Potential Questions for the Final Exam — 332: 416 Control Systems Design

Wednesday, May 12, 2004, **12–2pm**, SEC **117**

1. Present discrete-time linear observers (CN, pages –2 to –1).
2. State and present the solution to the deterministic continuous-time linear optimal regulator problem (GL96, 441–443 *and* class notes=CN, page 0).
3. For a continuous-time linear stochastic system driven by a zero-mean stationary Gaussian stochastic white noise process derive the expression for the state mean. Define the state variance and write the state variance equation (GL96, pages 437–439).
4. Find the mean of state variables for a discrete-time system driven by white noise. Derive the variance equation of a discrete-time linear system driven by white noise (GL96, pages 439–440 and class notes).
5. Formulate the Kalman filtering problem, present its solution and draw the corresponding block diagram. (GL96, pages 444–447).
6. Explain the technique for linearization of nonlinear systems about nominal points (GL96, pages 22–28).
7. Describe the method for controlling nonlinear systems through linearization about set points and draw the corresponding block diagram (F96, pages 98–101, *distributed with* Project #4).
8. Define system equilibrium points (CN, pages 1–2).
9. Explain nonlinear phenomena called: finite escape time and existence of harmonics and subharmonics, and compare them to the corresponding phenomena of linear systems (CN, pages 2–3).
10. Present all possible shapes for state trajectories of second-order linear time invariant systems (CN, pages 4–7).
11. Define the limit cycle of nonlinear systems (CN, page 8).
12. Explain chaos as a phenomenon characterizing certain behavior of nonlinear systems (CN, pages 9–10).
13. State the first method of Lyapunov for examining stability of nonlinear systems by using the linearization technique. Apply it to a simple pendulum (CN, pages 12–13).
14. Define precisely stability in the sense of Lyapunov (CN, pages 10–11).
15. Present the second stability method of Lyapunov based on the existence of Lyapunov functions and demonstrate it on a simple pendulum equation (CN, pages 13–15).
16. Apply the second stability method of Lyapunov to a linear time invariant continuous-time system and derive the algebraic Lyapunov equation (GL96, 179–181, and Definition C.1 on page 475).
17. Apply the second stability method of Lyapunov to a linear time invariant discrete-time system and derive the algebraic Lyapunov equation (GL96, 182–183, and Definition C.1 on page 475).
18. Derive the least-square formula for identification of coefficients of discrete-time time-invariant n-th order linear systems (GL96, pages 450–452).
19. Explain the describing function method for studying periodicity (limit cycles) of linear systems with static nonlinearities (F96, pages 57–62, *distributed in class*).
20. Define the characteristic equation for linear systems with static nonlinearities and demonstrate on a very simple example how to find the frequency and magnitude of oscillations (CN, page 16).
21. Using the Nyquist diagram explain the method for examining stability of limit cycles (CN, pages 17–18).
22. Explain the passivity property of linear electrical networks containing a nonlinear resistor and explain how to use this property for studying stability of control systems (CN, pages 19–21).
23. Present the extended linearization technique for control of nonlinear systems (CN, pages 22–23).
24. Present the feedback linearization technique for control of nonlinear systems (CN, pages 24–25).
25. Present observers for nonlinear systems and draw the corresponding block diagram (CN, pages 26–28).
26. Present the extended (nonlinear) Kalman filter and draw the corresponding block diagram (CN, pages 30–31, *disctributed in class*).

Variance Derivations in Question #4, see also (10.25)

$$x(k+1) = Ax(k) + Gw(k)$$
$$m(k+1) = Am(k)$$

Subtracting the second equation from the first one we have

$$x(k+1) - m(k+1) = A(x(k) - m(k)) + Gw(k)$$

The variance is defined by

$$Q(k) = E\Big\{(x(k) - m(k))(x(k) - m(k))^T\Big\}$$

which implies

$$\begin{aligned} Q(k+1) &= E\Big\{(x(k+1) - m(k+1))(x(k+1) - m(k+1))^T\Big\} \\ &= E\Big\{[A(x(k) - m(k)) + Gw(k)][A(x(k) - m(k)) + Gw(k)]^T\Big\} \\ &= E\big\{A(x(k) - m(k))(x(k) - m(k))A^T\big\} + E\big\{Gw(k)w^T(k)G^T\big\} \\ &= AQ(k)A^T + GWG^T = Q(k+1) \end{aligned}$$

Note that due to the fact that the white noise process is uncorrelated with the state variables the cross product terms are zero, that is $E\big\{(x(k) - m(k))w^T(k)\big\} = 0 = E\Big\{w(k)(x(k) - m(k))^T\Big\}$

Sample Final Exam – 332: 416 Control System Design

Answer questions 1 and 11, and any eight out of the remaining nine questions.

1. State and present the solution to the deterministic continuous-time linear optimal regulator problem.

2. Explain the technique for linearization of nonlinear systems about nominal points.

3. Explain nonlinear phenomena called: finite escape time and existence of harmonics and subharmonics, and compare them to the corresponding phenomena of linear systems.

4. Present all possible shapes for state trajectories of second-order linear time invariant systems.

5. State the first method of Lyapunov for examining stability of nonlinear systems by using the linearization technique. Apply it to a simple pendulum.

6. Present the second stability method of Lyapunov based on the existence of Lyapunov functions and demonstrate it on a simple pendulum equation.

7. Apply the second stability method of Lyapunov to a linear time invariant continuous-time system and derive the algebraic Lyapunov equation.

8. Explain the describing function method for studying periodicity (limit cycles) of linear systems with static nonlinearities.

9. Explain the passivity property of linear electrical networks containing a nonlinear resistor and explain how to use this property for studying stability of control systems.

10. Reduced-order observers for nonlinear systems.

11. Present the extended (nonlinear) Kalman filter and draw the corresponding block diagram.