Project # 3 — 332: 406 Control System Design
Optimal Control and Kalman Filtering for a Passenger Car

Project due Thursday April 1, 2004

A mathematical model of a passenger car is given by (Salman, Lee, Boustany, General Motors Research Labs, *Transactions of ASME J. on Dynamics Measurements, and Control*, 604–610, 1990)

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0.8755 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1.79 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 60.435 & 0 & 0 \\ 0 & 0 & 0 & 0 & -60.435 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 60.435 \\ 0 & 0 & 0 & 0 & 0 & 0 & -60.435 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1.7021 & -1.7021 \\ -0.0891 & -1.8219 \\ 0 & 0 \\ 20.812 & 0 \\ 0 & 0 \\ 0 & 20.812 \end{bmatrix} \tag{2}$$

$$C = \begin{bmatrix} 29.814 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 29.814 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 14.2857 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 14.2857 \end{bmatrix}$$

a) Using MATLAB examine controllability and observability of this control system.

b) By following presentation in Section 10.3.1 (GL96) and using MATLAB (see Example 10.2) find the optimal feedback gain such that a quadratic performance criterion with

$$R_1 = C^T C, \quad R_2 = \begin{bmatrix} 73.826 & 25.475 \\ 25.475 & 124.33 \end{bmatrix} \tag{3}$$

is minimized. Find the optimal value of the performance criterion as defined in (10.34, GL96) assuming that the initial state is given by $\mathbf{x}(t_0) = [1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1]^T$. Determine the closed-loop system eigenvalues.

c) Design an observer for this car with the observer poles (eigenvalues) being much faster than the system optimal closed-loop poles obtained in part (b). In that direction, for the desired observer eigenvalues find the observer gain through the pole placement technique by using the MATLAB function `place` as `KT=place(A',C',evdesired); K=KT'`.

d1) Use SIMULINK and build the block diagram for the system-observer configuration using the numerical values previously obtained. Plot the difference between the system and observer outputs. *Hint:* Handout on observer implementation using SIMULINK (see also pages 21–22 of the Observers file posted on the class homepage).

d2) Use SIMULINK and build the block diagram for the system-reduced-order-observer configuration. Plot the difference between the system and observer outputs. *Hint:* See pages 39–40 of the Observers file posted on the class homepage.

e) Assume that the car dynamic equation and its measurements are corrupted by Gaussian white noise stochastic processes (unevenness of the road disturbances) whose intensities are $W = 0.00235 I_2$, $V = 0.1 I_4$, where I_2 and I_4 are identity matrices of corresponding dimensions. Also assume that the system noise matrix is equal to the system input matrix ($G = B$), that is

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Bw(t) \\ y(t) &= Cx(t) + v(t) \end{aligned} \tag{4}$$

Design the corresponding Kalman filter. Find the variance matrix of the estimation error by solving the filter algebraic Riccati equation. From the result obtained read the estimation errors for the car's front and rear wheel velocities, respectively represented the state variables x_6 and x_8. See Example 10.3 (GL96).