Potential Questions for Exam I in Nonlinear and Adaptive Control

Exam Time: March 13, 6:10–9:00
(Last Update: March 6, 1998)

1. Describe the most common nonlinear phenomena (finite escape time, multiple isolated equilibria, limit cycles, harmonics, chaos).
2. Discuss phase portraits of linear (and nonlinear systems) and comment on their stability in terms of eigenvalues.
3. Present the method for linearization of nonlinear systems. Demonstrate the procedure on the example of a simple pendulum described by $ml\ddot{\theta} = -mg\sin\theta - kl\dot{\theta}$
4. State and explain the use of the implicit function theorem.
5. State the Gronwall-Bellman inequality lemma.
6. State and prove the contraction mapping theorem.
7. State the local existence and uniqueness theorem (Theorem 2.2) and discuss its relationship with the global existence and uniqueness theorem (Theorem 2.3).
8. Define the Lipshitz continuity and discuss local and global Lipshitz continuity in view of Lemmas 2.2–2.4.
9. State the closeness of solutions theorem (Theorem 2.5) and present the idea of its proof.
10. State the theorem (Theorem 2.6) about the nonlinear system continuous dependence on initial conditions and parameters.
11. Derive the formula for system sensitivity functions with respect to system parameters.
12. Define stable, asymptotically stable, and unstable equilibrium points in the sense of Lyapunov.
13. Present the second (direct) stability method of Lyapunov and demonstrate it on the simple pendulum without friction example. The pendulum equation is given by $\dot{x}_1 = x_2, \dot{x}_2 = -(g/l)\sin x_1$ and a Lyapunov function candidate is $V(x) = (g/l)(1-\cos x_1) + 0.5x_2^2$.
14. State the global asymptotic theorem of Barbashin and Krasovski and demonstrate it on the following example: $\dot{x}_1 = -x_1 - x_2;\ \dot{x}_2 = x_1 - x_2^3$ with $V(x) = ax_1^2 + bx_2^2,\ a > 0, b > 0$.
15. State instability theorem of Chetaev and demonstrate it on the following example: $\dot{x}_1 = -x_1 + x_2^6;\ \dot{x}_2 = x_1^6 + x_2^3$ with $V(x) = -\frac{1}{6}x_1^6 + \frac{1}{4}x_2^4,$.
16. Present the first (indirect) stability method of Lyapunov and demonstrate it on the simple pendulum with friction example given by $ml\ddot{\theta} = -mg\sin\theta - kl\dot{\theta}$.
17. Present LaSalle's invariance principle and demonstrate it on the following example: $\dot{x}_1 = x_2$, $\dot{x}_2 = -x_1 - x_2\mathrm{sat}\left(x_2^2 - x_3^2\right), \dot{x}_3 = x_3\mathrm{sat}\left(x_2^2 - x_3^2\right)$ with $V(x) = x^T x$.
18. State stability definitions in the sense of Lyapunov for time varying systems for stable, uniformly stable, asymptotically stable, uniformly asymptotically stable, globally uniformly asymptotically stable, and exponentially stable equilibrium points.
19. Present the Lyapunov stability direct method for time varying systems and demonstrate it on the following example: $\ddot{y} + \dot{y} + (2+\sin t)y = 0$ with $V(x_1, x_2, t) = x_1^2 + x_2^2/(2+\sin t)$.
20. Prove that the function $V(x,t) = x^T P(t) x$ is the Lyapunov function for a time varying linear system $\dot{x}(t) = A(t)x(t)$ with continuous and bounded $A(t)$, where the positive definite matrix $P(t),\ 0 < c_1 I \leq P(t) = P^T(t) \leq c_2 I,\ \forall t,$ satisfies the Lyapunov differential equation $-\dot{P}(t) = P(t)A(t) + A^T(t)P(t) + Q(t)$ with $Q(t) = Q^T(t) \geq c_3 I > 0, \forall t$.
21. Present the indirect stability method of Lyapunov for time varying nonlinear systems.
22. State Bendixson criterion for examining the existence of limit cycles and demonstrate it on the following example: $\dot{x}_1 = x_2,\ \dot{x}_2 = ax_1 + bx_2 - x_1^2 x_2 - x_1^3$.

23. Explain the describing function method for studying periodicity (limit cycles) of linear systems with static nonlinearities.
24. Define the characteristic equation for linear systems with static nonlinearities and demonstrate how to find the frequency and magnitude of oscillations on the following example

$$G(s) = \frac{1}{s(s+1)(s+2)}, \quad \Psi(a) = \frac{4}{\pi a}$$

Discuss a method based on the Nyquist plot for examining stability of limit cycles.
25. State the Kalman-Yacubovich-Popove-Lemma and discuss its application for examining passivity of linear control systems with static nonlinearities.
26. Define Lure's stability problem and give its solution based on the circle criterion and Popov's criterion.
27. Discuss the general control problem of nonliear systems based on linearization and demonstrate it on a simple pendulum problem (Example 11.2 from the textbook).
28. Present nonlinear system regulation through linearization and integral control technique.
29. Present the input-state feedback linearization technique.
30. Present the input-output feedback linearization technique.