

Example 1.3: Consider the mathematical model of a single-link robotic manipulator with a flexible joint (Spong and Vidyasagar, 1989)

$$I\ddot{\theta}_1 + mgl \sin \theta_1 + k(\theta_1 - \theta_2) = 0 \\ J\ddot{\theta}_2 - k(\theta_1 - \theta_2) = u$$

where θ_1, θ_2 are angular positions, I, J are moments of inertia, m and l are, respectively, the link's mass and length, and k is the link's spring constant. Introducing the change of variables as

$$x_1 = \theta_1, \quad x_2 = \dot{\theta}_1, \quad x_3 = \theta_2, \quad x_4 = \dot{\theta}_2$$

the manipulator's state space nonlinear model equivalent to (1.66) is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{mgl}{I} \sin x_1 - \frac{k}{I}(x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k}{J}(x_1 - x_3) + \frac{1}{J}u \end{aligned}$$

Take the nominal points as $(x_{1n}, x_{2n}, x_{3n}, x_{4n}, u_n)$, then the matrices \mathbf{A} and \mathbf{B} defined in (1.71) are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k+mgl \cos x_{1n}}{I} & 0 & \frac{k}{I} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J} & 0 & -\frac{k}{J} & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J} \end{bmatrix}$$

In Spong (1995) the following numerical values are used for system parameters: $mgl = 5$, $I = J = 1$, $k = 0.08$.

Assuming that the output variable is equal to the link's angular position, that is

$$y = x_1$$

the matrices \mathbf{C} and \mathbf{D} , defined in (1.76), are given by

$$\mathbf{C} = [1 \quad 0 \quad 0 \quad 0], \quad \mathbf{D} = 0$$