

## Solutions to Homework Problems from Chapter 2

### Problem 2.4

The required signals are presented respectively in Figures 2.4a,b,c,d.

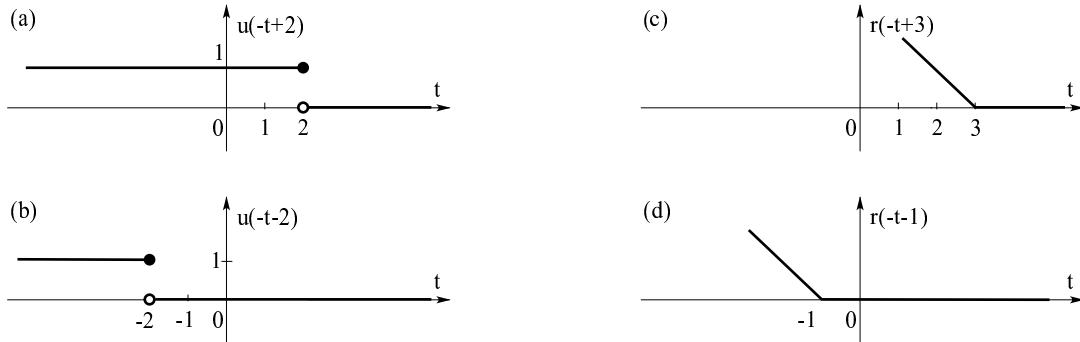


Figure 2.4

### Problem 2.6

The signal  $f(t) = u(t+1) + u(t-1) - r(t-1) + r(t-3)$  is presented in Figure 2.6 with dashed lines denoting particular signals.

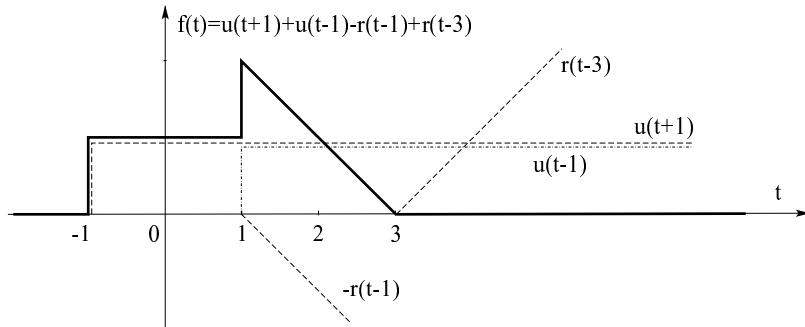


Figure 2.6

### Problem 2.8

See comments made in the solutions to Problem 2.7. In this problem, one of possible several solutions is given by

$$f(t) = u(t) - r(t-1) + r(t-3) + u(t-5)$$

The corresponding signals are plotted in Figure 2.7 using dashed lines.

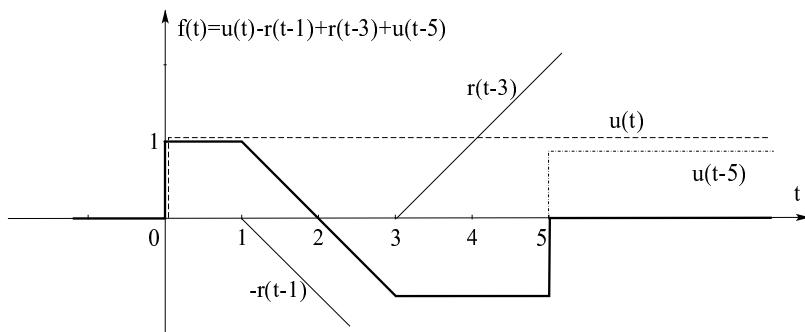


Figure 2.7

**Problem 2.20**

$$\begin{aligned}
& \int_{-\infty}^{\infty} (t^2 + \sin(t) + e^{-2t}) [\delta(t) + 3\delta(2t-1) + 4\delta^{(1)}(t-2)] dt \\
&= \int_{-\infty}^{\infty} f(t) [\delta(t) + 3\delta(2t-1) + 4\delta^{(1)}(t-2)] dt \\
&= f(0) + 3\frac{1}{2}f\left(\frac{1}{2}\right) + 4(-1)f^{(1)}(2) = 1 + \frac{3}{2}\left(\frac{1}{4} + \sin\left(\frac{1}{2}\right) + e^{-1}\right) - 4(4 + \cos(2) - 2e^{-4})
\end{aligned}$$

**Problem 2.21**

$$\begin{aligned}
& \int_{-\infty}^{\infty} (t\sin(t) + e^{-t}) [\delta^{(3)}(t) - \delta^{(2)}(t-2)] dt = \int_{-\infty}^{\infty} f(t) [\delta^{(3)}(t) - \delta^{(2)}(t-2)] dt \\
&= (-1)^3 f^{(3)}(0) - (-1)^2 f^{(2)}(2) = -(-3\sin(0) - 0\cos(0) - e^0) - (2\cos(2) - 2\sin(2) + e^{-2}) \\
&= 1 - 2\cos(2) + 2\sin(2) - e^{-2}
\end{aligned}$$

**Problem 2.22**

$$\begin{aligned}
(a) \quad & \int_{-3}^5 e^{-4t} u(t) \delta(t-4) dt = e^{-4 \times 4} u(4) = e^{-16} \\
(b) \quad & \int_{-3}^5 e^{-t} \delta(t-6) dt = 0 \quad \text{since } \delta(t-6) \text{ is outside of the integration limits} \\
(c) \quad & \int_{-3}^5 e^{-2t} \sin(t-3) \delta(t-5) dt = \int_{-3}^5 f(t) \delta(t-5) dt = \frac{1}{2} f(5) = \frac{1}{2} e^{-10} \sin(2) \\
(d) \quad & \int_{-3}^5 e^{-t} \delta(-t+3) dt = \int_{-3}^5 f(t) \delta(-t+3) dt = f(3) = e^{-3}
\end{aligned}$$

**Problem 2.26**

(a) The scaled and shifted rectangular pulse signals are defined by

$$\begin{aligned}
p_2(3t) &= \begin{cases} 1, & -1 \leq 3t \leq 1 \Leftrightarrow -1/3 \leq t \leq 1/3 \\ 0, & \text{elsewhere} \end{cases}, \quad p_2(3t-2) = \begin{cases} 1, & -1 \leq 3t-2 \leq 1 \Leftrightarrow 1/3 \leq t \leq 1 \\ 0, & \text{elsewhere} \end{cases} \\
p_4(4(t-5)) &= \begin{cases} 1, & -2 \leq 4(t-5) \leq 2 \Leftrightarrow -0.5 + 5 \leq t \leq 0.5 + 5 \Leftrightarrow 4.5 \leq t \leq 5.5 \\ 0, & \text{elsewhere} \end{cases}
\end{aligned}$$

The plots of these signals are presented in Figure 2.10.

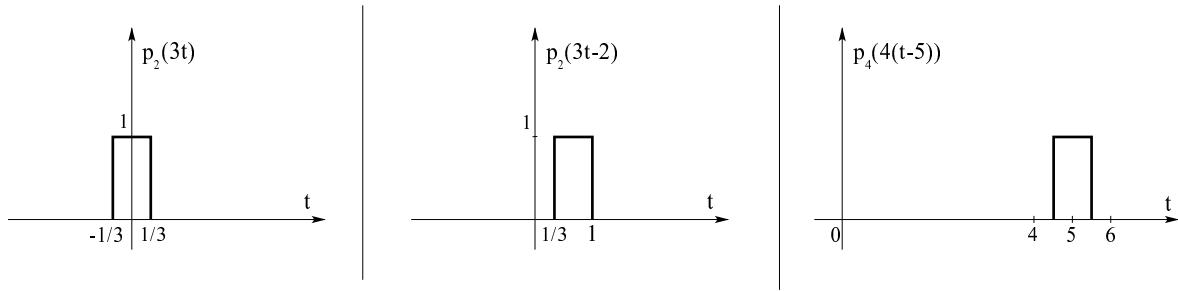


Figure 2.10

(b) The analytical expressions for the scaled and shifted step signals are given by

$$u(2t-3) = \begin{cases} 1, & 2t-3 \geq 0 \Leftrightarrow t \geq 3/2 \\ 0, & \text{elsewhere} \end{cases}, \quad u(-3t+2) = \begin{cases} 1, & -3t+2 \geq 0 \Leftrightarrow t \leq 2/3 \\ 0, & \text{elsewhere} \end{cases}$$

Graphical presentations of these signals are given in Figure 2.11.

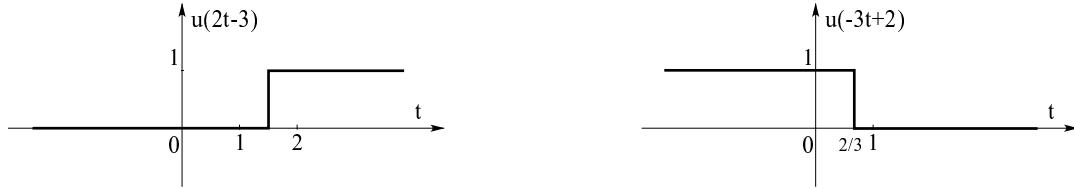


Figure 2.11

### Problem 2.29

Using the derivative product rule and the property of the delta impulse function, which states that  $f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$ , we obtain

$$\begin{aligned} \frac{Df(t)}{Dt} &= u(t-2) + (t+2)\delta(t-2) - \sin(t)u(t-3) + \cos(t)\delta(t-3) - 4e^{-4t}\sin(t) + e^{-4t}\cos(t) \\ &= u(t-2) + 4\delta(t-2) - \sin(t)u(t-3) + \cos(3)\delta(t-3) + e^{-4t}(\cos(t) - 4\sin(t)) \end{aligned}$$

### Problem 2.30

The graphs are presented in Figures 2.13 and 2.14

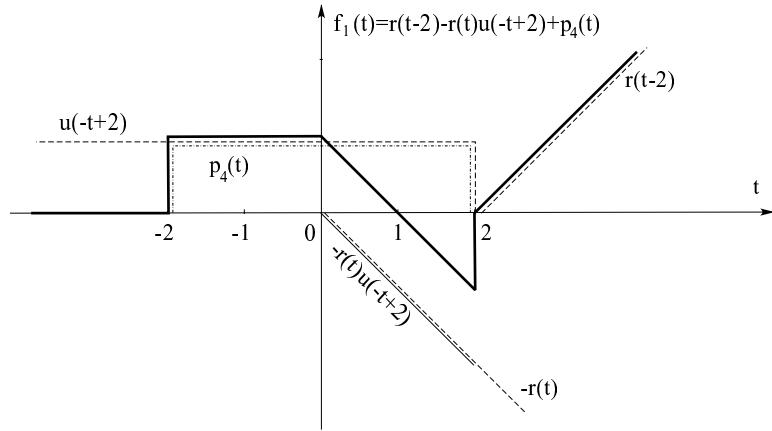


Figure 2.13

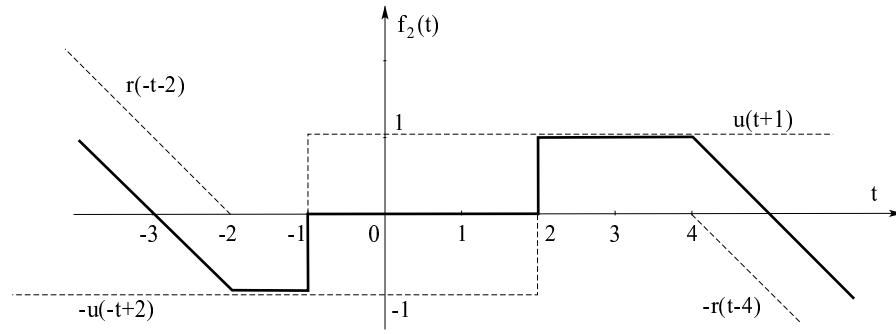


Figure 2.14

The corresponding generalized derivatives are given by

$$\frac{Df_1(t)}{Dt} = \begin{cases} 0, & t < -2 \\ \delta(t+2), & t = -2 \\ 0, & -2 < t < 0 \\ \text{undefined}, & t = 0 \\ -1, & 0 < t < 2 \\ \delta(t-2), & t = 2 \\ 1, & t > 2 \end{cases}, \quad \frac{Df_2(t)}{Dt} = \begin{cases} -1, & t < -2 \\ \text{undefined}, & t = -2 \\ 0, & -2 < t < -1 \\ \delta(t+1), & t = -1 \\ 0, & -1 < t < 2 \\ \delta(t-2), & t = 2 \\ 0, & 2 < t < 4 \\ \text{undefined}, & t = 4 \\ -1, & t > 4 \end{cases}$$

### Problem 2.38

$$\begin{aligned} \frac{D}{Dt}\{e^{\alpha t}u(t)\} &= \alpha e^{\alpha t}u(t) + e^{\alpha t}\delta(t) = \alpha e^{\alpha t}u(t) + \delta(t) \\ \frac{D^2}{Dt^2}\{e^{\alpha t}u(t)\} &= \frac{D}{Dt}\left\{\frac{D}{Dt}\{e^{\alpha t}u(t)\}\right\} = \frac{D}{Dt}\{\alpha e^{\alpha t}u(t) + \delta(t)\} = \alpha^2 e^{\alpha t}u(t) + \alpha\delta(t) + \delta^{(1)}(t) \\ \frac{D^3}{Dt^3}\{e^{\alpha t}u(t)\} &= \frac{D}{Dt}\left\{\frac{D^2}{Dt^2}\{e^{\alpha t}u(t)\}\right\} = \frac{D}{Dt}\{\alpha^2 e^{\alpha t}u(t) + \alpha\delta(t) + \delta^{(1)}(t)\} \\ &= \alpha^3 e^{\alpha t}u(t) + \alpha^2\delta(t) + \alpha\delta^{(1)}(t) + \delta^{(2)}(t) \\ \frac{D^4}{Dt^4}\{e^{\alpha t}u(t)\} &= \frac{D}{Dt}\left\{\frac{D^3}{Dt^3}\{e^{\alpha t}u(t)\}\right\} = \frac{D}{Dt}\{\alpha^3 e^{\alpha t}u(t) + \alpha^2\delta(t) + \alpha\delta^{(1)}(t) + \delta^{(2)}(t)\} \\ &= \alpha^4 e^{\alpha t}u(t) + \alpha^3\delta(t) + \alpha^2\delta^{(1)}(t) + \alpha\delta^{(2)}(t) + \delta^{(3)}(t) \end{aligned}$$

Following the same pattern, we have

$$\frac{D^n}{Dt^n}\{e^{\alpha t}u(t)\} = \alpha^n e^{\alpha t}u(t) + \alpha^{n-1}\delta(t) + \alpha^{n-2}\delta^{(1)}(t) + \cdots + \alpha\delta^{(n-2)}(t) + \delta^{(n-1)}(t), \quad \alpha > 0$$

## Answers to Problems Similar to Homework Problems from Chapter 2

### Answer 2.5

The graphs of the required signals are plotted respectively in Figures 2.5a,b,c,d.

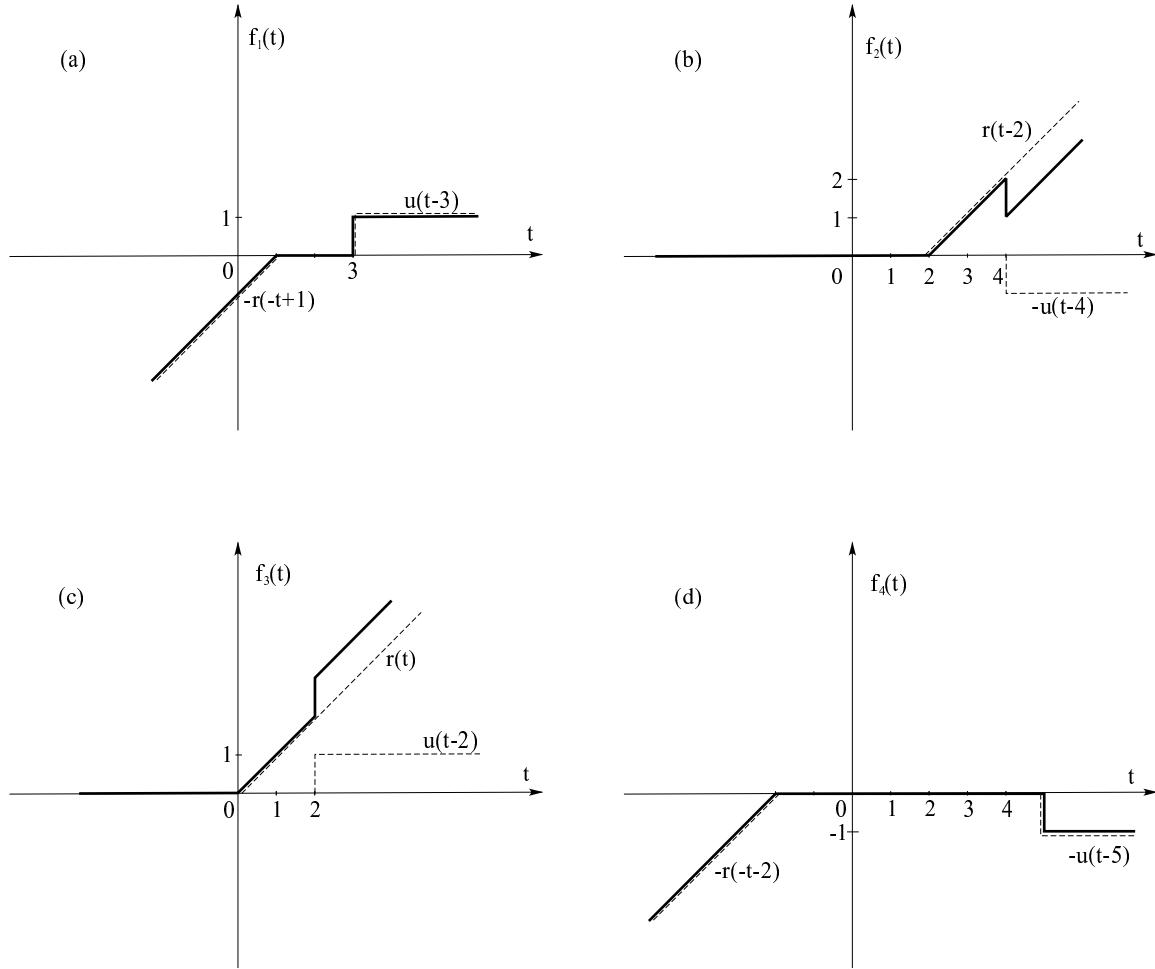


Figure 2.5

### Answer 2.7

$$(a) f_1(t) = r(t) - 2r(t-1) + r(t-2); \quad (b) f_2(t) = r(t-2) - r(t-3) - u(t-1) \\ (c) f_3(t) = r(t-1) - r(t-2) - u(t-2)$$

**Answer 2.23** Given in the textbook.

**Answer 2.24**

$$e^{-2t}\delta(2t-1) + \int_{-\infty}^{\infty} \sin(\pi(t-1)) [\delta^{(2)}(t-1) + \delta(t-2)] dt + \int_{-2}^{+2} \tan(2t)\delta(2t-4) dt + \int_{-3}^{4} 5\delta(t+2) dt \\ = e^{-2\frac{1}{2}}\delta(2t-1) + (-\pi^2)(-1)^2 \sin(\pi(1-1)) + \frac{1}{2} \sin(\pi(1-1)) + \frac{1}{2} \tan(4) + 5 \\ = e^{-1}\delta(2t-1) + \frac{1}{2} \tan(4) + 5$$

Note that the first term was evaluated using the following result

$$f(t)\delta(at - t_0) = f\left(\frac{t_0}{a}\right)\delta(at - t_0)$$

**Answer 2.25** Given in the textbook.

**Answer 2.27**

The ramp signal is defined by  $r(t) = tu(t)$ ,  $t > 0$  and  $r(t) = 0$ ,  $t < 0$ . Using this definition, we have

$$r(-4(t+2)) = \begin{cases} -4(t+2)u(-4(t+2)), & -4(t+2) > 0 \Leftrightarrow t < -2 \\ 0, & \text{elsewhere} \end{cases} = \begin{cases} -4t-8, & t < -2 \\ 0, & \text{elsewhere} \end{cases}$$

Note also that

$$u(-4(t+2)) = \begin{cases} 1, & -4(t+2) > 0 \Leftrightarrow t < -2 \\ 0, & \text{elsewhere} \end{cases} = u(-t-2)$$

The scaled and shifted step and rectangular pulse signals are analytically given by

$$u(-3t-1) = \begin{cases} 1, & -3t-1 \geq 0 \Leftrightarrow t \leq -1/3 \\ 0, & \text{elsewhere} \end{cases}, \quad p_2(-2t-4) = \begin{cases} 1, & -1 \leq -2t-4 \leq 1 \Leftrightarrow -5/2 \leq t \leq -3/2 \\ 0, & \text{elsewhere} \end{cases}$$

The above signals are plotted in Figure 2.12.

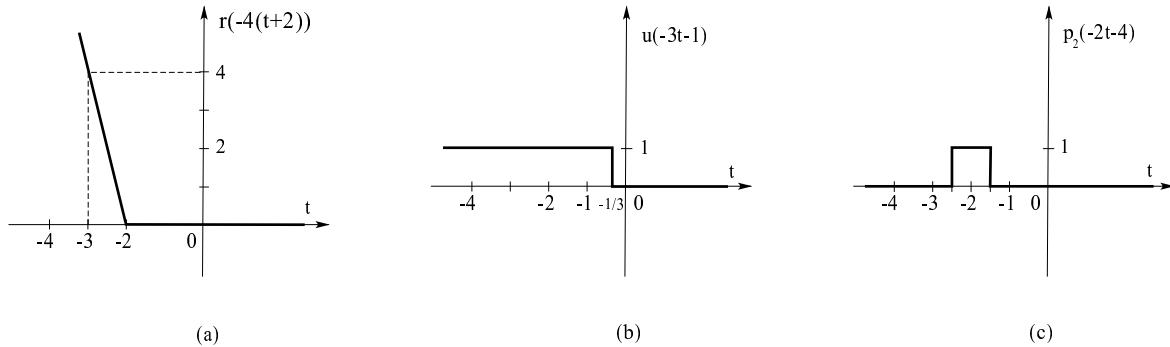


Figure 2.12

**Answer 2.28**

$$\frac{Df_1(t)}{Dt} = \begin{cases} 1, & t < 1 \\ \text{undefined}, & t = 1 \\ 0, & 1 < t < 3, \\ \delta(t-3), & t = 3 \\ 0, & t > 3 \end{cases} \quad \frac{Df_2(t)}{Dt} = \begin{cases} 0, & t < 2 \\ \text{undefined}, & t = 2 \\ 1, & 2 < t < 4 \\ -\delta(t-4), & t = 4 \\ 1, & t > 4 \end{cases}$$

$$\frac{Df_3(t)}{Dt} = \begin{cases} 0, & t < 0 \\ \text{undefined}, & t = 0 \\ 1, & 0 < t < 2, \\ \delta(t-2), & t = 2 \\ 1, & t > 2 \end{cases} \quad \frac{Df_4(t)}{Dt} = \begin{cases} 1, & t < -2 \\ \text{undefined}, & t = -2 \\ 0, & -2 < t < 5 \\ -\delta(t-5), & t = 5 \\ 0, & t > 5 \end{cases}$$

$$\frac{Df(t)}{Dt} = \begin{cases} 0, & t < -1 \\ \delta(t+1), & t = -1 \\ 0, & -1 < t < 1 \\ \delta(t-1), & t = 1 \\ -1, & 1 < t < 3 \\ \text{undefined}, & t = 3 \\ 0, & t > 3 \end{cases}$$

**Answer 2.35** Given in the textbook.

**Answer 2.36**

$$\begin{aligned} \frac{D}{Dt}\{tu(t)\} &= t\delta(t) + u(t) = 0\delta(t) + u(t) = u(t) \\ \frac{D^2}{Dt^2}\{tu(t)\} &= \frac{D}{Dt}\left\{\frac{D}{Dt}\{tu(t)\}\right\} = \frac{D}{Dt}\{u(t)\} = \delta(t) \\ \frac{D^3}{Dt^3}\{tu(t)\} &= \frac{D}{Dt}\left\{\frac{D^2}{Dt^2}\{tu(t)\}\right\} = \frac{D}{Dt}\{\delta(t)\} = \delta^{(1)}(t) \\ \frac{D^4}{Dt^4}\{tu(t)\} &= \frac{D}{Dt}\left\{\frac{D^3}{Dt^3}\{tu(t)\}\right\} = \frac{D}{Dt}\{\delta^{(1)}(t)\} = \delta^{(2)}(t) \end{aligned}$$

**Answer 2.37**

$$\begin{aligned} \frac{D}{Dt}\{\cos(t)u(t)\} &= -\sin(t)u(t) + \cos(t)\delta(t) = -\sin(t)u(t) + \delta(t) \\ \frac{D^2}{Dt^2}\{\cos(t)u(t)\} &= \frac{D}{Dt}\left\{\frac{D}{Dt}\{\cos(t)u(t)\}\right\} = \frac{D}{Dt}\{-\sin(t)u(t) + \delta(t)\} = -\cos(t)u(t) + \delta^{(1)}(t) \\ \frac{D^3}{Dt^3}\{\cos(t)u(t)\} &= \frac{D}{Dt}\left\{\frac{D^2}{Dt^2}\{\cos(t)u(t)\}\right\} = \frac{D}{Dt}\{-\cos(t)u(t) + \delta^{(1)}(t)\} = \sin(t)u(t) - \delta(t) + \delta^{(2)}(t) \\ \frac{D^4}{Dt^4}\{\cos(t)u(t)\} &= \frac{D}{Dt}\left\{\frac{D^3}{Dt^3}\{\cos(t)u(t)\}\right\} = \frac{D}{Dt}\left\{\sin(t)u(t) - \delta(t) + \delta^{(2)}(t)\right\} \\ &= \cos(t)u(t) - \delta^{(1)}(t) + \delta^{(3)}(t) \end{aligned}$$