Project # 2 — 332:416 Control System Design
A Pitch Controller for a BOEING Aircraft

Project due: Monday March 5, 2001

The linearized equations governing the motion of a BOEING's commercial aircraft are given by (Messner and Tilbury, 1998)[1]

$$\begin{aligned} \frac{d\alpha(t)}{dt} &= -0.313\alpha(t) + 56.7q(t) + 0.232\delta_e(t) \\ \frac{dq(t)}{dt} &= -0.0139\alpha(t) - 0.426q(t) + 0.0203\delta_e(t) \\ \frac{d\theta(t)}{dt} &= 56.7q(t) \end{aligned} \tag{1}$$

where $\theta(t)$ represents the pitch angle. The corresponding open-loop transfer function obtained from (1) is given by

$$\frac{\Theta(s)}{\Delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} = \frac{1.151(s + 0.1541)}{s(s^2 + 0.739s + 0.921)} = 1.151G(s) \tag{2}$$

In this project we design an autopilot that controls the pitch angle $\theta(t)$ of this aircraft. The autopilot is obtained by forming the closed-loop system with a unity feedback and a controller of the form $KG_c(s)$. For simplicity we assume that $K = 1.151K'$ so that the open-loop feedback transfer function is $KG_c(s)G(s)$.

(a) Find the steady state error due to a unit ramp input of the original ($K = 1, G_c(s) = 1$) closed-loop system ($G(s)/1 + G(s)$). Plot the closed-loop system ramp response and observe (check) the corresponding steady state error. Hint: In order to find the ramp response use the MATLAB function `y=lsim(cnum,cden,t,t)` with `t=0:0.1:30`.

(b) Find the value for the static gain K such that the steady state error due to the unit ramp is at most 10% ($e_{ss}^{ramp} = 0.1$). For the obtained value of K plot the corresponding closed-loop system ramp response and notice the steady state error improvement. Hint: Use the same time range as in part (a).

(c) For the obtained value of K find the phase and gain stability margins and observe that the phase margin is pretty pure. Design the phase-lead controller $G_c(s)$ to improve the phase stability margin such that the compensated system has the phase stability margin close to $50°$. Find the step response of the compensated system and compare it to the step response of the uncompensated system whose static gain K is found in part (b). Comment on the transient response improvement of the compensated system. Hint: In order to be able to estimate the value for ω_{wax} use the following frequency range `w=0.1:0.1:100` with `bode(K*num,den,w)`. MATLAB will produce, for this particular example, the Bode plot in the frequency range up to $10\,\mathrm{rad/s}$. However, in the formulated design problem ω_{wax} is greater than $10\,\mathrm{rad/s}$.

(d) Design the phase-lag controller to satisfy the stability requirement imposed in (c). Find the step response of the system compensated (controlled) by the phase-lag controller and compare it to the step response of the system compensated by the phase-lead controller. Which one has a smaller rise time? Which one do you prefer?

[1] Note that in the second part of this course we will study the so-called linearization procedure that produces a set of linear differential equations from a set of nonlinear differential equations.