Potential Questions for Exam I in 330:519 — Kalman Filtering

Exam I: Friday March 28, 6:10–9:00

1. Derive the relationship between the input and output spectra of linear time-invariant stochastic systems.
2. Define and describe the white noise stochastic process.
3. Derive the orthogonality principle of the mean-squared estimation problem.
4. Derive the variance equation for state variables of a linear continuous-time stochastic system driven by white noise (you can follow either the text-book or the journal paper by Athans and Tse)..
5. Derive the variance equation for state variables of a linear discrete-time stochastic system driven by white noise.
6. Using the orthogonality principle derive the expression for the optimal gain of the discrete-time Kalman filter in terms of the estimation error a priori covariance.
7. Derive the relationship that relates the a posteriori and a priori estimation error covariance matrices for the discrete-time Kalman filter.
8. Starting with the Wiener-Hopf equation derive the expression for the optimal continuous-time Kalman filter gain.
9. Derive the form of the continuous-time Kalman filter (formula (I) on page 103 of the original paper by Kalman and Bucy).
10. Derive the dynamic eqquation for the estimation errror of the continuous-time Kalman filter (fformula (II) inthe original paper).
11. Using the fact that the optimal Kalman filter is given by $K(t) = P(t)H^T(t)R^{-1}(t)$ derive the continuous-time filter differential Riccati equation whose solution represents the optimal variance of the estimation error.
12. Explain how to handle the colored measurement noise (wide band noise or exponentially correlated noise) of the discrete-time Kalman filter. Comment on the corresponding continuous-time problem.
13. Present the solution of the continuous-time differential Riccati equation in terms of the solutions of a system of linear differential equations.
14. Derive the Newton algorithm for solving the continuous-time algebraic Riccati equation.
15. Explain the eigenvector method for solving the algebraic Riccati equation.
16. Assuming that the continuous-time system white noise intensity matrix is known derive the expression for the corresponding discrete-time white noise intensity matrix.
17. Show that the discrete-time innovation process is a white noise stochastic process and derive the expression for its intensity (Mehra's paper, Section III).
18. Relationship between Kalman and Wiener filters.