330:345 Exam II Information — Fall 2003

Exam II will be based on the material from Chapters 4 and 5 covered in class and outlined in the course syllabus.

Closed book and notes. No calculators allowed.

Tables 4.1, 4.2, 5.1 and 5.2 will be distributed to the students before the exam.

Study Guide:
Do all homework problems and read the study guides given in the chapter summaries. No proofs of the Laplace and $\mathcal{Z}$–transform properties.

Exam Time:
Monday, November 24, 2003; 8:10–9:30am (regular class hours)

Place:
SEC 111, **A-MA** (129 students — Random sitting chart will be used)
SEC 117, **MB-S** (62 students — Random sitting chart will be used)
SEC 210 **T-Z** (35 students — Random sitting chart will be used)

Attachments:
Sample Exam II, Fall 1999
Tables 4.1, 4.2, 5.1, and 5.2

Point distribution for Exam II: (30 points = 30% of the course grade)
Chapter 4: 15 points (50%)
Chapter 5: 15 points (50%)

SERC Library:
Copies of all solutions to homework problems from Chapters 4 and 5 and Exam II Information are *also* on reserve reading in the SERC Library.

332:345 — Linear Systems and Signals — Sample Exam II

#1a) 5pts. Find the Laplace transform of the following function

$$f(t) = 3t^2 e^{-2t} u(t) + (t+1)e^{-t} u(t-2) + t\cos\left(\pi(t-2)\right) u(t)$$

#1b) 5pts. Find the inverse Laplace transform of the following function

$$F(s) = \frac{e^{-2s}}{s^2(s+2)^2}$$

#2a) 5pts. Find the $\mathcal{Z}$-transform of the following function

$$f[k] = 3^k(k+2)u[k-2] + \cos\left[k\frac{\pi}{2}\right]u[k-2] + f_1[k], \quad f_1[k] = \begin{cases} 5, & k=2 \\ 9, & k=4 \\ 0, & \text{otherwise} \end{cases}$$

#2b) 5pts. Find the inverse $\mathcal{Z}$-transform of the following function

$$F(z) = \frac{3z(z+1)}{(z+2)(z+4)(z+6)}$$

Using the initial and final values theorems find $f[0]$ and $f[+\infty]$.

#3a) 5pts. Consider a continuous-time system

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = \frac{df(t)}{dt} + 3f(t)$$
$$y\left(0^-\right) = 1, \quad y^{(1)}\left(0^-\right) = 2, \quad f(t) = tu(t)$$

Find its transfer function (1pt), impulse response (1pt), step response (1pt), zero-state (1pt), and zero-input (1pt) responses.

#3b) 5pts. Consider a discrete-time system

$$y[k+2] + \frac{1}{6}y[k+1] - \frac{1}{6}y[k] = f[k+1] + 2f[k], \qquad y[0] = 1, \;\; y[1] = 2$$

with $f[k] = (-1)^k u[k]$. Find its transfer function (1pt), impulse response (1pt), step response (1pt), and complete response (2pts).

Hint: $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$.

Table 4.1: Properties of the Laplace Transform

$\mathcal{L}\{\alpha_1 f_1(t) + \alpha_2 f_2(t)\}$	$\alpha_1 F_1(s) + \alpha_2 F_2(s)$
$\mathcal{L}\{f(t-t_0)u(t-t_0)\}$	$e^{-st_0}F(s), \quad t_0 > 0$
$\mathcal{L}\{f(at)\}$	$\frac{1}{a}F\left(\frac{s}{a}\right), \quad a > 0$
$\mathcal{L}\{t^n f(t)\}$	$(-1)^n \frac{d^n}{ds^n}F(s)$
$e^{\lambda t}f(t)$	$F(s-\lambda)$
$f(t)\cos(\omega_0 t)$ $f(t)\sin(\omega_0 t)$	$\frac{1}{2}[F(s+j\omega_0) + F(s-j\omega_0)]$ $\frac{j}{2}[F(s+j\omega_0) - F(s-j\omega_0)]$
$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\}$	$sF(s) - f\left(0^-\right)$
$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\}$	$s^2F(s) - sf\left(0^-\right) - f^{(1)}\left(0^-\right)$
$\mathcal{L}\left\{\frac{d^n}{dt^n}f(t)\right\}$	$s^nF(s) - s^{n-1}f\left(0^-\right) - s^{n-2}f^{(1)}\left(0^-\right)$ $- \cdots - f^{(n-1)}\left(0^-\right)$
$\mathcal{L}\{f_1(t) * f_2(t)\}$	$F_1(s)F_2(s)$
$\mathcal{L}\{\int_0^t f(\tau)d\tau\}$	$\frac{1}{s}F(s)$
$\lim_{t\to 0^+}\{f(t)\}$	$\lim_{s\to\infty}\{sF(s)\}$
$\lim_{t\to\infty}\{f(t)\}$	$\lim_{s\to 0}\{sF(s)\}$

Table 4.2: Common Laplace transform pairs

$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$t^n e^{-\alpha t}u(t)$	$\frac{n!}{(s+\alpha)^{n+1}}$
$u(t)\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$u(t)\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$e^{-\alpha t}u(t)\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
$e^{-\alpha t}u(t)\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
$tu(t)\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
$tu(t)\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
$te^{-\alpha t}u(t)\cos(\omega t)$	$\frac{(s+\alpha)^2-\omega^2}{\left((s+\alpha)^2+\omega^2\right)^2}$
$te^{-\alpha t}u(t)\sin(\omega t)$	$\frac{2\omega(s+\alpha)}{\left((s+\alpha)^2+\omega^2\right)^2}$

Table 5.1: Properties of the $\mathcal{Z}$-transform

$\mathcal{Z}\{a_1 f_1[k] \pm a_2 f_2[k]\}$	$a_1 F_1(z) \pm a_2 F_2(z)$
$\mathcal{Z}\{f[k-k_0]u[k-k_0]\}$	$\frac{1}{z^{k_0}} F(z)$
$\mathcal{Z}\{f[k-1]u[k]\}$	$\frac{1}{z} F(z) + f[-1]$
$\mathcal{Z}\{f[k-2]u[k]\}$	$\frac{1}{z^2} F(z) + \frac{1}{z} f[-1] + f[-2]$
$\mathcal{Z}\{f[k-k_0]u[k]\}$	$\frac{1}{z^{k_0}} F(z) + \frac{1}{z^{k_0-1}} f[-1] + \cdots + \frac{1}{z} f[-k_0+1] + f[-k_0]$
$\mathcal{Z}\{f[k+1]u[k]\}$	$zF(z) - zf[0]$
$\mathcal{Z}\{f[k+2]u[k]\}$	$z^2 F(z) - z^2 f[0] - zf[1]$
$\mathcal{Z}\{f[k+k_0]u[k]\}$	$z^{k_0} F(z) - z^{k_0} f[0] - z^{k_0-1} f[1] - \cdots - zf[k_0-1]$
$\mathcal{Z}\{kf[k]\}$	$-z\frac{d}{dz} F(z)$
$\mathcal{Z}\{k^2 f[k]\}$	$z\frac{d}{dz} F(z) + z^2 \frac{d^2}{dz^2} F(z)$
$\mathcal{Z}\{a^k f[k]\}$	$F\left(\frac{z}{a}\right)$
$\mathcal{Z}\{f[k]\cos(\omega kT)\}$	$\frac{1}{2}\left[F\left(ze^{j\omega T}\right) + F\left(ze^{-j\omega T}\right)\right]$
$\mathcal{Z}\{f[k]\sin(\omega kT)\}$	$\frac{j}{2}\left[F\left(ze^{j\omega T}\right) - F\left(ze^{-j\omega T}\right)\right]$
$\mathcal{Z}\{f_1[k] * f_2[k]\}$	$F_1(z)F_2(z)$
$\lim_{k\to 0} f[k]$	$\lim_{z\to\infty}\{F(z)\}$
$\lim_{k\to\infty} f[k]$	$\lim_{z\to 1}\left\{\frac{z-1}{z} F(z)\right\}$

Table 5.2: Common $\mathcal{Z}$-transform pairs

$\delta[k]$	1
$u[k]$	$\frac{z}{z-1}$
$a^k u[k]$	$\frac{z}{z-a}$
$ku[k]$	$\frac{z}{(z-1)^2}$
$k^2 u[k]$	$\frac{z(z+1)}{(z-1)^3}$
$ka^k u[k]$	$\frac{az}{(z-a)^2}$
$k^2 a^k u[k]$	$\frac{az(z+1)}{(z-a)^3}$
$\frac{1}{m!}k(k-1)(k-2)\cdots(k-m+1)u[k]$	$\frac{z}{(z-1)^m}$
$\frac{1}{m!}k(k+1)(k+2)\cdots(k+m)a^k u[k]$	$\frac{z^{m+1}}{(z-a)^{m+1}}$
$u[k]\cos(\omega kT)$	$\frac{z^2 - z\cos(\omega T)}{z^2 - 2z\cos(\omega T)+1}$
$u[k]\sin(\omega kT)$	$\frac{z\sin(\omega T)}{z^2 - 2z\cos(\omega T)+1}$
$a^k u[k]\cos(\omega kT)$	$\frac{z^2 - az\cos(\omega T)}{z^2 - 2az\cos(\omega T)+a^2}$
$a^k u[k]\sin(\omega kT)$	$\frac{az\sin(\omega T)}{z^2 - 2az\cos(\omega T)+a^2}$