

## Sample Exam 1: Chapters 1, 2, and 3

**#1)** Consider the linear-time invariant system represented by

$$\frac{dy(t)}{dt} + 2y(t) = 3, \quad y(0) = 4$$

Find the system response and its zero-state and zero-input components. What are the response steady state and transient components.

**#1b)** Using the linearity and time invariance principle (note that you have to assume zero initial conditions) find the response of the system defined in #1a) due to the input signal equal to  $u(t) - u(t - 2)$ .

**#2a)** Using the properties of the impulse delta function simplify the following expressions

$$(i) \quad e^{-5t}\delta(t - 2), \quad (ii) \quad \int_{-\infty}^{\infty} (t + 2)\delta(t - 0.5)dt$$

$$(iii) \quad \int_{-\infty}^1 e^{-2t} \sin(\pi t)\delta^{(1)}(t - 2)dt, \quad (iv) \quad \int_{-5}^5 e^{-5t} \cos(t - 5)\delta(t - 5)dt$$

**#2b)** Plot the graph of the signal represented in terms of unit step and unit ramp signals as

$$x(t) = u(-t + 2) + u(t + 2) + r(-t - 1)$$

and find its generalized derivative.

**#3a)** Find the Fourier series of a periodic signal represented by

$$x(t) = x(t + 2) = \begin{cases} 1, & -1 \leq t \leq 0 \\ 2, & 0 \leq t \leq 1 \end{cases}$$

**#3b)** Find the system response due to the periodic input signal given in #3a) of a linear system whose transfer function is  $H(j\omega) = \frac{1}{1+j\omega}$ .

**#3c)** Using the tables of common pairs and properties of the Fourier transform find Fourier transforms of the following signals:

$$(i) \quad te^{-2|t|}, \quad (ii) \quad e^{-2t}u_h(t) \cos(t), \quad (iii) \quad \int_{-\infty}^t p_2(3\tau)d\tau, \quad (iv) \quad u_h(t - 2)$$

**#3d)** Find the inverse Fourier transform of the signals

$$(i) \quad \frac{1}{1 + j\omega} \cos(2\omega)e^{-j5\omega} \quad (ii) \quad p_6(\omega - 2)$$

## Sample Exam 2: Chapters 1, 2, and 3

**#1)** Consider the linear-time invariant system represented by

$$\frac{dy(t)}{dt} + y(t) = \sin(t - 3), \quad y(0) = 0$$

Find the system response and its zero-state and zero-input components. What are the response steady state and transient components.

**#2a)** Using the properties of the impulse delta signal simplify the following expressions

$$(i) \quad (t-1)^2\delta(t-1), \quad (ii) \int_{-\infty}^{\infty} \cos(\pi t)\delta^{(3)}(t-1)dt, \quad (iii) \int_{-\infty}^2 \sin(\pi t)\delta(3t-2)dt$$

$$(iv) \int_{-\infty}^3 e^{-5t} \sin(3t)\delta(t-3)dt, \quad (v) \int_{-3}^4 f(t)\delta^{(1)}(t+5)dt$$

**#2b)** Plot the graph of the signal represented in terms of unit step, rectangular, and unit ramp signals as

$$x(t) = r(t-3) + 2p_4(2t-5) - u(-4t+2)$$

and find its generalized derivative.

**#3a)** Find the Fourier series for the sawtooth signal with  $T = 1, E = 1$ . Note it is an odd signal.

*Hint:*

$$\int t \sin(\alpha t) dt = \frac{1}{\alpha^2} \sin(\alpha t) - \frac{t}{\alpha} \cos(\alpha t)$$

**#3b)** Find the liner system response due to the periodic input signal defined in 3a). The system transfer function is  $H(\omega) = \frac{j\omega}{1+j\omega}$ .

**#3c)** Using the tables of common pairs and properties of the Fourier transform find Fourier transforms of the following signal:

$$(i) \quad \sin(2\pi t)[u_h(t-2) - u_h(t-1)], \quad (ii) \quad t^2 e^{-3t} u_h(t), \quad (iii) \quad \text{sinc}(3t-4)$$

**#3d)** Find the inverse Fourier transform of the signal

$$X(j\omega) = 2p_4(\omega) - p_2(\omega - 1)$$

**#3e)** Find the response of the system defined in 3b) due to the input signal  $5 \cos(10t + \frac{\pi}{3})$ .

### Sample Exam 3: Chapters 4 and 5

**1)** Find the Laplace transform of the following signals

$$f_1(t) = (t+2)e^{-t}u(t-1), \quad f_2(t) = \cos(\pi t)u(t-2), \quad f_3(t) = 6te^{-2t}\sin(3t)u(t)$$

**2)** Find the inverse Laplace transform of the following function

$$F(s) = \frac{2s + e^{-s}}{s^2(s+1)}$$

**3)** Find the  $\mathcal{Z}$ -transform of the following signals

$$f_1[k] = 3^{k+2}(k+2)u[k-2], \quad f_2[k] = k \cos\left[k\frac{\pi}{2}\right]u[k], \quad f_3[k] = \begin{cases} 2, & k=5 \\ 4, & k=9 \\ 0, & \text{otherwise} \end{cases}$$

**4)** Find the inverse  $\mathcal{Z}$ -transform of the following function

$$F(z) = \frac{4(z+2)}{(z+1)(z+3)(z+5)}$$

Using the initial and final values theorems verify the values obtained for  $f[0]$  and  $f[\infty]$ .

**5)** Consider a continuous-time system

$$\frac{d^2y(t)}{dt^2} + 6\frac{dy(t)}{dt} + 9y(t) = 3\frac{df(t)}{dt} - f(t), \quad y(0^-) = 1, \quad y^{(1)}(0^-) = 0, \quad f(t) = e^{-t}u(t)$$

Find its transfer function (1pt), impulse response (1pt), step response (1pt), zero-state (1pt), and zero-input (1pt) responses.

**6)** Consider a discrete-time system

$$\begin{aligned} y[k] - \frac{1}{6}y[k-1] - \frac{1}{6}y[k-2] &= f[k-1] + f[k-2] \\ y[-2] &= -2, \quad y[-1] = 0, \quad f[k] = (-1)^k u[k] \end{aligned}$$

Find the system transfer function (1pt), impulse response (1pt), zero-state (1pt), and zero-input (1pt) responses. What is the system steady state response due to  $f[k] = 5u[k]$  (1pt)?

Hint:  $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$ .

## Sample Exam 4: Chapters 4 and 5

**#1a)** Find the Laplace transform of the following signals

$$f_1(t) = e^{-t}u(3t - 4), \quad f_2(t) = \cos(\pi t)[u(t) - u(t - 2)], \quad f_3(t) = (t^2 + 1)u(t - 1)$$

**#1b1)** Find the time domain component of the inverse Laplace transform that corresponds to the pole located at the origin

$$F_1(s) = \frac{e^{-2s}}{s(s+1)(s+2)(s+3)(s+4)(s+5)}$$

**#1b2)** Find the inverse Laplace transform of the function

$$F_2(s) = \frac{e^{-3s}}{s(s^2 + 1)}$$

**#2a)** Find the  $\mathcal{Z}$ -transform of the following signals

$$f_1[k] = 3^{k+2}(k+2)u[k], \quad f_2[k] = \cos\left[k\frac{\pi}{2}\right]u[k-1], \quad f_3[k] = \begin{cases} 3, & k = 15 \\ -4, & k = 29 \\ 0, & \text{otherwise} \end{cases}$$

**#2b)** Find the inverse  $\mathcal{Z}$ -transform of the following function

$$F(z) = \frac{5(z+2)}{(z+1)^2(z-1)}$$

Can you apply the initial and final values theorems to the function  $F(z)$ . If yes, find  $f[0]$  and  $f[\infty]$ .

**#3a)** Consider a continuous-time system

$$\frac{d^2y(t)}{dt^2} + 5\frac{dy(t)}{dt} + 4y(t) = f(t), \quad y(0^-) = 2, \quad y^{(1)}(0^-) = 0, \quad f(t) = 5u(t)$$

Find its transfer function, impulse response, step response, zero-state, zero-input, and steady state responses.

**#3b)** Consider a discrete-time system

$$\begin{aligned} y[k] + y[k-1] + \frac{1}{4}y[k-2] &= f[k-1] + f[k-2] \\ y[-2] = 2, \quad y[-1] = 1, \quad f[k] &= \left(-\frac{1}{4}\right)^k u[k] \end{aligned}$$

Find the system transfer function, impulse response, zero-input, and zero-state responses.

Hint:  $\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$ .

## Sample Exam 5: Chapters 6, 8, and 12

- 1)** Convolve graphically continuous-time signals  $p_2(t)$  and  $\Delta_2(t - 1)$ .  
**2)** Using the sliding tape method find the discrete-time convolution  $f_1[k] * f_1[k]$ , where

$$f_1[k] = \begin{cases} 1 & k = 0 \\ -2 & k = 2 \\ 3 & k = 3 \\ -1 & k = 4 \\ -2 & k = 5 \\ 0 & \text{otherwise} \end{cases}$$

- 3)** Consider the continuous-time linear system represented in the state space form by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t), \quad \begin{bmatrix} x_1(0^-) \\ x_2(0^-) \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$y(t) = [0 \quad 1] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

- a)** Find the system transfer function.
  - b)** Find the system impulse response.
  - c)** Find the system transition matrix using the Laplace transform.
  - d)** Find the system output response ( $y(t)$ ) due to the system input  $f(t) = e^{-3t}u(t)$ .
- 4)** Consider the discrete-time linear system represented in the state space form by

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f[k], \quad \begin{bmatrix} x_1[0] \\ x_2[0] \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$y[k] = [1 \quad 0] \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

- a)** Find the system transfer function.
  - b)** Find the system transition matrix.
  - c)** Find the system output response ( $y[k]$ ) due to the system input  $f[k] = (0.5)^k u[k]$ .
- 5)** The open-loop system transfer function is given by

$$H(s) = \frac{1}{s + \alpha}$$

Assuming the unity feedback system, find the sensitivity function of the closed-loop system with respect to parameter  $\alpha$ .

*Hints:*

$$e^{-at}u(t) \leftrightarrow \frac{1}{s + a}, \quad te^{-at}u(t) \leftrightarrow \frac{1}{(s + a)^2}, \quad a^k u[k] \leftrightarrow \frac{z}{z - a}, \quad ka^k u[k] \leftrightarrow \frac{az}{(z - a)^2}$$

## Sample Exam 6: Chapters 6, 8, and 12

**#1a)** Convolve graphically continuous-time signals  $p_3(t - 1.5)$  and  $p_2(t) + \Delta_2(t)$ .

**#1b)** Using the sliding tape method find the discrete-time convolution  $f_1[k] * f_2[k]$ , where

$$f_1[k] = \begin{cases} 1 & k = 0 \\ 2 & k = 2 \\ 3 & k = 3 \\ 2 & k = 4 \\ 0 & \text{otherwise} \end{cases}, \quad f_2[k] = \begin{cases} 2 & k = 2 \\ 3 & k = 3 \\ -2 & k = 4 \\ -1 & k = 5 \\ 0 & \text{otherwise} \end{cases}$$

**#2A)** Consider the continuous-time linear system represented in the state space form by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f(t), \quad \begin{bmatrix} x_1(0^-) \\ x_2(0^-) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$y(t) = [3 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

- a) Find the system transition matrix  $\Phi(s)$ , and obtain  $\Phi(t)$  using the Laplace inverse transform.
- b) Find the system transfer function.
- c) Find the system state response for  $f(t) = \delta(t)$  and the given initial conditions.
- d) Find the system output response ( $y(t)$ ) due to  $f(t) = e^{-3t}u(t)$  and the given initial conditions.

**#2B)** Consider the discrete-time linear system represented in the state space form by

$$\begin{bmatrix} x_1[k+1] \\ x_2[k+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{6} & -\frac{5}{6} \end{bmatrix} \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \end{bmatrix} f[k], \quad \begin{bmatrix} x_1[0] \\ x_2[0] \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$y[k] = [0 \quad 2] \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix}$$

- a) Find the system transition matrix  $\Phi(z)$ . Obtain  $\Phi[k]$  via the  $\mathcal{Z}$ -transform
- b) Find the system transfer function.
- c) Find the system output response ( $y[k]$ ) due to  $f[k] = (-\frac{1}{4})^k u[k]$  and the given initial conditions.
- d) Given the system represented by

$$y[k+5] + y[k+4] - y[k+3] - y[k+2] + 2y[k+1] - 2y[k] = f[k+3] + 4f[k]$$

Find the system state space form.

**#3)** The system open-loop system transfer function is given by

$$H(s) = \frac{K}{s(s+1)(s+2)(s+5)}$$

Assuming the unity feedback system and the system asymptotic stability for some values of the static gain  $K$ , find the steady state step, ramp, and parabolic errors in terms of  $K$ .

*Hint: Some common pairs:*

$$u(t) \leftrightarrow \frac{1}{s}, \quad e^{-at}u(t) \leftrightarrow \frac{1}{s+a}, \quad te^{-at}u(t) \leftrightarrow \frac{1}{(s+a)^2}, \quad a^k u[k] \leftrightarrow \frac{z}{z-a}, \quad ka^k u[k] \leftrightarrow \frac{az}{(z-a)^2}$$