converge in $O(h)$ rounds and generates only $O(h)$ messages per node. It represents, therefore, a scalable clustering procedure. The 1-hop clustering case is equivalent to an *asynchronous* version of the LCA. It is an asynchronous version of the LCA that is assumed to be in effect for election of clusterheads, known here as asynchronous LCA (ALCA).

To better understand the ALCA, the ALCA election process is described briefly. Essentially, a level-k node v_k is elected as a level-k clusterhead by a neighbor $u_k \in N_k(v_k)$ if its node ID v_k is the largest among all nodes in the closed neighborhood of u_k (i.e., $u_k \cup N_k(u_k)$). For example, in the level-0 topology of Fig. 1, node 97 is elected to serve as a clusterhead because it is the largest node in its neighborhood. As another example, node 68 is also elected to serve as a clusterhead because it has the largest node ID in the level-0 neighborhood of node 63, even though 68 is *not* the largest node in its own level-0 neighborhood. The recursive application of this election process is illustrated in Fig. 1 by the level-1 and level-2 topologies. Thus, yielding a 3-level clustered hierarchy for this example network.

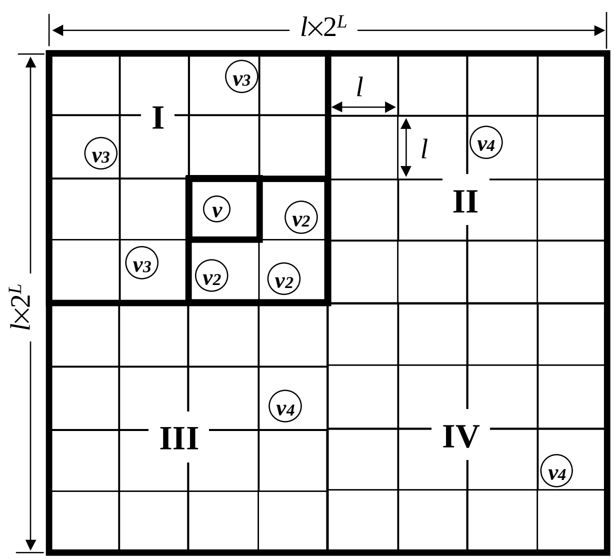

Fig. 2: Example of grid-based hierarchy.

3. Hierarchical Location Management

3.1. Grid Location Service

The Grid Location Service (GLS), proposed in [5], represents an efficient means by which a distributed database of geographic positioning information can be created, maintained and queried. As shown in Fig. 2, GLS relies on a grid-based hierarchy overlaying the network area. A large square area divided recursively into smaller square areas. The smallest square areas, l-by-l squares, are referred to as level-1 squares. The largest square consisting of the entire network area is a level-

(L+1) square and is $l\times 2^L$-by-$l\times 2^L$. The bold squares show the hierarchical grid areas to which a particular node v belongs for each level of the grid hierarchy.

To understand GLS, an arbitrary node v is considered. The salient features of the distributed database maintaining the geographic coordinates of v are as follows:

a) The set of nodes functioning as LM servers for v are based on the relation of their node ID to v and their location in the grid hierarchy
b) The density of LM servers for v in regions near v is high and low in the regions far from v
c) The frequency at which v updates its location to nearby LM servers is high while servers situated far from v receive updates at a low frequency

Feature (a) ensures that for each grid zone a node can be selected unambiguously to function as the location server for v. The ID-based rule for selecting the server set consists of selecting the zone node $z \in Z$ whose ID minimizes the following for all nodes belonging to a level-k grid zone:

$$\mathrm{mod}_{v+|V|}\left(z+|V|\right) \tag{5}$$

The ID-based selection of the distributed location database serves two objectives. First, it provides a means for unambiguously selecting a server set and to subsequently direct *queries* properly to the server set. Second, it tends to distribute the load of server functionality equitably throughout V as it will be typically rare that any node will satisfy (5) for a disproportionately large number of nodes.

The combination of features (b) and (c) provides favorable scalability to GLS. Intuitively, these features effectively summarize location detail about v in regions of the network that are far from v.

3.2. Clustered Hierarchy Location Management

The technique of [5] to unambiguously select location servers is applied to clustered hierarchy location management (CHLM). Instead, however, of updating a particular node in the grid hierarchy, a node v updates a peer in the member cluster to which it belongs.

Considering node 63 of Fig. 1, as example, 63 is a member of the level-1 cluster 68. Since complete topology information within a level-1 cluster, no LM messaging is required for level-1 server maintenance, as in GLS. Now considering level-2 server maintenance for 63, the level-1 clusters (45, 59, 68, 74, 75, 97) of the level-2 cluster to which 63 belongs are candidates for housing the level-2 LM server of 63. As in GLS, a hashing function based on the ID of v and the cluster IDs of the level-2 cluster is needed. The hashing function of (5) can not be used here as it would result in a disproportionately large number of nodes in this cluster selecting 45 as the level-1 cluster to house their level-2 server. Thus, a slightly more complex hashing function is required in CHLM. Otherwise, equitable distribution of server functionality will not be achieved. The specific implementation is not crucial to understand, as long as the goals of unambiguous server selection and equitable distribution of server load are realized.

A particular CHLM hashing function happens to yield cluster 59 as the level-1 cluster with which 63 must register. Another function is then applied to nodes within cluster 59 (i.e., nodes 20, 33, 47, 53, 58 and 59) to yield node 33 as the level-0 node serving as the level-2 LM server for node 63.

Finally, 63 belongs to the level-3 cluster with ID 100 (top level cluster). Applying a CHLM hashing function yields 85 as the level-2 cluster to which node 63 must register level-3 server updates. Another function is then applied to the level-1 members of 85 (34, 37, 61, 85 and 100) yielding cluster 37. The detail of Fig. 1 does not show the membership of the level-1 cluster 37, but a hashing function is also used to select a member of cluster 37 to be the level-0 node acting as the level-3 LM server for 63.

Further detailed discussion of CHLM is omitted. However, it should be evident that the features of (a)-(c) relevant to GLS are also achieved for CHLM via modest modifications to the GLS procedure. Lastly, it should also be evident that by maintaining LM data at $L = \Theta(\log|V|)$ levels in the clustered hierarchy means that each node acts as a LM server for $\Theta(\log|V|)$ nodes, on average. This is an important concept for evaluating handoff overhead as it quantifies the *magnitude of data transferred* as a result of single node handoff.

4. Handoff Due to Node Migration

In a completely distributed hierarchical LM system, every node maintains LM information for, on average, $\Theta(\log|V|)$ other nodes. The peers for which a node is assigned as the LM server is based on its hierarchical address and the relative proximity of other nodes in the network hierarchy to it. Therefore, when it migrates from a particular level-k cluster to which it belongs it must transfer $\Theta(\log|V|)$ LM entries to the appropriate nodes of the cluster it just exited. Alternatively, when a node joins a new level-k cluster it must acquire $\Theta(\log|V|)$ LM entries from nodes in the cluster it just joined.

There is a trade off in handoff overhead when considering node mobility at increasingly higher levels in the clustered hierarchy. On one hand, the average path length over which handoff data must be communicated increases at each successive level in the hierarchy. Specifically, the path length grows in accordance with square root of the cluster size. This augments the cost of handoff. On the other hand, the distance a node must migrate in order to trigger a handoff event increases at each successive level in the hierarchy. This mitigates the cost of handoff by reducing the frequency of handoff events at successively higher levels in the clustered hierarchy.

$$\phi_k = \Theta\left(f_k \cdot h_k \cdot \log|V|\right) \tag{6a}$$

$$\phi_k = \Theta\left(f_k \cdot \sqrt{\prod_{j=1}^{k} \alpha_j} \cdot \log|V| \right) \tag{6b}$$

$$\rightarrow \phi = \sum_{k=1}^{L} \phi_k \tag{6c}$$

Here, $L = \Theta(\log|V|)$. Eq. (6b) follows from (6a) by applying (3). Clearly, if $f_{k-1}/f_k = \Omega\left(\sqrt{\alpha_k}\right)$ then $\phi_k = \mathrm{O}(\log|V|)$ and $\phi = \mathrm{O}(\log^2|V|)$. This condition is equivalent to requiring $f_k = \mathrm{O}(1/h_k)$ as given by (6a).

Since the average area of coverage of a level-k cluster is $\Theta(c_k)$, the average relative distance (δ_k) required for a node to migrate out of range of its level-k clusterhead is:

$$\delta_k = \Theta\left(R_{TX} \cdot \sqrt{c_k}\right) = \Theta\left(\sqrt{c_k}\right) = \Theta\left(\prod_{j=1}^{k} \sqrt{\alpha_j} \right) = \Theta(h_k) \tag{7}$$

Extending (4) for the general case of f_k yields:

$$f_k = \Theta(f_0/\delta_k) = \Theta(\mu/(R_{TX} \cdot \delta_k)) = \Theta(1/\delta_k) \tag{8}$$

Combining (4), (7) and (8) implies that:

$$f_0/f_k = \Theta\left(\prod_{j=1}^{k} \sqrt{\alpha_j} \right) \Rightarrow f_k = \Theta(1/h_k) \tag{9}$$

Thus, the condition required for $\phi_k = \mathrm{O}(\log|V|)$ is satisfied and $\phi = \Theta(\log^2|V|)$.

5. Handoff Due to Cluster Reorganization

When a level-k cluster link state change occurs between a pair of level-k clusters u and v, and either u, v or both u and v are level-(k+1) nodes, then handoff occurs between the nodes in the two level-k clusters. When considering a level-0 cluster link state change between a node and a clusterhead, this is equivalent to handoff due to node migration (i.e., ϕ_1). However, when a level-k ($k \geq 1$) cluster link state change occurs between a level-k cluster and a level-(k+1) clusterhead, then all nodes in the level-k cluster participate in handoff with the relevant level-(k+1) cluster.

Two pairs of opposing phenomenon are present to complicate the analysis of handoff due to cluster reorganization. One is similar to the pair mentioned already for handoff due to node migration. That is, the opposing effects of increasing path length for handoff messaging with increasing level and the reduced frequency at which cluster link state events occur as a result of greater relative distance that nodes must traverse before a high level link state change event occurs. The other pair relates to the increasing node count within each cluster that must undergo handoff at each successive level in the hierarchy and the decreasing number of cluster links at higher levels. That is, the average number of nodes within a level-k cluster is *larger* by a factor of α_k than that in a level-(k–1) cluster, as indicated by (1b). However, this is counteracted by the fact that the number of level-k cluster links ($|E_k|$) is *smaller* by a factor of α_k than the number of level-(k–1) links ($|E_{k-1}|$), as indicated by (1b).

$$\gamma_k = \Theta\left(g_k \cdot c_k \cdot h_k \cdot \log|V|\right) \tag{10a}$$

$$\gamma_k = \Theta\left(g_k \cdot \log|V| \cdot \prod_{j=1}^{k} \alpha_j^{3/2} \right) \tag{10b}$$

$$\gamma = \sum_{k=1}^{L} \gamma_k \tag{11}$$

Eq. (10b) follows from (10a) by applying (2a) and (3). Clearly, if $g_{k-1}/g_k = \Omega\left(\alpha_k^{3/2}\right)$ then $\gamma_k = \mathrm{O}(\log|V|)$ and $\gamma = \mathrm{O}(\log^2|V|)$. This condition is equivalent to:

$$g_k = \mathrm{O}(1/(c_k \cdot h_k)) \tag{12}$$

Recalling (1a) it follows from (2b) that $|E_k|$ is inversely proportional to c_k:

$$|E_k| = \frac{d_k}{2} \cdot \frac{|V|}{\prod_{j=1}^{k} \alpha_j} = \frac{d_k}{2} \cdot \frac{|V|}{c_k} \tag{13a}$$

$$\rightarrow \frac{|E_k|}{|V|} = \frac{d_k}{2 \cdot c_k} = \Theta(1/c_k) \tag{13b}$$

The implication of (13) is that the c_k term appearing in the denominator of (12) is accounted for by the fact that the number of cluster links decreases by a factor of α_k for each increment in k. That is, although cluster size increases at each level in the clustered hierarchy, the number of links in the level-k topology decreases by a similar factor. Defining now g'_k as the frequency of change per (level-k) cluster link, it is evident that the condition required for (12) to hold, reduces to the following:

$$g'_k = \Theta(g_k / c_k) = O(1/h_k) \tag{14}$$

Thus, combining (12), (13) and (14), it is evident that what remains to be shown to validate the supposition of $\gamma_k = O(\log|V|)$ is for the frequency of individual cluster link state change events that trigger cluster reorganization, and hence handoff, to be inversely proportional to h_k. This is shown in Section 5.3.

5.1. Cluster Dynamics Model

When a level-k cluster link state change occurs between a level-k node and a level-k clusterhead (or, between a pair of clusterheads) occurs, handoff of LM data may also occur. The link state change event may occur simply because of node mobility that results in the two level-k nodes migrating sufficiently near to (or, far from) one another to incur a link state change. Such a cluster link state change event is very similar to the effect of node migrations that cause link state change events to occur among level-0 nodes as they migrate within and outside R_{TX} m from one another.

Another effect that triggers level-k link state changes is the *election* of a new level-(k–1) clusterhead via the ALCA. As a result, a new level-k node is created and subsequent level-k links are created between a subset of nodes in V_k and their new neighbor. Similarly, the *rejection* of an existing clusterhead as a result of its failure to be reelected as a level-(k–1) clusterhead causes link state changes in the level-k topology.

The ALCA *state* of any level-k node $v_k \in V_k$ can be characterized by the number of level-k neighbors of v_k that have elected it as their level-k clusterhead. For the node with the highest ID in V, this state will always be $n_{k,v}$, where $n_{k,v}$ is the number of level-k neighbors of a level-k node v. For the lowest ID node in V_k, this will always be 0 (i.e., the node is *never* elected as level-k clusterhead).

As shown in Fig. 3, each node in V_k exists in one of $1+n_{k,v}$ possible states (subscripts omitted in figure). States 1 through $n_{k,v}$ are in bold to indicate that they are clusterhead states. That is, one or more neighbors have elected v_k to serve as a level-k clusterhead. State 0, of course, corresponds to non-clusterhead status. Another feature of Fig. 3 is that a node will only make transitions between adjacent states. This means that at any instant in time, the state of a node will only be incremented (or decremented) by one. The justification for this unit transition effect is that within an arbitrarily small time interval, the probability that *more* than one node either elects (or rejects) v_k as it level-k clusterhead also becomes arbitrarily small. Thus, for the continuous time state transition diagram, instantaneous transitions occur only between adjacent states.

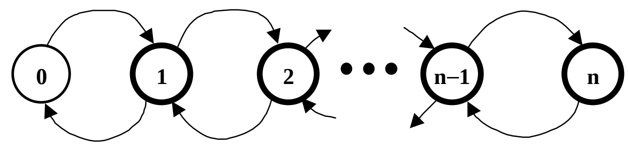

Fig. 3: ALCA cluster state transitions.

An implication of adjacent state transitions with respect to the ALCA is that states 0 and 1 represent *critical clustering* states. That is, a node can change its clusterhead status, either from non-clusterhead to clusterhead or visa versa, only when it is in state 0 or 1, respectively. If a node v_{k-1} is in state 0, it becomes a level-(k–1) clusterhead (i.e., level-k node) when a single neighbor elects v_{k-1} to be its clusterhead, in the process transitioning from state 0 to state 1. If v_{k-1} is in state 1, it loses its level-k status when the single level-(k–1) node that had previously elected v_{k-1} to be its clusterhead elects a different neighbor to serve as its clusterhead, in the process v_{k-1} transitions from state 1 to state 0. If v_{k-1} is in any of the states $\{2,3,\ldots,n_{k,v}\}$, a single increment or decrement in node state does not alter its clusterhead status. This feature of the cluster dynamics model is important for the assessment of handoff overhead due to recursive clusterhead election/rejection.

5.2. Reorganization Factors

In Section 4, packet transmission overhead is isolated at each level $k \in \{1,2,\ldots,L\}$ of the clustered hierarchy. This approach is refined further here by considering in isolation the handoff overhead of a single, arbitrary level-k cluster $v_k \in V_k$. The assessment of v_k applies to all $|V_k|$ level-k clusters.

The cluster reorganization events that trigger handoff are enumerated now. These events are considered in turn to determine whether their frequencies decrease sufficiently with increasing cluster level so as to satisfy (14). Cluster reorganization incurs handoff for a level-k cluster $v_k \in V_k$ whenever one of the following occur:

i. A new level-k link is formed between v_k and $u_k \in V_k$, where v_k or $u_k \in V_{k+1}$, because v_k and u_k move from 2 to 1 level-k hops from one another. This incurs handoff as LM data is redistributed to the level-k cluster(s) joining the level-(k+1) cluster(s).

ii. A level-k link is broken between v_k and $u_k \in V_k$, where v_k or $u_k \in V_{k+1}$, because v_k and u_k move from 1 to 2 level-k hops from one another. This incurs handoff as LM data is redistributed to the remaining level-k clusters of the level-(k+1) cluster(s).

iii. The cluster v_{k-1} becomes a level-k cluster (v_k) as a result of a cluster $u_{k-1} \in V_{k-1}$ electing v_{k-1} as its clusterhead by moving from 2 to 1 level-(k–1) hops from v. This incurs handoff between v_k and the level-(k+1) cluster(s) it then joins.

iv. The cluster v_{k-1} loses its level-k cluster status as a result of a cluster $u_{k-1} \in V_{k-1}$ not electing v_{k-1} as its clusterhead by moving from 1 to 2 level-(k–1) hop from v. This incurs handoff between v_k and the level-(k+1) cluster(s) with which it has membership prior to relinquishing its role as a level-k cluster.

v. The cluster v_{k-1} becomes a level-k cluster (v_k) as a result of a cluster $u_{k-1} \in V_{k-1}$, that is 1 level-(k–1) hop from v, electing v_{k-1} as its clusterhead after itself (u) is elected as a level-(k–1) cluster. This incurs handoff between v_k and the level-(k+1) cluster(s) it then joins.

vi. The cluster v_{k-1} loses its level-k cluster status as a result of a cluster $u_{k-1} \in V_{k-1}$ not reelecting v_{k-1} as its clusterhead after itself (u) was not reelected as a level-(k–1) cluster. This incurs handoff between v_k and the level-(k+1) cluster(s) with which it has membership prior to relinquishing its role as a level-k cluster.

vii. A level-k neighbor of v_k, $u_k \in N_k(v)$, is elected as a level-(k+1) clusterhead. This incurs handoff between v_k and the new level-(k+1) cluster, u_{k+1}.

In all above cases, $\Theta(c_k)$ nodes are involved in redistributing LM data between v_k and the appropriate level-(k+1) cluster(s). Each communication session must traverse a level-(k+1) cluster, incurring $\Theta(h_{k+1}) = \Theta(h_k)$ hops (as $h_{k+1} = \alpha_{k+1} \cdot h_k$). Hence, the requirement of (14) to satisfy $\gamma_k = O(\log|V|)$.

Events (i) and (ii) are due to *cluster migration*. These events are similar to node migration in that the distance a pair of nodes must move relative to one another to trigger a cluster link state change is $\Theta(h_k)$. Events (iii) through (vii) relate to *clusterhead election/rejection*. Their impact is more complex to quantify.

Interestingly, the converse of (vii) does not incur handoff overhead. This is because if the cluster u_{k+1} ceases to exist, the level-k clusters that previously comprised u_{k+1} either already belong to another level-(k+1) cluster that contains the complete LM hierarchy or have elected already another level-(k+1) clusterhead with which handoff occurs and is subsumed by event type (iii).

Further, event (iv) may seem to imply a recursive clusterhead rejection process. That is, because v can lose its level-k clusterhead status as a result of the migration of the level-(k–1) node that elected it, v might also have lost its level-k and level-(k–1) status due to the migration of a single level-(k–2) node. The weakness of this recursive argument is as follows. If the ID of v is sufficiently high for it to be elected, at some time, by at least one of its neighbors to serve as a level-(k–1) clusterhead, then it is likely to be sufficiently high to be elected as a level-(k–2) clusterhead by *more* than one of its level-(k–2) neighbors. Thus, the instability of the clusterhead status of v is dominated by cluster status changes at the highest level in the clustered hierarchy it has achieved (in this case, level-k)).

Another possible recursive argument to consider is that if v is elected by a single level-(k–1) node u, and u itself is elected by a single level-(k–2) node w, then rejection of u as a level-(k–1) node because of the rejection of w as a level-(k–2) node would also result in v losing its level-k status. That is, the rejection of w results in the rejection of u, which results in the rejection of v. This argument can be extended recursively by considering that the level-(k–3) node x that elected w may also exist because of single level-(k–4) node that has elected it, and so on. The significance of this domino effect is analyzed in the next section, where it is shown that its impact can be summarized by a scaling constant.

Lastly, concerning (vii), events (iii) and (v) apply to each node $u_k \in N_k(v)$. Thus, the analysis of (iii) and (v) determine the magnitude of handoff overhead due to (vii) by a scaling factor of $|N_k(v)|$.

5.3. Frequency Assessment

In the section, the frequency of (i)-(vii), described in section 5.2, is considered for a level-k cluster, $k \in \{1,2,\ldots,L\}$. The goal is to determine whether the frequency for each event is $O(1/h_k)$.

5.3.1. Cluster Migration Events. Events (i) and (ii) are due to relative mobility between level-k clusters. The higher the cluster is in the clustered hierarchy, the larger will be its node count, as well as the geographical area covered by the cluster. Recalling Section 1.2, the area of coverage is proportional to c_k. Thus, with each increment in the clustered hierarchy, the relative distance separating neighbor clusterheads also increases. This distance is $\Theta\left(\sqrt{c_k}\right)$. Thus, the relative distance a pair of level-k

clusterheads must migrate in order for a cluster link to break is also $\Theta\left(\sqrt{c_k}\right)=\Theta(h_k)$.

Since the relative distance between a pair of neighboring clusterheads must migrate in order to break an existing level-k link is $\Theta(h_k)$, the expected duration to elapse prior to link breakage is $\Theta(h_k/\mu)=\Theta(h_k)$. Similarly, the expected duration required for a pair of clusterheads situated 2 level-k hops from one another to migrate within 1 level-k hop is also $\Theta(h_k)$. Hence, the frequency of level-k cluster link state changes per level-k node cluster link is $\Theta(1/h_k)$, as required by (14).

5.3.2. Cluster Election/Rejection Events. Events (iii) through (vii) are related to the clusterhead maintenance of the ALCA. Again, the analysis is conducted with respect to an arbitrary level-k node, v_k.

An argument similar to that employed in Section 5.3.1 is applied here to show that the frequency of events (iii) and (iv) is $\Theta(1/h_k)$. However, rather than a level-k link being created or broken, what is at stake is the status of level-(k–1) link. Following the argument of Section 5.3.1, the frequency of a cluster link state change between a pair of level-(k–1) nodes is simply $\Theta(1/h_{k-1})=\Theta(\alpha_k/h_k)=\Theta(1/h_k)$, as $\alpha_k=\Theta(1)$. Hence, the frequency of (iii) and (iv) are $O(1/h_k)$ per node per level-k cluster link, as required by (14). Letting f_M be the frequency at which the migration of a node u_{k-1} impacts level-k election or rejection summarizes this result:

$$f_M=\Theta\left(\frac{1}{h_{k-1}}\right)=\Theta\left(\frac{1}{h_k}\right)$$

The recursive election/rejection process of events (v) and (vi) requires a more sophisticated analysis. The analysis considers the frequency of event (vi), the rejection of a *critical*[1] existing level-k node as a result of the node electing it u_{k-1} failing to be elected to the set of level-(k–1) nodes. A treatment of event (v) is omitted as the steady state average frequency of election events must equal that of rejection events.

In order to assess the frequency of event (vi), additional parameters are defined:

– $p_j \equiv$ Probability that a level-j node is elected to serve as a level-j clusterhead by exactly 1 of its n_j neighbors (i.e., is ALCA state 1).
– $T_R \equiv$ Expected duration for a level-k node v_k to persist in state 1 prior to a recursive rejection process of the level-(k–1) node u_{k-1} that elected it to incur event (vi) with respect to v_k.
– $T_m \equiv$ Expected duration prior to rejection for a critical level-m node via (iv).

The following relations quantify T_R:

$$q_j=\begin{cases}\left(1-p_{k-j-1}\right)\cdot\prod_{i=1}^{j}p_{k-i} & j\in\{1,2,...,k-2\}\\ \prod_{i=1}^{j}p_{k-i} & j=k-1\end{cases} \tag{15a}$$

$$Q\equiv\sum_{j=1}^{k-1}q_j \tag{15b}$$

$$T_R=\frac{1}{f_R}=\frac{1}{Q}\cdot\sum_{j=1}^{k-1}q_j\cdot T_{k-j} \tag{16}$$

From (16), it is evident that T_R is dependent on lower level cluster migration frequencies. The key issue to address, is to determine by how much. Setting $T_j=0$ for $j\in\{2,3,\ldots,k-1\}$, T_R can be under bounded as follows:

$$T_R=\frac{1}{Q}\cdot\sum_{j=1}^{k-1}q_j\cdot T_{k-j}\geq\frac{q_1\cdot T_{k-1}}{Q} \tag{17}$$

The ratio q_1/Q represents the fraction of time recursive rejection stops at level-(k–1) and T_1 represents the expected duration prior to rejection of a level-(k–1) node u_{k-1}, that has elected v_k. Thus, to under bound T_R requires q_1 and T_1 to be quantified.

Quantification of q_1/Q consists of a formulation of another lower bound. To formulate a lower bound, a dummy parameter p is defined as follows:

$$p\equiv\max\{p_1,p_2,...,p_{k-1}\} \tag{18}$$

Further, a dummy aggregate $P\geq Q$ is defined:

$$P\equiv q_1+\sum_{j=2}^{k-2}p^j\cdot(1-p)+p^{k-1}\geq Q \tag{19}$$

The summation of (19) is manipulated as follows:

$$\sum_{j=2}^{k-2}p^j\cdot(1-p)=(1-p)\cdot\frac{p^2-p^{k-1}}{1-p}=p^2-p^{k-1} \tag{20}$$

Substituting (20) into (19) yields P and a lower bound for q_1/Q:

[1] Here forward, a node is considered to be a critical node if it is in ALCA state 1.

$$P = p^2 + q_1 \geq Q \quad (21a)$$

$$\rightarrow \frac{q_1}{Q} \geq \frac{q_1}{p^2 + q_1} \quad (21b)$$

Before proceeding, it is worth noting that (21) is independent of the level-k under consideration or any of the lower levels $j < k$ in the clustered hierarchy. That is, the parameters q_1 and p appear as constants, invariant with respect to $|V|$ (and $L = \Theta(\log|V|)$). In particular, the following relation is of interest at each level of the hierarchy:

$$\lim_{|V|\to\infty} q_1 > \varepsilon > 0 \quad (22)$$

Here, ε can be any constant greater than 0.

Provided (22) holds, the order of magnitude of the lower bound for T_R given by (17) is due solely to T_1. Justification of (22) is based on the observation that the recursive clustering procedure of the ALCA tends to yield multiple levels of clustered hierarchy that are similar in terms of average cluster arity and degree. Thus, if q_1 on *average* satisfies (22) for some level k in the hierarchy, then the relation should hold at all levels. Actual quantification of q_1 via simulation represents a direction for future work.

Now, T_1 is considered. Since the recursive rejection processed stopped at level-(k–1), the time for a level-(k–2) neighbor that has elected $u_{k-1} \in V_{k-1}$ to migrate from 1 to 2 level-(k–2) hops from u_{k-1} is of interest. Therefore, T_1 is just the expected duration for a specific level-(k–2) cluster link to be created or broken. That is, $T_1 = \Theta\left(\sqrt{c_{k-2}}\right) = \Theta(h_{k-2})$. Applying this fact and (21b) to (17) yields:

$$T_R \geq \Theta\left(\frac{q_1}{p^2 + q_1} \cdot h_{k-2}\right) = \Theta(h_{k-2}) \quad (23a)$$

$$f_R = \frac{1}{T_R} = \Theta\left(\frac{1}{h_{k-2}}\right) = \Theta\left(\frac{\alpha_k \cdot \alpha_{k-1}}{h_k}\right) = \Theta\left(\frac{1}{h_k}\right) \quad (23b)$$

The assessment of recursive cluster creation (v), follows similar logic. Summing f_M and f_R yields the net effect of cluster election on v_k (events (iii) and (v)) or cluster rejection (events (iv) and (vi)):

$$\begin{aligned} f_{ELECT} = f_{REJECT} &= (1 - p_1) \cdot f_M + p_1 \cdot f_R \\ &= (1 - p_1) \cdot \Theta\left(\frac{\alpha_k}{h_k}\right) + p_1 \cdot \Theta\left(\frac{\alpha_k \cdot \alpha_{k-1}}{h_k}\right) \end{aligned} \quad (24a)$$

$$\rightarrow f_{ELECT} = f_{REJECT} = \Theta\left(\frac{1}{h_k}\right) \quad (24b)$$

Again $p_1 \leq 1$ and $\alpha_j = \Theta(1)\ \forall\, j$.

Lastly, event (vii) applies to each of the $n_{k,v}$ neighbors of v_k. The frequency of such an event per neighbor is summarized by (24). Since $n_{k,v} = O(1)$, the frequency of event (vii) is also given by (24). This means all 7 of the events contributing to cluster reorganization handoff occur with frequency that is $\Theta(1/h_k)$ per level-k cluster link and the requirement of (14) is satisfied. Thus, the condition required for $\gamma_k = O(\log|V|)$ is satisfied and $\gamma = \Theta(\log^2|V|)$.

6. Conclusions

This paper has evaluated location management handoff overhead in the context hierarchically organized mobile ad hoc networks. The factors that trigger a handoff event have been identified and evaluated. Specifically the trigger events have been broken down into node migration and cluster reorganization. In both cases, overhead is $\Theta(\log^2|V|)$ packet transmissions per node per second. The significance of this result is that the capacity of MANET links need only to grow at a *polylogarithmic* rate in order to scale gracefully with increasing node count.

Of course, LM handoff is not the only factor contributing to control overhead in hierarchically organized MANETs. For example, there are the issues of cluster maintenance, dissemination of the hierarchical topology to cluster members, location registration and location queries. However, in [16] and [17] it is shown that the factors of cluster maintenance, flooding of the hierarchical topology and location registration incur packet transmission counts that are only logarithmic in $|V|$. Further, the overhead associated with a location query is of the same order of magnitude as the hop count between the requesting node and the target node, and occurs only once per communication session. Hence, query overhead is arguably absorbed in the associated session. Thus, the result here combined with those of [16] and [17] indicate that IP-based MANETs can scale well using hierarchical organization.

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