Potential Exam Questions in Control Theory II — Fall 2002

Exam I: October 16, 2002, at 6:10–7:30 in Control Systems Lab, ELE 202

1. Define bounded-input bounded-output stability of linear time invariant *continuous* systems, state the corresponding theorem (Theorem 5.1) and give its proof. (C99, p. 121–122; Class Notes p. 1–2).
2. Find the steady state response of a bounded-input bounded-output linear time invariant *continuous* system due to (a) constant input, (b) sinusoidal input (C99, p. 123–124; Class Notes p. 2–3).
3. State and justify bounded-input output-stability of time invariant linear *continuous* systems in terms of system poles (C99, p. 124; Class Notes, p. 3–4)—Theorem 5.3.
4. Define bounded-input bounded-output stability of linear time invariant *discrete* systems, state the corresponding theorem (Theorem 5.D1) and give its proof. (C99, p. 126–127; Class Notes p. 5–6).
5. Find the steady state response of a bounded-input bounded-output linear time invariant *discrete* system due to (a) constant input, (b) sinusoidal input (C99, p. 127–128; Class Notes p. 6–7).
6. State and justify bounded-input output-stability of time invariant linear *continuous* systems in terms of system poles (C99, p. 128; Class Notes, p. 7–8)—Theorem 5.D3.
7. State internal stability theorem for time invariant linear *continuous* systems and demonstrate by an example stability of multiple eigenvalues on the imaginary axis (C99, 130–131; Class Notes p. 9–11).
8. State internal stability theorem for time invariant linear *discrete* systems (C99, 131; Class Notes, p. 12).
9. Define system stability in sense of Lyapunov and state the general Lyapunov stability theorem (also known as direct or second method of Lyapunov), (Class Notes, p. 13–15).
10. State the Lyapunov stability theorem for linear time invariant linear *continuous* systems and show that the theorem is the direct consequence of the second method of Lyapunov (C99 p. 132; Class Notes p. 15–16).
11. State and Prove Theorem 5.6 (the form of a unique solution of the algebraic Lyapunov equation), (C99, p. 134; Class Notes, p. 17)
12. State the second Lyapunov theorem for general discrete time invariant systems and specialize the theorem to linear discrete time invariant systems (Class Notes, p. 18–19).
13. Define internal stability of time varying linear continuous-time systems and present an example that indicates that the statement: $Re\{\lambda_i(A(t))\} < 0, \forall t, \forall i$ implies asymptotic stability is false (C99, p. 137–139; Class Notes, p. 20–21).
14. State and justify the Lyapunov test for time varying continuous linear systems (Example 3.21 from Khalil's textbook (Class Notes p. 9) or Theorem 7.2, p. 116, from Rugh's textbook).
15. State and justify the Lyapunov test for time varying discrete linear systems (Rugh's textbook, p. 438–439).
16. State four controllability theorems (C99, p. 145; Class Notes, p. 2).
17. Prove the theorem that states that the linear time invariant system is controllable if and only if its controllability Grammian is nonsingular (C99, p. 145–146; Class Notes, p. 3–4).
18. Prove the equivalence between the full controllability matrix rank test and nonsingularity of the corresponding Grammian (C99, p. 145–146; Class Notes, p. 5–6).
19. Derive the Popov-Belevitch test from the controllability matrix rank condition (C99, p. 147; Class Notes, p. 6–7).
20. Proof the equivalence between the Popov-Belevitch rank test and the Popov-Belevitch eigenvector test (Class Notes p. 7–8).
21. Show that system controllability is invariant under similarity transformation (C99, p. 152; Class Notes, p. 8).
22. Define system stabilizability and state main stabilizability theorems for linear continuous time invariant systems (Class Notes, p. 9).