

# **Optimal Control of Weakly Coupled Systems and Applications**

HIGH ACCURACY TECHNIQUES

**Z. Gajić, M-T. Lim, D. Skatarić  
W-C. Su, and V. Kecman**

**Taylor & Francis (CRC Press, Dekker)**

**2008**



## Preface

This book is intended for engineers, mathematicians, physicists, and computer scientists interested in control theory and its applications. The book studies a special class of linear and bilinear control systems known as weakly coupled systems. These systems, characterized by the presence of weak coupling among subsystems, describe dynamics of many real physical systems such as chemical plants, power systems, aircraft, satellites, machines, cars, computer/communication networks.

Weakly coupled control systems have become an extensive research area since the end of the 1960s when the original papers of Professor Kokotovic and his coworkers and graduate students were published. A relatively large number of journal papers on weakly coupled control systems were published during the 1970s, 1980s, and 1990s. The approaches taken during the 1970s and 1980s were based on expansion methods (power series, asymptotic expansions, Taylor series). These approaches were in most cases accurate only with an  $O(\epsilon^2)$  accuracy, where  $\epsilon$  is a small weak coupling parameter. Generating higher order expansions for those methods has been analytically cumbersome and numerically inefficient, especially for higher dimensional control systems. Even more, it has been demonstrated in the control literature that for some applications the  $O(\epsilon^2)$  accuracy either is not satisfactory or even in some cases has not solved considered weakly coupled control problems.

The development of high accuracy efficient techniques for weakly coupled control systems started at the end of the 1980s in the papers by Professor Gajic and his graduate students and coworkers. The corresponding approach was recursive in nature and based on fixed-point iterations. At the beginning of the 1990s, the fixed-point recursive approach culminated in the so-called Hamiltonian approach for the *exact* decomposition of weakly coupled, linear-quadratic, deterministic and stochastic, optimal control and filtering problems. In the 2000s Professor Kecman developed the generalized Hamiltonian approach based on the eigenvector method. At the same time, Professor Mukaidani and his

coworkers discovered a new approach for studying various formulations of optimal linear weakly coupled control systems.

This book represents a comprehensive overview of the current state of knowledge of both the recursive approach and the Hamiltonian approach to weakly coupled linear and bilinear optimal control systems. The book devises unique powerful methods whose core results are repeated and slightly modified over and over again, while the methods solve more and more challenging problems of linear and bilinear weakly coupled optimal continuous- and discrete-time systems. It should be pointed out that some related problems still remain unsolved, especially corresponding problems in the discrete-time domain, and the optimization problems over a finite horizon. Such problems are identified in the book as open problems for future research.

The presentation is based on the research work of the authors and their coworkers. The book presents a unified theme about the exact decoupling of the corresponding optimal control problems and decoupling of the nonlinear algebraic Riccati equation into independent, reduced-order, subsystem-based algebraic Riccati equations.

Each chapter is organized to represent an independent entity so that readers interested in a particular class of linear and bilinear weakly coupled control systems can find complete information within the particular chapter. The book demonstrates theoretical results on many practical applications using examples from aerospace, chemical, electrical, and automotive industries. To that end, we apply theoretical results obtained to optimal control and filtering problems represented by real mathematical models of aircraft, power systems, chemical reactors, and so on.

The authors are thankful for support and contributions from their colleagues, Professors S. Bingulac, H. Mukaidani, D. Petkovski, B. Petrović, N. Prljaca, and X. Shen, and Drs. Z. Aganović, D. Arnautović, I. Borno, Y-J. Kim, M. Qureshi and V. Radisavljević.

*Zoran Gajić*  
*Myo-Taeg Lim*  
*Dobrica Skatarić*  
*Wu-Chung Su*  
*Vojislav Kecman*

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