4.6 Laplace Transform MATLAB Laboratory Experiment

Purpose: This experiment presents the frequency domain analysis of continuous-time linear systems using MATLAB. The impulse, step, sinusoidal, and exponential responses of continuous-time systems will be examined using the transfer function method based on the Laplace transform. In addition, MATLAB will be used to perform the partial fraction expansion and to find the inverse Laplace transform.

Part 1. Consider the linear system represented by the transfer function

$$H(s) = \frac{s+1}{s^2+5s+6}$$

Using MATLAB, find and plot:

(a) The system impulse response.

(b) The system step response.

(c) The system zero-state response due to the input signal $f(t) = \sin(2t)u(t)$.

(d) The system zero-state response due to the input signal $f(t) = e^{-t}u(t)$.

Part 2. Consider the transfer function

$$H(s) = \frac{2s^5+s^3-3s^2+s+4}{5s^8+2s^7-s^6-3s^5+5s^4+2s^3-4s^2+2s-1}$$

(a) Find the factored form of the transfer function by using the MATLAB function `[z,p,k]=tf2zp(num,den)`.

(b) The partial fraction expansion of rational functions can be performed using the MATLAB function `residue`. Find the Laplace inverse of the given transfer function using the MATLAB function `residue`; that is, find analytically the system impulse response.

Part 3. Consider the system defined by

$$y^{(2)}(t) + 5y^{(1)}(t) + 4y(t) = f(t)$$

and the input signal represented in Figure 4.13. Use MATLAB to plot the zero-state response of this system. (*Hint:* See Example 4.24.)

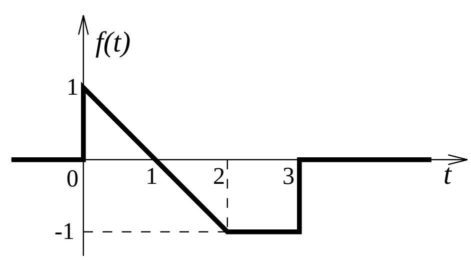

FIGURE 4.13: An input signal

Part 4. Find and plot the zero-input response of a flexible beam [9], whose transfer function is given by

$$H(s) = \frac{1.65s^4-0.331s^3-576s^2+90.6s+19080}{s^6+0.996s^5+463s^4+97.8s^3+12131s^2+8.11s}$$

with the initial conditions $y^{(4)}(0^-) = 1,\ y^{(j)}(0^-) = 0,\ j = 0,1,2,3,5$. (*Hint:* Find $I(s)$ and $\Delta(s)$ as defined in formulas (4.36) and (4.52) and use the MATLAB function `impulse`.)

Submit four plots for Part 1, one plot for Part 3, and one plot for Part 4, and present analytical results obtained in Parts 2–4.

SUPPLEMENT:

$$
\begin{aligned}
I(s) = &\Big(a_1 y(0^-) + a_2 y^{(1)}(0^-) + \cdots + a_{n-1} y^{(n-2)}(0^-) + y^{(n-1)}(0^-)\Big) \\
&+ s\Big(a_2 y(0^-) + a_3 y^{(1)}(0^-) + \cdots + a_{n-1} y^{(n-3)}(0^-) + y^{(n-2)}(0^-)\Big) \\
&+ s^2\Big(a_3 y(0^-) + a_4 y^{(1)}(0^-) + \cdots + a_{n-1} y^{(n-4)}(0^-) + y^{(n-3)}(0^-)\Big) \\
&+ \cdots + s^{n-2}\Big(a_{n-1} y(0^-) + y^{(1)}(0^-)\Big) + s^{n-1} y(0^-)
\end{aligned}
\tag{4.36}
$$

$$
\Delta(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 \tag{4.52}
$$

MATLAB Solution Program

```
% Experiment 4
%
% PART 1
%
num=[1 1]; den=[1 5 6];
% (a) Impulse response
t=0:0.01:5; h=impulse(num,den,t);
%
figure (1)
plot(t,h); grid; xlabel('Time [s]'); ylabel('Impulse response')
print -deps figure4_1.eps
% (b) Step response
ystep=step(num,den,t);
%
figure (2)
plot(t,ystep); grid; xlabel('Time [s]'); ylabel('Step response')
print -deps figure4_2.eps
% (c) Sinusiodal zero-state response
time=0:0.01:10; f=sin(2*time); yzs=lsim(num,den,f,time);
%
figure (3)
plot(time,yzs); xlabel('Time [s]'); ylabel('Sinusoidal zero-state response');
grid; print -deps figure4_3.eps
% (d) Exponential zero-state response
f=exp(-t); yzs=lsim(num,den,f,t);
%
figure (4)
plot(t,yzs); xlabel('Time [s]'); ylabel('Exponential zero-state response');
grid; print -deps figure4_4.eps
%
% PART 2
%
% (a) Transfer function factored form
num=[2 0 1 -3 1 4]; den=[5 2 -1 -3 5 2 -4 2 -1]; [z,p,k]=tf2zp(num,den)
% H(s)=k*((s-z(1))*(s-z(2))*...*(s-z(5)))/((s-p(1))*(s-p(2))*...*(s-p(8)))
% (b) Transfer function partial fraction form
```

```
[R,p,K]=residue(num,den);
% H(s)=num(s)/den(s)=R(1)/(s-p(1))+R(2)/(s-p(2))+...+R(8)/(s-p(8))+K(s)
R
K
r1=abs(R(1))
r3=R(3)
r4=abs(R(4))
r6=R(6)
r7=abs(R(7))
phi1=angle(R(1))
phi4=angle(R(4))
phi7=angle(R(7))
h=2*r1*exp(real(p(1))*t).*cos(imag(p(1))*t+phi1)+r3*exp(p(3)*t)...
+2*r4*exp(real(p(4))*t).*cos(imag(p(4))*t+phi4)+r6*exp(p(6)*t)...
+2*r7*exp(real(p(7))*t).*cos(imag(p(7))*t+phi7);
%
figure (5)
plot(t,h); grid; xlabel('Time [s]'); ylabel('Impulse response')
print -deps figure4_5.eps
%
% PART 3
%
num=1; den=[1 5 4]; dens=[1 5 4 0]; denr=[1 5 4 0 0];
ystep=step(num,den,time); % also ystep=impulse(num,dens)
yramp=lsim(num,den,time,time); % also yramp=impulse(num,denr)
ystepshift=[zeros(300,1); ystep(1:701)];
yrampshift=[zeros(200,1); yramp(1:801)];
y=ystep-yramp+ystepshift+yrampshift;
%
figure (6)
plot(time,y); grid; xlabel('Time [s]'); ylabel('Zero-state response')
print -deps figure4_6.eps
%
% PART 4
%
num=[1.65 -0.331 -576 90.6 19080]; den=[1 0.996 463 97.8 12131 8.11 0];
% using (4.36) we have
t=0:1:10000; y4th0=1; a5=den(2); numI=[1 a5*y4th0];
yzi=impulse(numI,den,t);
%
figure (7)
plot(t,yzi); grid; xlabel('Time [s]'); ylabel('Zero-input response')
print -deps figure4_7.eps
%
figure (8)
yzi=impulse(numI,den,time);
plot(time,yzi); grid; xlabel('Time [s]'); ylabel('Zero-input response');
axis([0 10 0 0.001]); print -deps figure4_8.eps
```

MATLAB Results

```
>>
z =
-0.5462 + 1.3017i
```

```
-0.5462 - 1.3017i
0.9509 + 0.5795i
0.9509 - 0.5795i
-0.8094
p =
-0.8144 + 0.9415i
-0.8144 - 0.9415i
-0.9718
0.6412 + 0.6253i
0.6412 - 0.6253i
0.6734
0.1225 + 0.4805i
0.1225 - 0.4805i
k = 0.4000
R =
-0.0875 - 0.0188i
-0.0875 + 0.0188i
0.0767
0.2652 + 0.0400i
0.2652 - 0.0400i
0.7293
-0.5807 + 0.7724i
-0.5807 - 0.7724i
K = []
r1 = 0.0895
r3 = 0.0767
r4 = 0.2682
r6 = 0.7293
r7 = 0.9663
phi1 = -2.9303
phi4 = 0.1496
phi7 = 2.2154
>>
```

Figures Generated by the MATLAB Solution Program

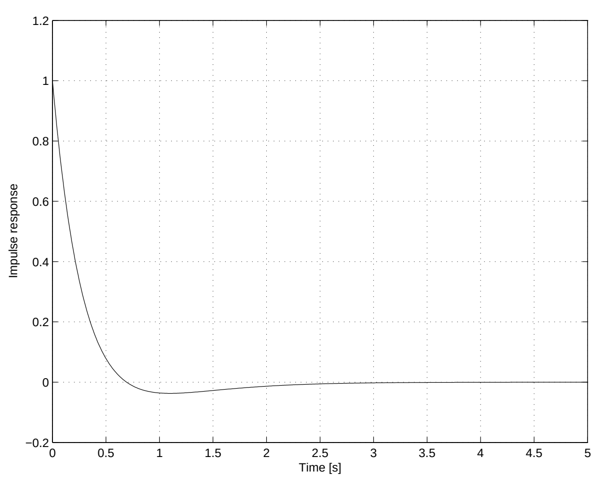

Figure 4.1

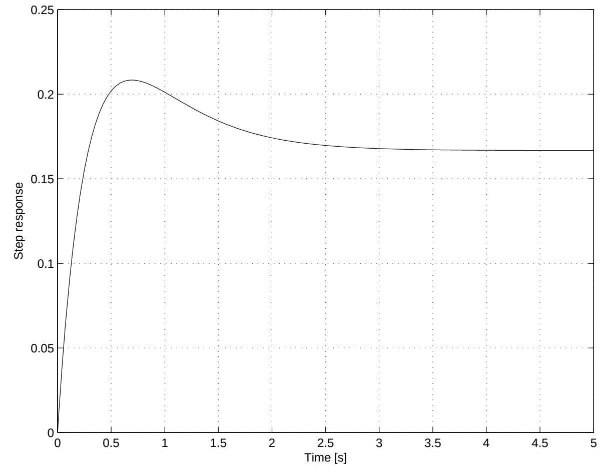

Figure 4.2

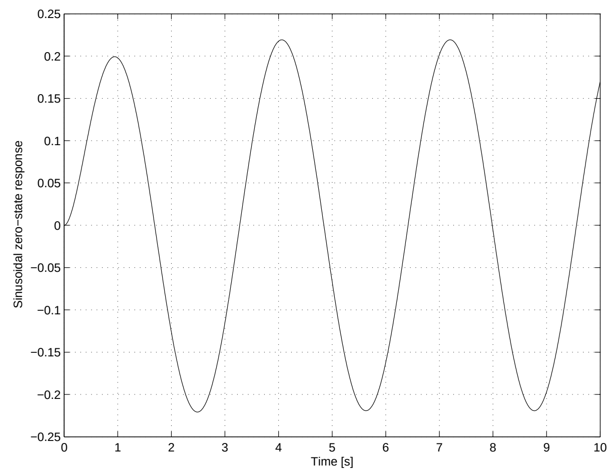

Figure 4.3

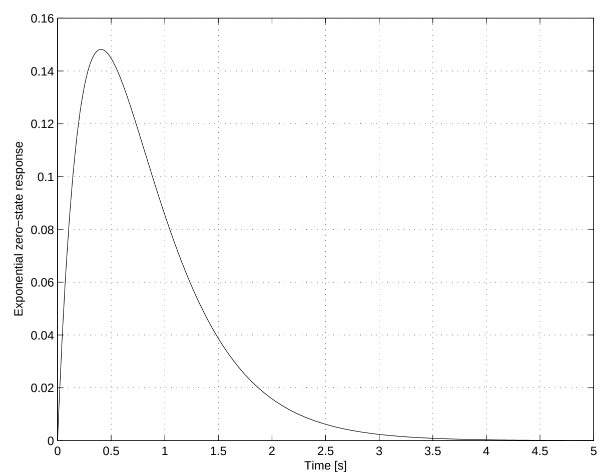

Figure 4.4

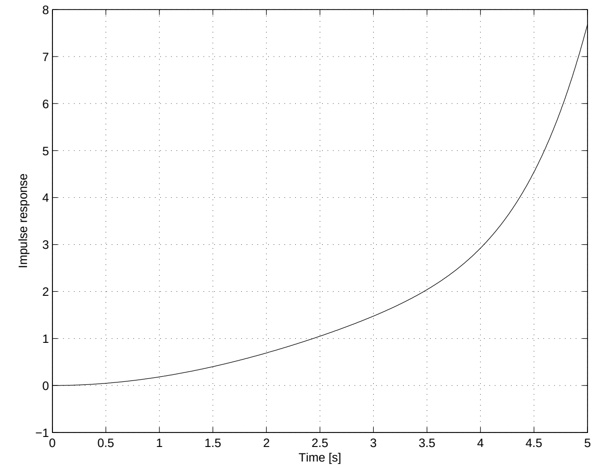

Figure 4.5

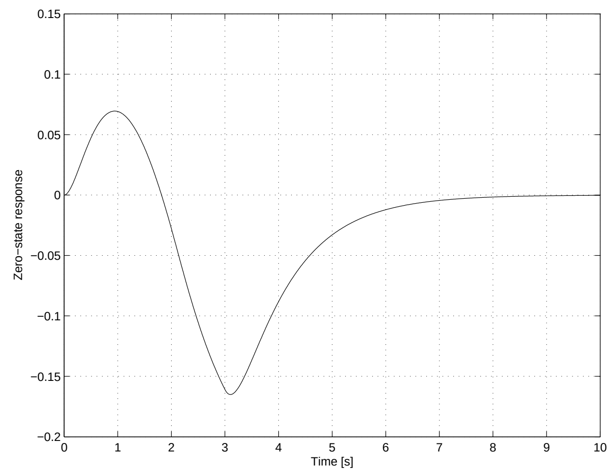

Figure 4.6

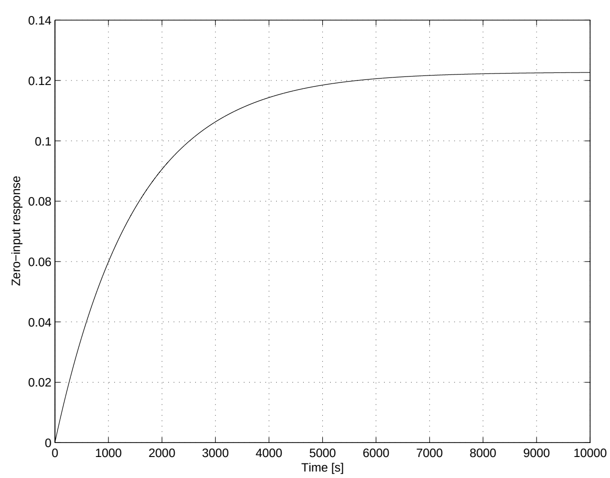

Figure 4.7

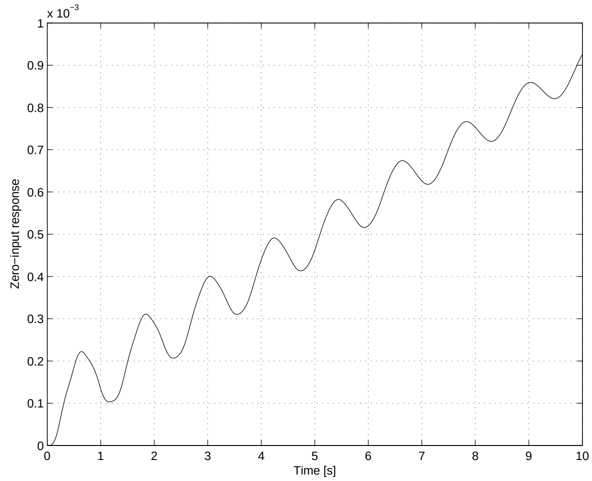

Figure 4.8

Note a different range for the time axis in Figure 4.8 comparing to Figure 4.7. Both figures represent the same signal, but in the different time spans.

Analytical Results

Part 2. (b) The system poles obtained with the MATLAB are

$$p_{1,2} = \alpha_1 \pm j\beta_1,\ p_3 = \alpha_3,\ p_{4,5} = \alpha_4 \pm j\beta_4,\ p_6 = \alpha_6,\ p_{7,8} = \alpha_7 \pm j\beta_7$$

with the corresponding numerical values given previously in MATLAB Results section. The analytical expression for the impulse response is

$$H(s) = \frac{2s^5 + s^3 - 2s^2 + s + 4}{5s^8 + 2s^7 - s^6 - 3s^5 + 5s^4 + 2s^3 - 4s^2 + 2s + 1} = \frac{R(1)}{s - p(1)} + \frac{R^*(1)}{s - p^*(1)}$$

$$+\frac{R(3)}{s - p(3)} + \frac{R(4)}{s - p(4)} + \frac{R^*(4)}{s - p^*(4)} + \frac{R(6)}{s - p(6)} + \frac{R(7)}{s - p(7)} + \frac{R^*(7)}{s - p^*(7)}$$

$$\mathcal{L}^{-1}\{H(s)\} = h(t) = 2|R(1)|e^{\alpha_1 t}\cos\left(\beta_1 t + \measuredangle R(1)\right) + R(3)e^{\alpha_3 t}$$

$$+2|R(4)|e^{\alpha_4 t}\cos\left(\beta_4 t + \measuredangle R(4)\right) + R(6)e^{\alpha_6 t} + 2|R(7)|e^{\alpha_7 t}\cos\left(\beta_7 t + \measuredangle R(7)\right)$$

$$= 2 \times 0.0895e^{-0.8144t}\cos\left(0.9415t - 2.9303\right) + 0.0767e^{-0.9718t}$$

$$+2 \times 0.2682e^{0.6412t}\cos\left(0.6253t + 0.1496\right) + 0.7293e^{0.6734t}$$

$$+2 \times 0.9663e^{0.1225t}\cos\left(0.4805t + 2.2154\right)$$

Note that the used notation is consistent with the MATLAB program and the results obtained. All angles in the above formula are expressed in radians.

Part 3. Using the linearity principle we have

$$f(t) = u(t) - r(t) + r(t-2) + u(t-3) \quad \Rightarrow \quad y(t) = y_{step}(t) - y_{ramp}(t) + y_{ramp}(t-2) + y_{step}(t-3)$$

Part 4. From formula (4.36), we have for $n = 6$ and the given initial conditions

$$I(s) = a_5 y^{(4)}(0^-) + s y^{(4)}(0^-) = s + 0.996 \quad \Rightarrow \quad Y_{zi}(s) = \frac{s + 0.996}{\Delta(s)}$$

Using formula (4.55), we obtain

$$y_{zi}(t) = \mathcal{L}^{-1}\{Y_{zi}(s)\} = \mathcal{L}^{-1}\left\{\frac{s + 0.996}{s^6 + 0.996s^5 + 463s^4 + 97.8s^3 + 12131s^2 + 8.11s}\right\}$$