C. Some Results from Linear Algebra

Linear algebra plays a very important role in linear system control theory and applications (Laub, 1985; Skelton and Iwasaki, 1995). Here we review some standard and important linear algebra results.

Definite Matrices

Definition C.1: A square matrix $\mathbf{M}$ is *positive definite* if all of its eigenvalues have positive real parts, $Re\{\lambda_i(\mathbf{M})\} > 0$. It is *positive semidefinite* if $Re\{\lambda_i(\mathbf{M})\} \geq 0, \forall i$. In addition, *negative definite* matrices are defined by $Re\{\lambda_i(\mathbf{M})\} < 0, \forall i$ and *negative semidefinite* by $Re\{\lambda_i(\mathbf{M})\} \leq 0, \forall i$.

Null Space

Definition C.2: The null space of a matrix $\mathbf{M}$ of dimensions $m \times n$ is the space spanned by vectors $\mathbf{v}$ that satisfy $\mathbf{A}\mathbf{v} = \mathbf{0}$.

Systems of Linear Algebraic Equations

Theorem C.1 *Consider a consistent (solvable) system of linear algebraic equations in n unknowns*

$$\mathbf{M}\mathbf{x} = \mathbf{b} \tag{c.1}$$

with $\dim\{\mathbf{M}\} = m \times n$. Equation (c.1) has a solution if and only if (consistency condition)

$$\mathrm{rank}\{[\mathbf{M} \;\; \mathbf{b}]\} = \mathrm{rank}\{\mathbf{M}\} \tag{c.2}$$

In addition, if $\mathrm{rank}\{\mathbf{M}\} = m$, then (c.1) always has a solution. For $n = m$ and $\mathrm{rank}\{\mathbf{M}\} = m$ the solution obtained is unique.

Determinant of a Matrix Product

The following results hold for the determinant of a matrix product

$$\det\{\mathbf{M}_1\mathbf{M}_2\} = \det\{\mathbf{M}_1\}\det\{\mathbf{M}_2\} \tag{c.3}$$

For the proof of the above statement the reader is referred to Stewart (1973). This result can be generalized to the product of a finite number of matrices.

Determinant of Matrix Inversion

By using the rule for the determinant of a product we are able to establish the following formula

$$\det\left\{\mathbf{M}^{-1}\right\} = \frac{1}{\det\{\mathbf{M}\}} \tag{c.4}$$