P 4.1 [a] Five
[b] Three

[c] Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

\[-i_3 + i_1 + i_2 = 0\]
\[-i_1 - i_4 + i_3 = 0\]
\[i_6 - i_2 - i_3 = 0\]

[d] Two.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

\[R_{14}I_1 + R_{34}I_3 - R_{24}I_2 = 0\]
\[R_{34}I_3 + R_{45}I_4 - R_{43}I_2 = 0\]

P 4.6 [a]

\[
\frac{v_1 - 110}{2} + \frac{v_3 - v_2}{8} + \frac{v_1 - v_3}{16} = 0 \quad \Rightarrow \quad 11v_1 - 2v_3 - v_3 = 880 \\
\frac{v_2 - v_1}{8} + \frac{v_2 - v_3}{3} + \frac{v_2 - v_3}{24} = 0 \quad \Rightarrow \quad -3v_1 + 12v_2 - 3v_3 = 0 \\
\frac{v_3 + 110}{2} + \frac{v_3 - v_2}{24} + \frac{v_3 - v_1}{16} = 0 \quad \Rightarrow \quad -3v_1 - 2v_2 + 29v_3 = -2640 \\
\]

Solving, \(v_1 = 74.64\) V, \(v_2 = 11.79\) V, \(v_3 = -82.5\) V

Thus,
\[i_1 = \frac{110 - v_1}{2} = 17.68\ A \quad i_4 = \frac{v_1 - v_2}{5} = 7.85\ A \]
\[i_2 = \frac{v_2 - v_3}{3} = 3.93\ A \quad i_5 = \frac{v_3 - v_3}{24} = 3.93\ A \]
\[i_3 = \frac{v_3 + 110}{2} = 13.75\ A \quad i_6 = \frac{v_1 - v_3}{16} = 9.82\ A \]

[b] \[\sum P_{\text{dev}} = 110I_1 + 110I_4 = 3457.3\ W\]
\[\sum P_{\text{aux}} = I_1^2(2) + I_2^2(3) + I_3^2(2) + I_4^2(8) + I_5^2(24) + I_6^2(16) = 3457.3\ W\]
P 4.8 [a]

\[ v_1 + \frac{v_1 - 640}{5} + \frac{v_1 - v_2}{2.5} = 0 \quad \text{so} \quad 31v_1 - 20v_2 + 6v_3 = 6400 \]
\[ \frac{v_2 - v_1}{2.5} + \frac{v_2 - v_3}{5} + 12.8 = 0 \quad \text{so} \quad -2v_1 + 3v_2 - v_3 = -64 \]
\[ \frac{v_3}{2.5} + \frac{v_3 - v_2}{5} - 12.8 = 0 \quad \text{so} \quad 0v_1 - v_2 + 3v_3 = 64 \]

Solving, \( v_1 = 380 \) V, \( v_2 = 269 \) V, \( v_3 = 111 \) V,

\[ v = \frac{640 - 380}{5} = 52 \text{ A} \]
\[ P_{el} = (640)(52) = 33,280 \text{ W} \]

P 4.10

\[ v_1 - \frac{75}{20} + \frac{v_1 - v_2}{50} + \frac{v_1 - v_2}{40} = 0 \]
\[ \frac{v_2 - v_1}{40} + \frac{v_2 - 75}{800} - 6 + \frac{v_2}{200} = 0 \]

Solving, \( v_1 = 115 \) V; \( v_2 = 287 \) V

\[ \therefore v_2 = 115 - 75 = 40 \text{ V} \]

CHECK:

\[ i = (75 - 115)/20 + (75 - 287)/800 = -2.255 \text{ A} \]
\[ P_{MW} = -75i = 169.875 \text{ W (abs)} \]
\[ P_{HA} = -6v_2 = -6(287) = -1722 \text{ W (del)} \]

\[ P_{power} = (v_2 - 75)^2/800 = 58.18 \text{ W} \]
\[ P_{MW} = \frac{v_2^2}{20} = 1600/20 = 80 \text{ W} \]
\[ P_{HA} = (115)^2/50 = 264.5 \text{ W} \]
\[ P_{MW} = (v_2 - v_1)^2/40 = (287 - 115)^2/40 = 739.6 \text{ W} \]
\[ P_{HA} = (287)^2/200 = 411.845 \text{ W} \]
\[ \sum P_{abs} = 169.875 + 58.18 + 80 + 264.5 + 739.6 + 411.845 = 1722 \text{ W} \]
\[ V_u = 4 \text{ V} \]

\[ \frac{7 + V_a}{3} + \frac{V_b - V_a}{1} = 0 \]

\[ -2V_e + \frac{V_c - V_b}{1} + \frac{V_e - V_a}{2} = 0 \]

\[ V_e = V_b - V_a = V_e - 4 \]

Solving, \( V_a = V_b = 1.5 \text{ V} \)

\[ P 4.25 \]

\[ i_o = \frac{V_a - V_b}{4} = \frac{235 - 222}{4} = 3.25 \text{ A} \]

\[ \beta_{1a} = 30(3.25) = 97.5 \text{ V} \]

\[ V_1 + \beta_{1a} = V_b \]

\[ V_1 = V_b - \beta_{1a} = 222 - 97.5 = 124.5 \text{ V} \]

\[ V_b = V_a = 250 \]

\[ \therefore V_\Delta = 250 - 235 = 15 \text{ V} \]

\[ \alpha_{1\Delta} = (3.2)(15) = 48 \text{ A} \]

\[ i_f = \frac{250 - 124.5}{2} + \frac{250 - 235}{1} = 77.75 \text{ A} \]

\[ P_{1\Delta} = -250i_f = -250(77.75) = -19,437.5 \text{ W(dei)} \]

\[ i_{1\Delta} = i_f + \frac{V_1}{40} + 48 = 0 \]

\[ i_{1\Delta} = i_f - 222/40 - 48 = 3.25 - 5.55 - 48 = -50.3 \text{ A} \]

\[ P_{1\Delta} = (30i_{1\Delta})^2 = (97.5)(-50.3) = -4904.25 \text{ W(dei)} \]

\[ P_{1\Delta} = (\alpha_{1\Delta})(V_1) = (48)(22) = 10,656 \text{ W(sabi)} \]

\[ \therefore \sum P_{1\Delta} = 19,437.5 + 4904.25 = 24,341.75 \text{ W} \]

\[ P_{1\Delta} = \frac{V_1^2}{10} = \frac{(124.5)^2}{10} = 1550.025 \text{ W} \]

\[ P_{1\Delta} = \frac{(250 - 124.5)^2}{2} = 7875.125 \text{ W} \]

\[ P_{1\Delta} = \frac{(250 - 235)^2}{1} = 225 \text{ W} \]
\[ p_{201} = \frac{(235)^2}{20} = 2761.25 \text{ W} \]

\[ p_{401} = (3.25)^2(4) = 42.25 \text{ W} \]

\[ p_{400} = \frac{(222)^2}{40} = 1232.10 \text{ W} \]

\[ \therefore \sum P_{\text{dis}} = 10,656 + 1550.025 + 7875.125 + 225 + 2761.250 + 42.25 + 1232.1 = 24,341.75 \text{ W} \]

Thus, \( \sum P_{\text{dev}} = \sum P_{\text{dis}} \)

P 4.27

![Diagram](image)

\[-18 + 3i_1 + 9i_2 - 15 + 6i_9 + 2i_1 = 0; \quad i_9 = i_1 = 3\]

Solving, \( i_1 = -0.6 \text{ A}; \quad i_2 = 2.4 \text{ A}; \quad p_{300} = -18i_1 = 10.8 \text{ W} \)

\( v_7 = 15i_2 - 15 = 36 - 15 = 21 \text{ V} \)

\( p_{30} = -3i_6 = -63 \text{ W}(\text{dev}) \)

\( p_{31} = -15i_2 = -36 \text{ W}(\text{dev}) \)

\( \sum P_{\text{dev}} = 99 \text{ W} \)

P 4.32 [a]

![Diagram](image)
\[110 + 12 = 17i_1 - 10i_2 - 3i_3\]
\[0 = -10i_1 + 28i_2 - 12i_3\]
\[-12 - 70 = -8i_1 - 12i_2 + 17i_3\]

Solving, \(i_1 = 8\ A; \quad i_2 = 2\ A; \quad i_3 = -2\ A\)

\[p_{11} = -110i_1 = -880\ \text{W (dev)}\]
\[p_{22} = -12(i_1 - i_2) = -120\ \text{W (dev)}\]
\[p_{33} = 70i_3 = -140\ \text{W (dev)}\]

\[\therefore \sum p_{\text{dev}} = 1140\ \text{W}\]

\[P_{\text{in}} = (8)^2(4) = 256\ \text{W}\]

\[P_{\text{mom}} = (6)^2(10) = 360\ \text{W}\]

\[P_{\text{xu}} = (-4)^2(12) = 192\ \text{W}\]

\[P_{\text{in}} = (-2)^2(2) = 8\ \text{W}\]

\[P_{\text{ex}} = (2)^2(6) = 24\ \text{W}\]

\[P_{\text{in}} = (10)^2(3) = 300\ \text{W}\]

\[\therefore \sum p_{\text{in}} = 1140\ \text{W}\]

P 4.36 [a]

\[200 = 85i_1 - 25i_2 - 50i_3\]
\[0 = -75i_1 + 35i_2 + 150i_3\]

\[i_3 - i_2 = 4.3(i_1 - i_2)\]

Solving, \(i_1 = 4.6\ A; \quad i_2 = 5.7\ A; \quad i_3 = 0.97\ A\)

\[i_a = i_2 = 5.7\ A; \quad i_b = i_1 = 4.6\ A\]

\[i_a = i_3 = 0.97\ A; \quad i_d = i_1 - i_2 = -1.1\ A\]

\[i_a = i_2 - i_3 = 3.83\ A\]

\[10i_2 + v_b + 25(i_2 - i_1) = 0\]

\[\therefore \ v_b = -57 - 27.5 = -84.5\ \text{V}\]

\[p_{\text{in}} = -v_b(4.3i_3) = -(84.5)(4.3)(-1.1) = -399.685\ \text{W (dev)}\]

\[p_{\text{dev}} = -200(4.5) = -920\ \text{W (dev)}\]

\[\sum p_{\text{dev}} = 1319.685\ \text{W}\]

\[\sum p_{\text{in}} = (5.7)^210 + (1.1)^2(25) + (0.97)^2100 + (4.6)^2(10) + (3.83)^2(50)\]

\[= 1319.685\ \text{W}\]

\[\therefore \sum p_{\text{dev}} = \sum p_{\text{in}} - 1319.685\ \text{W}\]
The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 5 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 5 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node Voltage method, it is the preferred approach.

Node voltage equations:

\[
\frac{v_1}{100} + \frac{v_2}{250} - 0.2 + 3 \times 10^{-3}v_3 = 0
\]

\[
\frac{v_3}{500} + \frac{v_4}{200} - 3 \times 10^{-3}v_3 + 0.2 = 0
\]

Constraints:

\[v_3 - v_1 = 20; \quad v_4 - v_3 = 0.4v_3; \quad v_e = v_3\]

Solving, \(v_3 = 44\ V\)

\[i_e = 200 - 44/0.25 = 24\ mA\]

\[p_{ppv} = 20i_e = 480\ mW\ (abs)\]
Applying a source transformation to each current source yields

Now combine the 20 V and 10 V sources into a single voltage source and the 5 \(\Omega\), 4 \(\Omega\) and 1 \(\Omega\) resistors into a single resistor to get

Now use a source transformation on each voltage source, thus

which can be reduced to

\[ i_a = \frac{1.25 \times 8}{10} = 1 \text{ A} \]

50i_a - 40i_b = 20 - 10 - 10 = 0

\[ -40i_a + 42i_b = 10 \]

Solving, \( i_b = \frac{N_b}{\Delta} = 1 \text{ A} = i_e \)
OPEN CIRCUIT

\[ v_2 = -40i_b \times 10^3 = -16 \times 10^3i_b \]

\[ 5 \times 10^{-6} v_2 = 80i_b \]

\[ 580i_b + 5 \times 10^{-5} v_2 = 900i_b \]

\[ 100(540 \times 10^{-8}) = 54 \text{ mV} \]

\[ \therefore i_b = \frac{54 \times 10^{-3}}{1000} = 54 \mu A \]

\[ v_{th} = -16 \times 10^3(54 \times 10^{-6}) = -86.40 \text{ V} \]

SHORT CIRCUIT

\[ v_2 = 0; \quad i_c = -40i_b \]

\[ i_c = \frac{54 \times 10^{-3}}{1080} = \frac{54}{1.08} \times 10^{-4} = 50 \mu A \]

\[ i_c = -40(50) = -2000 \mu A = -2 \text{ mA} \]

\[ R_{th} = \frac{-86.4}{-2} \times 10^3 = 43.2 \text{ k}\Omega \]
P 4.61 After making a source transformation the circuit becomes

\[ 300 = 48i_1 - 40i_2 \]
\[-450 = -40i_1 + 200i_2 \]
\[
\therefore \quad i_1 = 5.25 \text{ A and } i_2 = -1.2 \text{ A}
\]
\[ v_{RB} = 8i_1 + 10i_2 = 30 \text{ V} \]
\[ R_{RB} = (40)(8 + 10)(50) = 15 \Omega \]

P 4.65 [a] Use source transformations to simplify the left side of the circuit.

\[ i_s = \frac{10 - 7.5}{2.5} = 1 \text{ mA} \]
Let \[ R_s = R_{source} \parallel 10 \Omega = 7.5/0.8 = 9.375 \Omega \]
\[
\therefore \quad \frac{R_{source}(10)}{R_{source} + 10} = 0.375, \quad R_{source} = \frac{(9.375)(10)}{0.375} = 150 \Omega
\]

[b] Actual value of \( i_s \):
\[ i_s = \frac{10}{2.5 + (0.8)(10)} = 0.9524 \text{ mA} \]
\[ v_s = 0.8i_s(10) = 7.62 \text{ V} \]
\[
\% \text{ error} = \left( \frac{7.5 - 7.62}{7.62} \right) \times 100 = -1.57\%
\]
\[ i_1 = \frac{45}{15} = 3 \text{ mA} \]

\[ 45 = v_{TH} - 3R_{TH}, \quad v_{FB} = 45 + 3R_{TH} \]

\[ i_2 = \frac{25}{5} = 5 \text{ mA} \]

\[ 25 = v_{TH} - 5R_{TH}, \quad v_{TH} = 25 + 5R_{TH} \]

\[ 45 + 3R_{TH} = 25 + 5R_{TH} \quad \text{so} \quad R_{TH} = 10 \text{ k}\Omega \]

\[ v_{TH} = 45 + 30 = 75 \text{ V} \]

P 4.68 [a] Find the Thévenin equivalent with respect to the terminals of the ammeter. This is most easily done by first finding the Thévenin with respect to the terminals of the 50\Omega resistor. Thévenin voltage: note \( i_a \) is zero.

\[ \frac{v_{TH}}{80} + \frac{v_{TH}}{40} + \frac{v_{TH}}{240} + \frac{v_{TH} - 40}{18} = 0 \]
Solving, \( v_{th} = 24 \) V Short-circuit current:

\[
i_{sc} = 2.5 \text{ + } 6i_{ac}, \quad \therefore \quad i_{sc} = -0.5 \text{ A}
\]

\[
R_{th} = \frac{24}{-0.5} = -48 \Omega
\]

\[
R_{local} = \frac{24}{10} = 2.4 \Omega
\]

\[
R_{source} = 2.4 - 2 = 0.40 \Omega
\]

[b] Actual current:

\[
i_{actual} = \frac{24}{2} = 12 \text{ A}
\]

\[
\% \text{ error} = \frac{10 - 12}{12} \times 100 = -16.67\%
\]
We begin by finding the Thévenin equivalent with respect to $R_e$. After making a couple of source transformations, the circuit simplifies to

\[ i_\Delta = \frac{160 - 30i_\Delta}{50}; \quad i_\Delta = 2 \text{ A} \]

\[ v_{th} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100 \text{ V} \]

Using the test-source method to find the Thévenin resistance gives

\[ i_T = \frac{v_T}{30} + \frac{v_T - 30(-v_T/30)}{20} \]

\[ \frac{i_T}{v_T} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15} \]

\[ R_{th} = \frac{v_T}{i_T} = \frac{15}{2} = 7.5 \Omega \]

Thus our problem is reduced to analyzing the circuit shown below.

\[ 7.5 \Omega \]
\[ p = \left( \frac{100}{7.5 + R_o} \right)^2 \quad R_o = 250 \]

\[ \frac{10^4}{R_o^2 + 15R_o + 56.25} \quad R_o = 250 \]

\[ \frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25 \]

\[ 40R_o = R_o^2 + 15R_o + 56.25 \]

\[ R_o^2 - 25R_o + 56.25 = 0 \]

\[ R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10 \]

\[ R_o = 22.5 \Omega \]

\[ R_o = 2.5 \Omega \]

P 4.31 [a] Open circuit voltage

![Circuit Diagram]
Node voltage equation:
\[
\frac{v_1}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - v_2}{4} = 0
\]

Constraint equations:
\[
i_\Delta = \frac{100 - v_1}{2} \quad \frac{v_2 - v_1}{4} - v_\Delta = 0 \quad v_\Delta = v_1 - v_2
\]

Solving, \( v_2 = 90 \text{ V} = v_{Th} \)

Short circuit current:

\[
\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1}{4} = 0
\]

\[
i_\Delta = \frac{100 - v_1}{2}
\]

Solving, \( v_1 = 80 \text{ V} = v_\Delta \)

\[
i_{sc} = \frac{v_1}{4} + v_\Delta = 20 + 80 = 100 \text{ A}
\]

\[
R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{90}{100} = 0.9 \Omega
\]

\( \therefore R_e = R_{Th} = 0.9 \Omega \)

\[p_{max} = \frac{(45)^2}{0.9} = 2250 \text{ W}\]
\[ \frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - 45}{4} = 0 \]

\[ i_\Delta = \frac{100 - v_3}{2} \]

Solving, \( v_1 = 85 \) V; \( i_\Delta = 7.5 \) A; \( v_\Delta = v_1 - v_2 = 85 - 45 = 40 \) V

\[ i_{\text{loop}} = i_\Delta = 7.5 \text{ A} \]

\[ P_{\text{loop}} \ (\text{dev}) = 100(7.5) = 750 \text{ W} \]

\[ i_{12} = \frac{v_\Delta}{4} = \frac{40}{4} = 10 \text{ A} \]

\[ i_1 = i_{12} - i_\Delta - 10 - 7.5 = 2.5 \text{ A} \]

\[ P_{3\Delta} \ (\text{dev}) = (97.5)(2.5) = 243.75 \text{ W} \]

\[ P_{4\delta} \ (\text{dev}) = (45)(40) = 1800 \text{ W} \]

\[ \sum P_{\text{loop}} = 750 + 243.75 + 1800 = 2793.75 \text{ W} \]

\[ \% \text{ delivered} = \frac{2250}{2793.75} \times 100 = 80.54\% \]

---

P 4.91 Voltage source acting alone:

\[ \frac{v_{bl}}{20} + \frac{v_{bl} - 35}{5} - 5 \left( \frac{35 - v_{bl}}{5} \right) = 0 \]

\[ v_{bl} = 33.6 \text{ V} \]
Current source acting alone:

\[ \frac{v_{e2}}{20} + \frac{v_{e1}}{5} = 5 \left( \frac{-v_{e2}}{5} \right) = 0 \]

\[ \therefore v_{e2} = -5.6 \text{ V} \]

\[ v_o = v_{e1} + v_{e2} = 33.6 - 5.6 = 28 \text{ V} \]

P 4.92 [a] By hypothesis \( i'_o + i''_o = 1.5 \text{ mA} \).

\[ i''_o = 10 \left( \frac{2}{20} \right) = 1 \text{ mA}; \quad \therefore i_o = 1.5 + 1 = 2.5 \text{ mA} \]

[b] With all three sources in the circuit write a single node voltage equation:

\[ \frac{v_b}{18} + \frac{v_b - 20}{2} - 5 - 10 = 0 \]

\[ \therefore v_b = 45 \text{ V} \]

\[ i_o = \frac{v_b}{18} = 2.5 \text{ mA} \]

P 4.95 [a] In studying the circuit in Fig. P4.95 we note it contains six meshes and six essential nodes. Further study shows that by replacing the parallel resistors with their equivalent values the circuit reduces to four meshes and four essential nodes as shown in the following diagram.

The node Voltage approach will require solving three node Voltage equations along with equations involving \( v_A \) and \( i_e \).

The mesh-current approach will require writing one supermesh equation plus three constraint equations involving the three current sources. Thus at the outset we know the supermesh equation can be reduced to a single unknown current. Since we are interested in the power developed by the 1 V source, we will
retain the mesh current $i_b$ and eliminate the mesh currents $i_a$, $i_c$
And $i_d$.
The supermesh is denoted by the dashed line in the following figure.

[b] Summing the voltages around the supermesh yields

\[-9i_b + \frac{4}{3}i_a + 0.75i_b + 1 + 5i_b + 7(i_c - i_d) + 8i_c = 0\]

Note that $i_d = i_b$. And multiply the equation by 12:

\[-108i_b + 16i_a + 9i_b + 12 + 60i_b + 84(i_c - i_d) + 96i_c = 0\]
or

\[16i_a - 39i_b + 180i_c - 84i_d = -12\]

Now note:

\[i_b - i_c = 3i_b = 3i_b; \quad i_c = -2i_b\]

whence

\[16i_a - 39i_b - 360i_c - 84i_d = -12\]

Now use the constraint that

\[i_a = i_c = -2\]
\[i_a = -2 + i_c = -2 - 2i_b\]

Therefore

\[-32 - 32i_b - 399i_b - 84i_d = -12\]
\[-431i_b - 84i_d = 20\]

Now use the constraint

\[i_d = -6i_a = -6\left(\frac{4}{3}i_a\right) = 8i_a = -16 - 16i_b\]

Therefore

\[-431i_b - 84(-16 - 16i_b) = 20\]
or

\[913i_b = -1324\]

∴ $i_b \approx -1.45$ A

$p_{IV} = 1i_b \approx -1.45$ W; ∴ $p_{IV}$ (developed) ≈ 1.45 W