Preface

This book is intended for engineers, mathematicians, physicists, and computer scientists interested in control theory and its applications. The book studies a special class of linear control systems known as singularly perturbed systems. These systems, characterized by the presence of slow and fast variables, describe dynamics of many real physical systems such as aircraft, power systems, nuclear reactors, chemical reactors, electrical circuits, dc and induction motors, robots, large space flexible structures, synchronous machines, cars, and so on. In general, all systems that have components of different physical nature (for example, electrical vs mechanical components) display slow-fast phenomena. Mathematically, the slow and fast phenomena are characterized by small and large time constants, or by system eigenvalues that are clustered into two disjoint sets. The slow system variables correspond to the set of the eigenvalues closer to the imaginary axis, and the fast system variables are represented by the set of the eigenvalues that are far from the imaginary axis.

Mathematical theory of singularly perturbed systems, also known as theory of differential equations with small parameters multiplying certain derivatives, originated in the papers of A. Tikhonov, J. Levin, and N. Levinson at the beginning of the 1950s and gained its maturity during the 1960s and 1970s in the works of A. Vasileva, V. Butuzov, W. Wasow, F. Hoppensteadt, R. O'Malley, K. Chang, and their coworkers. One of the most important results in mathematical theory of linear singularly perturbed systems is the development of the Chang transformation, which facilitates exact decomposition of singularly perturbed *linear systems* into pure-slow and pure-fast subsystems.

Singularly perturbed *control* systems became an extensive subject of research by the end of the 1960s and during the 1970s in the papers published

by P. Kokotovic and his graduate students, among whom P. Sannuti, J. Chow, H. Khalil, and D. Young were the most productive. A large number of journal papers on singularly perturbed control systems were published during the 1970s, 1980s, and 1990s in both mathematics and engineering. The approaches taken in engineering during the 1970s and 1980s were based on the expansion methods (power series, asymptotic expansions, Taylor series)—the methods developed by previously mentioned mathematicians. The approaches were in most cases accurate only with an $O(\epsilon)$ accuracy, where ϵ is a small positive singular perturbation parameter. Generating higher order expansions for those methods has been analytically cumbersome and numerically inefficient, especially for higher dimensional control systems. Even more, it has been demonstrated in the control literature that for some applications the $O(\epsilon)$ accuracy either is not sufficient or in some cases has not solved the considered singularly perturbed control problems.

The development of high accuracy efficient techniques for singularly perturbed control systems started in the middle of the 1980s along the lines of slow-fast integral manifold theory of E. Fridman, V. Sobolev, and V. Strygin, and the recursive approach based on fixed-point iterations of Z. Gajic. At the beginning of the 1990s, the fixed-point recursive approach culminated in the so-called Hamiltonian approach for the *exact* slow-fast decomposition of singularly perturbed, linear-quadratic, deterministic and stochastic, optimal control and filtering problems.

This book represents a comprehensive overview of the current state of knowledge of the Hamiltonian approach to singularly perturbed linear optimal control systems. The book devises a unique powerful method whose core result seems to be repeated and slightly modified over and over again, while the method solves more and more challenging problems of linear singularly perturbed optimal continuous- and discrete-time systems, including nonstandard singularly perturbed linear systems, high gain feedback and cheap control problems, small measurement noise problem, sampled data systems, and H_∞ optimization and filtering problems. It should be pointed out that some related problems still remain unsolved, especially corresponding problems in the discrete-time domain, and the optimization problems over a finite horizon. These problems are identified in the book as open problems for future research.

The presentation is based on the recent research work of the authors and their coworkers. The book presents a unified theme about the exact pure-slow pure-fast decoupling of the corresponding optimal control problems owing to the existence of a transformation that exactly decouples the nonlinear algebraic Riccati equation into the pure-slow and pure-fast, reduced-order, independent, algebraic Riccati equations. In that direction, we show how to study independently in slow and fast time scales with very high accuracy (theoretically with perfect accuracy) deterministic and stochastic, continuous- and discrete-time,

linear-quadratic optimal control and filtering problems. Some of the results presented appear for the first time in this book.

Each chapter is organized to represent an independent entity so that the readers interested in a particular class of linear singularly perturbed control systems can find complete information within the particular chapter. The book demonstrates theoretical results on many practical applications using examples from aerospace, chemical, electrical, and automotive industries. In that direction, we apply theoretical results obtained to optimal control and filtering problems represented by real mathematical models of aircraft, cars, power systems, chemical reactors, and so on.

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