Project # 2 — 332:416 Control System Design
A Pitch Controller for a BOEING Aircraft
Project due: Thursday, March 2, 2006

PART A: SIMULATION (10 points)

The linearized equations governing the motion of a BOEING's commercial aircraft are given by (Messner and Tilbury, *Control Tutorials for MATLAB and Simulink*, Addison Wesley, 1998)[1]

$$\frac{d\alpha(t)}{dt} = -0.313\alpha(t) + 56.7q(t) + 0.232\delta_e(t)$$
$$\frac{dq(t)}{dt} = -0.0139\alpha(t) - 0.426q(t) + 0.0203\delta_e(t) \tag{1}$$
$$\frac{d\theta(t)}{dt} = 56.7q(t)$$

where $\theta(t)$ represents the pitch angle. The corresponding open-loop transfer function obtained from (1) is given by

$$\frac{\Theta(s)}{\Delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} = \frac{1.151(s + 0.1541)}{s(s^2 + 0.739s + 0.921)} = 1.151G(s) \tag{2}$$

In this project we design an autopilot that controls the pitch angle $\theta(t)$ of this aircraft. The autopilot is obtained by forming the closed-loop system with a unity feedback and a controller of the form $KG_c(s)$. For simplicity we assume that $K = 1.151K'$ so that the open-loop feedback transfer function is $KG_c(s)G(s)$.

(a) Find the steady state error due to a unit ramp input of the original ($K = 1, G_c(s) = 1$) closed-loop system ($G(s)/1 + G(s)$). Plot the closed-loop system ramp response and observe (check) the corresponding steady state error. Hint: In order to find the ramp response use the MATLAB function `y=lsim(cnum,cden,t,t)` with `t=0:0.1:30`.

(b) Find the value for the static gain K such that the steady state error due to the unit ramp is at most 10% ($e_{ss}^{ramp} = 0.1$). For the obtained value of K plot the corresponding closed-loop system ramp response and notice the steady state error improvement. Hint: Use the same time range as in part (a).

(c) For the obtained value of K find the phase and gain stability margins and observe that the phase margin is pretty pure. Design the phase-lead controller $G_c(s)$ to improve the phase stability margin such that the compensated system has the phase stability margin close to 50°. Find the step response of the compensated system and compare it to the step response of the uncompensated system whose static gain K is found in part (b). Comment on the transient response improvement of the compensated system. Hint: In order to be able to estimate the value for ω_{wax} use the following frequency range `w=0.1:0.1:100` with `bode(K*num,den,w)`. MATLAB will produce, for this particular example, the Bode plot in the frequency range up to $10\,\text{rad/s}$. However, in the formulated design problem ω_{wax} is greater than $10\,\text{rad/s}$.

(d) Design the phase-lag controller to satisfy the stability requirement imposed in (c). Find the step response of the system compensated (controlled) by the phase-lag controller and compare it to the step response of the system compensated by the phase-lead controller. Which one has a smaller rise time? Which one do you prefer?

(e) Using the SIMULINK package, build the block diagrams for the system controlled by phase-lead and phase-lag controllers, plot the step responses in both cases, and confirm the results obtained in Parts (c) and (d).

Hint: Use and appropriately modify MATLAB programs for Examples 9.4 and 9.5.

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[1] Note that in the second part of this course we will study the so-called linearization procedure that produces a set of linear differential equations from a set of nonlinear differential equations.

Part B: CONTROLLER ELECTRONIC BOARD DESIGN (10 points)

Design an electronic board for the best possible phase-lead controller obtained in Part A. Use operational amplifiers, resistors, and capacitors. Test the controller using the model for a the BOEING aircraft dynamics. Note that you have to build an electronic simulator for the lateral aircraft dynamics using operational amplifiers, resistors and capacitors.

RECOMMENDATION

Use the red wire for + (OpAmps' pin 7), green wire for – (OpAmps' pin 4), black wire for ground (OpAmps' pin 3), and blue wire for signals. Organize your board neatly. This will make the board testing easy.

TESTING STEP:

1. Check the circuit once more to see that all components are present and properly connected.
2. Use the square wave of magnitude one and large period to imitate the step reference input, and a train of triangular pulses (or the sawtooth wave) for the ramp signal.
3. Check the power supply voltages for OpAmps. Pin 4 should be $-15\,\mathrm{V}$ and pin 7 should be $+15\,\mathrm{V}$.
4. Check that the amplifiers are not saturated (saturating voltage between pins 2 and 3 is less than $1\,\mathrm{mV}$ (for OpAmp 741 used in this design).
5. Be sure to use the amplifier with the approximative derivative (the output impedance is a parallel combination of a resistor and inductor with a large time constant (CR small)) instead of the pure derivative.
6. Break connection between the controller and the system model and check which one is not performing properly.
7. Test your controller by using a triangular wave input and observing performances of the integrator, differentiator, multiplicator, and summing amplifiers. In that process open the feedback loop.
8. If necessary, run simulation in PSPICE.

REMARK 1:

Sometimes pure integrators saturate (integrators sum the area between the signal and the horizontal axis). Small errors in the input signal over long period of time are integrated into large errors in the output signal, which can cause amplifier saturation. To avoid this problem use the non-saturating integrator obtained by placing in parallel with the output capacitor a resistor such that the time constant is large (CR small)

REMARK 2:

Note that the resistors and capacitors used have accuracy of respectively 5% and 20%, and that their values are slightly off from the values used in the simulation phase. Despite to those inaccuracies, due to the feedback structure of your controller you will be able to achieve excellent results and the controlled system output will be very close to the one obtained in simulation.

PROJECT REPORT

After the successful design of the electronic board observe on the oscilloscope the following signals: error, control, output signals. Plot these signals and include the plots in your report and comment on their values. Estimate the transient response parameters and the steady state errors and compare them to the corresponding values obtained in simulation. Your report should in addition contain the electronic scheme of the controller and system, discussion about the necessity to use the approximative derivative and the need to filter out noise to avoid amplifier saturation. Attach to the report one set of the MATLAB/Simulink simulation results obtained in Part A. Give the number of hours that you needed to complete simulation, design, and testing phases.

The team project reports (prepared as technical reports) for Part B are due on Monday, March 27, 2006.