332: 418 — Project #4 — Spring 2005

Due Wednesday, May 11, 2005

Control of a Nonlinear Robot Arm via the Linearization Technique

Consider the motor driven robot arm described by [1] (the fourth-order robot arm with a flexible joint model can be found in your textbook on page 29)

$$\ddot{\theta}(t) = -\alpha\dot{\theta} - \Omega^2 \sin(\theta) + \beta u$$

with the numerical data given by

$$\alpha = 5$$
, $\beta = 1$, $\Omega^2 = 64$

It is desired to control the robot arm to hold any reference angle in the entire range $\begin{bmatrix} 0 & 2\pi \end{bmatrix}$.

- a) Design the set-point (steady state) controller for the robot arm to achieve the desired angle $(\theta = \theta_{desired} = \pi/4)$ with no angular speed $(\theta_{desired} = 0)$ using the linearization technique about given nominal (steady state, operating, set) points and build the corresponding Simulink block diagram. *Hint:* You must brush up the first seven pages of the handout on linearization of nonlinear systems (posted on the class web site and WebCT). See also the Simulink handout posted on WebCT. In addition, you will probably need to consult the class notes of April 21st and/or the handout from [1].
- b) Design any linear controller to improve the transient response (the settling time first of all and the overshoot). Search for a controller that facilitates the shortest settling time and the smallest overshoot. *Hint:* First find the transfer function of the linearized model at the nominal (steady state) points $\theta_{ss} = \pi/4$, $\dot{\theta}_{ss} = 0$ and then use any of the controller design algorithms from Chapters 8 and 9.

Simulate the performance of the controlled system starting in the state $\theta=0, \theta=0$. Originally consider the time interval of [0,500] seconds, for which the WebCT Simulink block diagram with the pure integral controller shows the convergence to the desired steady state value in more than 300 seconds (pure performance). To get credits for this project your controller must be at least better than the pure integral controller. Note that if you find the best controller, you will need to plot the response only in the time interval [0,10]. Replace the pure integral controller by your controller and plot the response for $\theta(t)$, $\omega(t)=\theta(t)$, and u(t)—the actual control signal applied to the robot arm. Estimate the overshoot and the settling time and comment on the transient response obtained (of course the steady state error must be zero).

[1] B. Friedman, Advanced Control System Design, Prentice Hall, 1996.