HW #1 (1.11), (1.15)
due Feb. 4
Verify 7 and 11 from Appendix 1

Chapter 1

(1)

CONTROLLED DYNAMIC SYSTEM

(1) $\dot{x} = a(x, u, t)$, $x(t_0) = x_0$

$x \in \mathbb{R}^n$ is the state vector

$u \in \mathbb{R}^m$ is the control vector

$t \in [t_0, t_f]$ time

physical constraints

$u \in U$ — U-set of admissible controls

$x \in X$ — X-set of admissible trajectories

t_0 - initial time — $x(t_0)$ - initial state

t_f - final time — $x(t_f)$ - final state

$x(t_f) \in S$ - target set

THE PERFORMANCE MEASURE

(2) $J = \int_{t_0}^{t_f} g(x(t), u(t), t)\,dt + h(x(t_f), t)$ — scalar

h, g scalar function

OPTIMAL CONTROL PROBLEM: find $u \in U$ which minimizes (2) subject to (1) and constraints $x \in X, u \in U$

$\min_{u \in U} J \Rightarrow u^*$ and x^* optimal control and optimal trajectory

$\Rightarrow J(x^*, u^*) \leq J(x, u)$ $x \in X, u \in U$

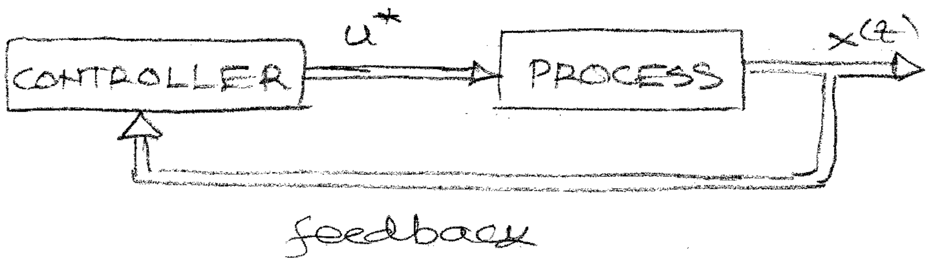
questions:

- does it (u^*) exist ?
- is it unique (useful) ?
- local or global minimum ?

OPEN-LOOP VS CLOSED LOOP

$u^*(t) = e(x(t_0), t) = e(t)$ $\Rightarrow$ open loop control

$u^*(x(t)) = f(x(t), t)$ $\Rightarrow$ closed loop (feedback)



STATE VARIABLES

$\dot{x} = a(x(t), u(t), t)$ nonlinear time varying

$\dot{x} = a(x(t), u(t))$ " " invariant

$\dot{x} = A(t)x + B(t)u$ linear time varying

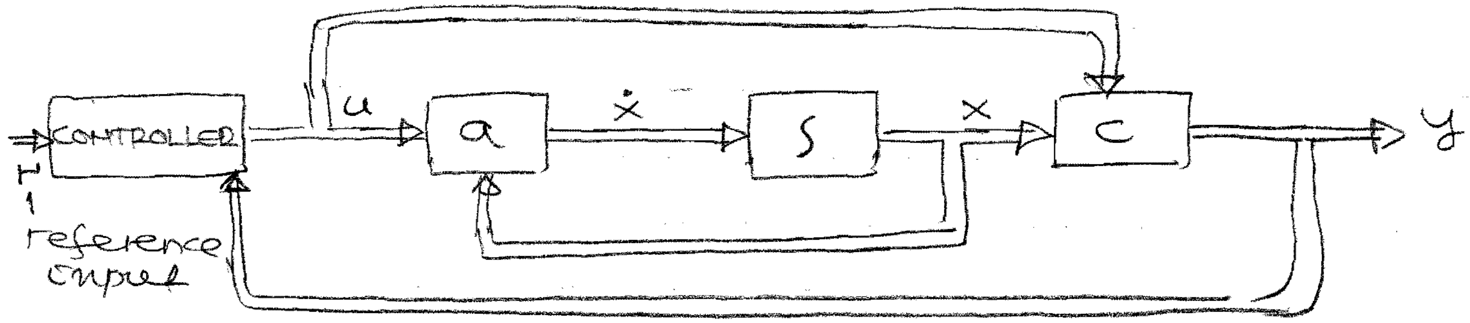
$\dot{x} = Ax + Bu$ " " invariant

OUTPUT EQUATION

$y(t) = c(x(t), u(t), t)$, $y \in \mathbb{R}^q$

$y(t) = Cx + Du$
linear

BLOCK DIAGRAM



LINEAR SYSTEMS.

$$\dot{x} = A(t)\,x(t) + B(t)\,u(t)$$

$$x(t) = \phi(t,t_0) + \int_{t_0}^{t} \phi(t,\tau)\,B(\tau)\,u(\tau)\,d\tau$$

$\phi(t,t_0)$ = system transition matrix

$\phi(t,t_0) = e^{A(t-t_0)}$ for time invariant sys.

$$e^{At} = I + At + \frac{A^2t^2}{2!} + \cdots$$

$$e^{At} = \mathcal{L}[(sI-A)^{-1}]$$

Properties of $\phi(t,t_0)$ TABLE 1. HW #1

1) $\phi(t,t) = I$

2) $\Phi(t_2,t_1)\,\phi(t_1,t_0) = \phi(t_2,t_0)$

3) $\Phi^{-1}(t_2,t_1) = \phi(t_1,t_2)$

4) $\dot{\phi}(t,t_0) = A(t)\,\phi(t,t_0)$, $\phi(t_0,t_0) = I$

CONTROLLABILITY

$$\dot{x}(t) = a(x(t), u(t), t) \qquad x(t_0) = x_0$$

If there is a finite $t_1 > t_0$ and a control $u \in \mathcal{U}$ which transfers $x(t_0)$ to the origin (with no lack of generality) $\Rightarrow$ controllability

If all components of $x(t_0)$ are controllable
$\Rightarrow$ completely controllable system

If all unstable components are controllabe
$\Rightarrow$ system is stabilizable

time linear system: CONTROLLABILITY TEST

$$\text{rank}\,[B \mid AB \mid \ldots A^{n-1}B] = n$$

OBSERVABILITY

If observing the output $y(t)$, $t \in [t_0, t_1]$ the state $x(t_0)$ can be completely determined,
$\Rightarrow$ observability.

If all component of $x(t_0)$ are observable
$\Rightarrow$ system completely observable

If unstable component are observable
$\Rightarrow$ system is detectable

$$\text{rank}\,[C^T \mid A^T C^T \mid \ldots \mid (A^T)^{n-1} C^T] = n \Rightarrow \text{observable}$$