# 332: 519 Advanced Topics in Systems Engineering
# Spring 1999
## Game Theory with Applications to Communication, Networking, and Control Systems

**Textbook:** T. Basar and J. Olsder, *Dynamic Noncooperative Game Theory*, SIAM, Philadelphia, 1999. (this is a slightly revised version of the *Academic Press* 2nd edition of the same book published in 1995.)

**Instructor:** Zoran Gajic, ELE Bld. 222, tel: (732) 445–3415, email: gajic@ece.rutgers.edu

**Office Hours:** M 7;30–9:00 pm, F 11:00–12:30 am

**Class Web Site:** http://www.ece.rutgers.edu/~gajic/519b.html

**TOPICS**

*Week 1:* Course overview. Introduction to games with historical development of games. Introduction to extensive game form. Examples. (Chapter 1, class notes).

*Week 2:* Introduction to normal game form. Introduction to static games in Euclidean spaces. Introduction to dynamic games. (class notes, Chapter 4).

*Week 3:* *Finite zero-sum static games* in normal form (matrix games). (Sections 2.1–2.4)

*Week 4:* *Finite zero-sum static games* in extensive form. (Sections 2.4–2.6).

*Week 5:* *Nash strategies for finite* noncooperative *static* games. (Sections 3.1–3.3, 3.5)

*Week 6:* *Stackelberg* strategies *for finite* noncooperative *static* games. (Section 3.6). Braess paradox (Section 4.7).

*Week 7:* *Nash strategies* in Euclidean spaces (*infinite static* games). (Section 4.3).

*Week 8:* *Stackelberg strategies* in Euclidean spaces (Section 4.4). Dynamic games and *dynamic optimization* (Section 5.5; class notes).

*Week 9:* *Open-loop dynamic Nash strategies in continous-time* (Section 6.5 and class notes).

*Week 10:* *Nash feedback strategies* for infinite *dynamic* games. (Section 6.5, class notes) Isaacs equation of differential zero-sum games. (Section 8.2, class notes).

*Week 11:* *Stackelberg strategies* in continuous-time for infinite *dynamic* games. (Section 7.6).

*Week 12:* Exam

*Week 13:* Applications to Communications and Networking (discussions of journal papers)*.

*Week 14:* Applications to Communications and Networking (discussions of journal papers)*.

**Grading:**

70% Exam (based on theory and simple demonstration examples)

30% Term Paper (in students' areas of interest), up to 4 printed, single spaced, two-column pages (conference paper format) with the classroom presentation on May 12, 1999, 6:30–9:20.pm.

* The papers of the following authors in communications and networking will be discussed: Lazar, Basar, Orda, Shenker, Altman, Korilis, Douligeris, Mazumdar, Shah, Mandayam, Goodman, and others time permitting. Papers dealing with applications of game theory to controls will not be discussed in class. Control students will write term papers based on important journal papers dealing with applications of game theory to control systems.

# EXAM I — 330:416 — Spring 1999

March 29, 1999

Answer eight out of ten questions. Each question carries 3 points.

**1)** Define the system transient response parameters and explain the procedure for finding these parameters.

**2)** Define and derive steady state response errors for linear feedback control systems.

**3)** Present design algorithms based on the root locus technique for PID and phase-lead-lag controllers and comment on their main features.

**4)** Show how to read from Bode diagrams the phase and gain margins, phase and gain crossover frequencies, and steady state constants.

**5)** Plot the Bode diagrams for the phase lead controller and find its maximal phase and the frequency at which the phase maximum occurs.

**6)** Derive the observability rank conditions for both discrete- and continuous-time linear systems.

**7)** Explain the full-order observer design technique and draw the corresponding block diagram. Give the observer equations, the equation for the observation error, and explain a rational choice for observer poles.

**8)** Explain and justify the separation principle in the context of the observer design problem.

**9)** For a discrete-time linear stochastic system driven by a zero-mean stationary Gaussian stochastic white noise process derive state mean and state variance equations.

**10)** State and present the solution to the optimal deterministic continuous-time linear regulator problem.

## Exam in Game Theory — Spring 1999

April 16, 1999

**Part I. Classic Games:** Answer 5 out of 5 questions

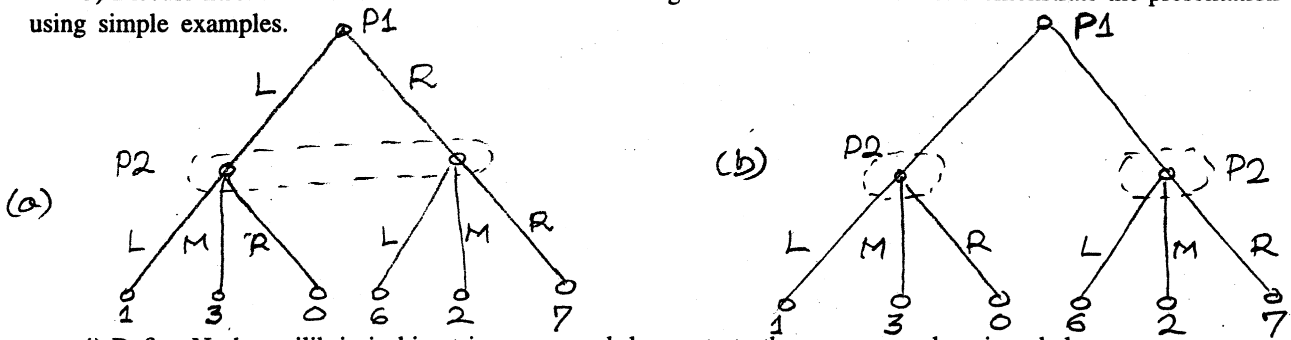
1) Define security strategies in normal (matrix) form zero-sum finite games and demonstrate them on a simple example

$$A = \begin{bmatrix} 1 & 3 & 3 & -2 \\ 0 & -1 & 2 & 1 \\ -2 & 2 & 0 & 1 \end{bmatrix}$$

Define the lower and upper game values and discuss the existence of a saddle-point solution .

2) State and prove the minimax theorem.

3) Discuss informational structures in zero-sum finite games in extensive forms. Demonstrate the presentation using simple examples.




4) Define Nash equilibria in bimatrix games and demonstrate them on examples given below

$$(a)\ A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix},\ B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}, \quad (b)\ A = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix},\ B = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}, \quad (c)\ A = \begin{bmatrix} 8 & 0 \\ 30 & 2 \end{bmatrix},\ B = \begin{bmatrix} 8 & 30 \\ 0 & 2 \end{bmatrix}$$

5) Discuss Stackelberg strategies in bimatrix form and demonstrate ~~it~~ them on the following example

$$A = \begin{bmatrix} 0 & 2 & 3/2 \\ 1 & 1 & 3 \\ -1 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & -2/3 \\ 2 & 0 & 1 \\ 0 & 1 & -0.5 \end{bmatrix}$$

Demonstrate a secured cost strategy on the following example

$$A = \begin{bmatrix} 0 & 1 & 3 \\ 2 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & -1 \end{bmatrix}$$

and give definitions of reaction sets and secured equilibrium Stackelberg strategy (class notes (6), pages 1–4).

**Part II. Static Games in Euclidean Spaces:** Answer 2 out of 3 questions

1) Define zero-sum games in Euclidean spaces (infinite games) with sequential decision making (minmax and maxmin solutions).

2) Plot the iso-cost curves for Stackelberg strategies for the case when either player does better as the follower than the leader.

3) Define Nash equilibria, state the necessary conditions for Nash equilibria, and give definitions of reaction sets and reaction curves (class notes (7), pages 1–3).

**Part III. ~~Classic~~ Dynamic Games:** Answer 3 out of 4 questions

1) Derive the Hamilton-Jacobi-Bellman equation and state necessary conditions for dynamic Nash games.

2) Solve the linear-quadratic Nash game when both players use the feedback strategies.

3) Solve the linear-quadratic zero-sum differential game.

4) Define dynamic Stackelberg strategies and state the necessary conditions for optimality.

**Potential Questions for Exam in Game Theory — Spring 1999**

Last update Appril 9, 1999 — Final list of questions

1. Define deterministic finite games in extensive form (class notes (1), pp. 5–6).
2. Define the finite deterministic game in normal form and give an example that relates game's extensive and normal forms. (class notes (2), pp. 1–2).
3. Define zero-sum games in Euclidean spaces (infinite games) with simultaneous decision making (saddle point solution). (class notes (2), pp. 3–5).
4. Define zero-sum games in Euclidean spaces (infinite games) with sequential decision making (minmax and maxmin solutions). (class notes (2), pp. 6–7, see also Chapter 4).
5. Define Nash strategies in Euclidean spaces, plot iso-cost curves and discuss the existence of Nash equilibria. (class notes (2), pp. 8–9, see also Chapter 4).
6. Define Stackelberg strategies in Euclidean spaces. (class notes (2), pp. 10–12).
7. Plot the iso-cost curves for Stackelberg strategies for the case when either player does better as the follower than the leader. (to be deduced from class notes (2), pages (8)-12)).
8. Define the Pareto (cooperative) strategies in Euclidean spaces (class notes (2), pages 12–13).
9. Define security strategies in normal (matrix) form zero-sum finite games and demonstrate them on a simple example. Define the lower and upper game values and discuss the existence of a saddle-point solution (class notes (3), pp. 1–4, Section 2.2).
10. Define mixed strategies in zero-sum finite games. Explain the need to mixed strategies, define mixed security strategies, and the saddle-point solution in mixed strategies. (class notes (3), pages 5–6, Section 2.2).
11. State and prove Lemma 2.1 (class notes (3), pp. 9–11, Section 2.2).
12. State and prove the minimax theorem (class notes (3), pp. 10–12a, see also comments from class notes (4), pp. 10) .
13. Show how to compute mixed strategies on a simple $2 \times 2$ or $2 \times 3$, $3 \times 2$ examples (class notes (3), pp. 13–14, Section 2.3).
14. Discuss informational structures in zero-sum finite games in extensive forms. Demonstrate the presentation using simple examples. (class notes (4), pages 1–3, Section 2.4).
15. Present the algorithm for solving zero-sum finite games in extensive form and demonstrate it on a simple example. (class notes (4), pp. 4–5, Section 2.4).
16. Discuss the need for introducing behavioral strategies and their relationship to mixed strategies (class notes (4), pp. 5–7, Section 2.4).
17. Define open-loop multi-act zero-sum finite games and demonstrate them using a simple tree (class notes (4), pages 7–8, Section 2.5).
18. Define feedback multi-act zero-sum finite games and demonstrate them using a simple tree (class notes (4), pages 9–10, Section 2.5).
19. Demonstrate a zero-sum finite game with chance moves using a simple tree (class notes (4), pp. 10, Section 2.6).
20. Define Nash equilibria in bimatrix games and demonstrate them on a simple example (class notes (5), pp. 1–3).
21. Define minimax security strategies in bimatrix game and demonstrate them on a simple example (class notes (5), pp. 4–5).
22. Define mixed strategies in Nash games and state Theorem 3.1 (class notes (5), pp. 5).
23. Define a Nash equilibrium for N-person finite Nash games in normal form and state the corresponding existence theorem for mixed strategies (class notes (5), pp. 6).
24. Discuss Nash strategies in extensive form with static and dynamic informational structure, draw the main conclusions, and present simple examples (class notes (5), pages 7–10).
25. Discuss Stackelberg strategies in bimatrix form, demonstrate a secured cost strategy on an example, and give definitions of reaction sets and secured equilibrium Stackelber strategy (class notes (6), pages 1–4).
26. Define reaction sets and equilibrium Stackelberg strategies in mixed strategies. Find the game cost for the leader in pure and mixed strategies (class notes (6), pages 5–7).
27. Explain the essence the Braess paradox without going into details (class notes (6), pages 8–10).
28. Define Nash equilibria, state the necessary conditions for Nash equilibria, and give definitions of reaction sets and reaction curves (class notes (7), pages 1–3).
29. Define stable and unstable Nash equilibrium strategies (class notes (7), page 4).
30. Discuss existence and uniqueness of Nash equilibria (class notes (7), pages 5–6).

31. Present main definitions for Stackelberg strategies (class notes (7), pages 7–8
32. Derive the Hamilton-Jacobi-Bellman equation and state the set of necessary conditions for dynamic optimization (class notes (8), pages 1–4).
33. Solve the linear-quadratic dynamic optimization problem and derive the Riccati equation (class notes (8), pages 5–6).
34. Derive the Hamilton-Jacobi-Bellman equation and state the set of necessary conditions for dynamic Nash games (class notes (9), pages 1–6).
35. Solve the linear-quadratic Nash game when both players use the open-loop strategies (class notes (9), pages 7–11).
36. Solve the linear-quadratic Nash game when both players use the feedback strategies (class notes (10), pages 12–15).
37. Find the optimal value for the performance criterion for the linear-quadratic Nash strategies (class notes (10), pages 17–18).
38. Derive the Isaacs equation of general zero-sum games and state necessary conditions for optimality (class notes (10), pages 18–21).
39. Solve the linear-quadratic zero-sum game (class notes (10), pages 21–22).
40. Define dynamic Stackelberg strategies and state the necessary conditions for optimality (class notes (11), pages 1–3).
41. Solve a linear-quadratic dynamic Stackelberg game (class notes (11), pages 3–4).

EXAM, APRIL 16, FRIDAY 6:30–9:20

# Instructions for Term Paper Preparation

## 332: 519 — Spring 1999
## Game Theory with Applications to Communications, Networking, and Control

Please follows these instructions as closer as possible:

1. Paper size = 8.5"x11".
2. Two-column format with column separation of no less than 3/8".
3. Left, right, top, and bottom margins = 1 inch.
4. Font sizes = 10, 11, or 12.
5. The title of the paper should be centered across the page.
6. The author's name and affiliation should be centered across the front page below the paper's title.
7. An abstract must be included.

Papers should contain up to **four** pages.
PAPERS ARE DUE ON MAY 12, 1999 by 9:00am to Professor Gajic's Office.