332: 416 — Project #4

Due Monday, April 26, 2004

Control of Nonlinear Systems with Application to a Robot Arm and a Single-Link Robotic Manipulator with a Flexible Joint

Part I. Consider the motor driven robot arm described by

$$\dot{\theta} = \omega$$
$$\dot{\omega} = -\alpha\omega - \Omega^2 \sin(\theta) + \beta u$$

with the numerical data given by

$$\alpha = 5, \quad \beta = 1, \quad \Omega^2 = 64$$

It is desired to control the robot arm to hold any reference (nominal, operating, trim, set) angle in the entire range $[0 \quad 2\pi]$.

(a) Design a full-state feedback control law for this control system using linearization about a set point. As a controller use the linear-quadratic optimal regulator from the corresponding SIMULINK library with unity penalty matrices. See the block diagram (a) of the handout on reserve reading in SEC library.

Simulate the performance of the system starting in the state $\theta = 0, \dot{\theta} = 0$ for the desired state $\theta = \pi/4, \quad \omega = \dot{\theta} = 0$. Draw the response for $\theta(t)$, $\omega(t) = \dot{\theta}(t)$, and $u(t)$—the actual control signal applied to the robot arm. Consider the time interval of $[0, 10]$ seconds.

(b) Use the PID controller from the corresponding SIMULINK library for the same problem. Note that the PID controller requires only one input, which is the actual system output, in this case the state variable $\theta(t)$.

Like in part (a) simulate the performance of the system starting in the state $\theta = 0, \dot{\theta} = 0$ for the desired state $\theta = \pi/4, \quad \omega = \dot{\theta} = 0$ for the following three choices of the PID controller parameters

$$K_P = 1, \quad K_D = 1, \quad K_I = 1$$
$$K_P = 1, \quad K_D = 5, \quad K_I = 1$$
$$K_P = 1, \quad K_D = 10, \quad K_I = 1$$

Note that our goal is to improve the transient response.

For the above three cases draw the response for $\theta(t)$, $\omega(t) = \dot{\theta}(t)$, and $u(t)$—the actual control signal applied to the robot arm. Consider the time interval of $[0, 10]$ seconds.

Comment on the transient responses obtained in parts (a) and (b).

Hint: See the handout for this project and the corresponding SIMULINK block diagrams.

Part II. Consider a single-link robotic manipulator with a flexible joint whose nonlinear model is defined in Example 1.3 of the textbook. Use the technique from Part Ia to design a controller that keeps the manipulator's link at steady state at $\theta_{1ss} = \pi/6$. Assume that the weighting matrices are $R_1 = I_4$ and $R_2 = 1$. Simulate the system response using SIMULINK and plot $\theta_1(t)$ as a function of time until it settles down to its steady state value. Comment on the result obtained.