Comments on Derivations of the Fourier Transforms of the Unit Step and Signum Signals

Using the notion of the generalized derivative, we have for the Heaviside unit step signal

$$F\{u_h(t)\} = \frac{1}{jw}, \quad w \neq 0$$

Formula (3.54) gives the Fourier transform for the signum signal

$$F\{\text{sgn}(t)\} = \frac{2}{jw}, \quad w \neq 0$$

Note that for $w = 0$, for the signum signal we obtain

$$F\{\text{sgn}(t)\}_{|w=0} = \int_{-\infty}^{\infty} \text{sgn}(t)dt = 0 \quad \Rightarrow \quad F\{\text{sgn}(t)\} = \begin{cases} \dfrac{2}{jw}, & w \neq 0 \\ 0, & w = 0 \end{cases}$$

However, for the unit step signal, we have

$$F\{u_h(t)\}_{|w=0} = \int_0^{\infty} 1dt = \frac{1}{2} \times \int_{-\infty}^{\infty} 1dt = \frac{1}{2} \times 2p\boldsymbol{d}(w) = p\boldsymbol{d}(w) \quad \Rightarrow \quad F\{u(t)\} = \begin{cases} \dfrac{1}{jw}, & w \neq 0 \\ p\boldsymbol{d}(w), & w = 0 \end{cases}$$

Having obtained the Fourier transform for the signum signal, we can use relationship (2.4) to find the Fourier transform for the unit step signal as given in (3.55).

Bracewell's derivations of the Fourier Transform for the Heaviside Unit Step Signal

He postulates the following result

$$jwF\{u(t)\} = 1 \quad \Rightarrow \quad F\{u(t)\} = \frac{1}{jw} + k\boldsymbol{d}(w)$$

To find the unknown constant, he uses the following arguments

$$u_h(t) = F^{-1}\{F(jw)\} = \frac{1}{2p} \int_{-\infty}^{\infty} F(jw)e^{jwt}dw = \frac{1}{2p} \int_{-\infty}^{\infty} (\frac{1}{jw} + k\boldsymbol{d}(w))e^{jwt}dw$$

For $t = 0$

$$u_h(0) = \frac{1}{2} = \frac{1}{2p} \int_{-\infty}^{\infty} \frac{1}{jw}dw + \frac{k}{2p} = \frac{k}{2p} \quad \Rightarrow \quad k = p$$