Text book: T. Basar and G. Olsder "Dynamic Noncooperative Games," SIAM, 1999

Office hours: M 7:30–9:00, F 11–12:30

Lgun → www.eden.rutgers.edu/~eggcom/game 4 comm. & trac
web page on games

Jan. 22, 99 (2)

NORMAL GAME FORM

The extensive game form described by a game tree displays explicitly the evolution of the game. However, the description by itself becomes complex in the case of many players, many alternatives, and many stages.

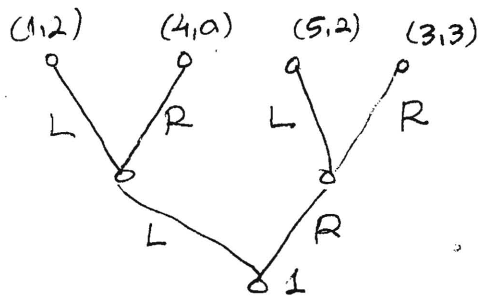
The normal game form suppresses information about game evolution and expresses the utility (loss) functions explicitly in terms of players' strategies.

$$J_i = J_i(u_1, u_2, \ldots, u_N), \quad i = 1, 2, \ldots, N$$

J_i = loss function (utility function, performance criterion) (cost function)

u_i = strategy of player i

(Example)



All possible strategies are

$\underset{u_1\ u_2}{LL}$, LR, RL, $\underset{u_1\ u_2}{RR}$

Hence

$$J_1 = J_1(u_1, u_2)$$

$$J_2 = J_2(u_1, u_2)$$

Rational behavior of each player is to minimize his (her) own losses assuming that the other player is doing the same.

$$J_1(u_1^*, u_2^*) \le J_1(u_1, u_2^*)$$
$$J_2(u_1^*, u_2^*) \le J_2(u_1^*, u_2)$$

Such a strategy (u_1^*, u_2^*), assuming that it exists is called the equilibrium strategy.

In general, for N-players, the loss functions are defined by

$$J_i = J_i(u_1, u_2, \ldots, u_N) \quad , \quad i = 1, 2, \ldots, N$$

with the equilibrium strategy satisfying

$$J_i(u_1^*, u_2^*, \ldots, u_i^*, \ldots, u_N^*) \le J_i(u_1^*, u_2^*, \ldots, u_i, u_{i+1}^*, \ldots u_N^*)$$

For $N=2$, normal form games are called the MATRIX games since they can be described using matrices

player 1 \ player 2	L	R
L	(1,2)	(4,0)
R	(5,2)	(3,3)

or

P1 \ P2	L	R
L	$J_1=1$ $J_2=2$	$J_1=4$ $J_2=0$
R	$J_1=5$ $J_2=0$	$J_1=3$ $J_2=3$

GAMES IN EUCLIDEAN SPACES
(Infinite games)

1) ZERO-SUM GAMES
   a) simultaneous decision making ⇒ saddle point solution
   b) sequential decision making ⇒ minmax or maxmin solution

2) CONFLICT GAMES
   a) simultaneous decision making ⇒ Nash strategies
   b) sequential decision making ⇒ Stackelberg "

3) COOPERATIVE GAMES ⇒ Pareto strategies

All games in E^n spaces are described by a static state equation

$$f(x,u) = 0$$

$x \in R^n$ = state of the game vector

$u \in R^m$ with $u = (u_1, u_2, \dots, u_M)$

$u_i \in R^{m_i}$ = strategy of player i

In addition, each player has a loss function (utility function, cost, performance criterion)

$$J_i(x,u) \quad , \quad i = 1,2,\dots,M$$

Assuming that $f(x,u)=0 \Rightarrow x = \varphi(u)$

$$\Rightarrow J_i(\varphi(u), u) = J_i(u_1, u_2, \dots, u_M)$$

Note that none of the players has complete control over the game outcome.

1) ZERO-SUM GAMES $(J_2 = -J_1)$

a) Saddle point strategies

From

$$\left.\begin{aligned} J_1(u_1^*, u_2^*) &\le J_1(u_1, u_2^*) \\ J_2(u_1^*, u_2^*) &\le J_2(u_1^*, u_2) \end{aligned}\right\} \quad (1)$$

and the fact that $J_2 = -J_1$, and

$$J_2(u_1^*, u_2^*) \le J_2(u_1^*, u_2) \implies -J_1(u_1^*, u_2^*) \le -J_1(u_1^*, u_2)$$

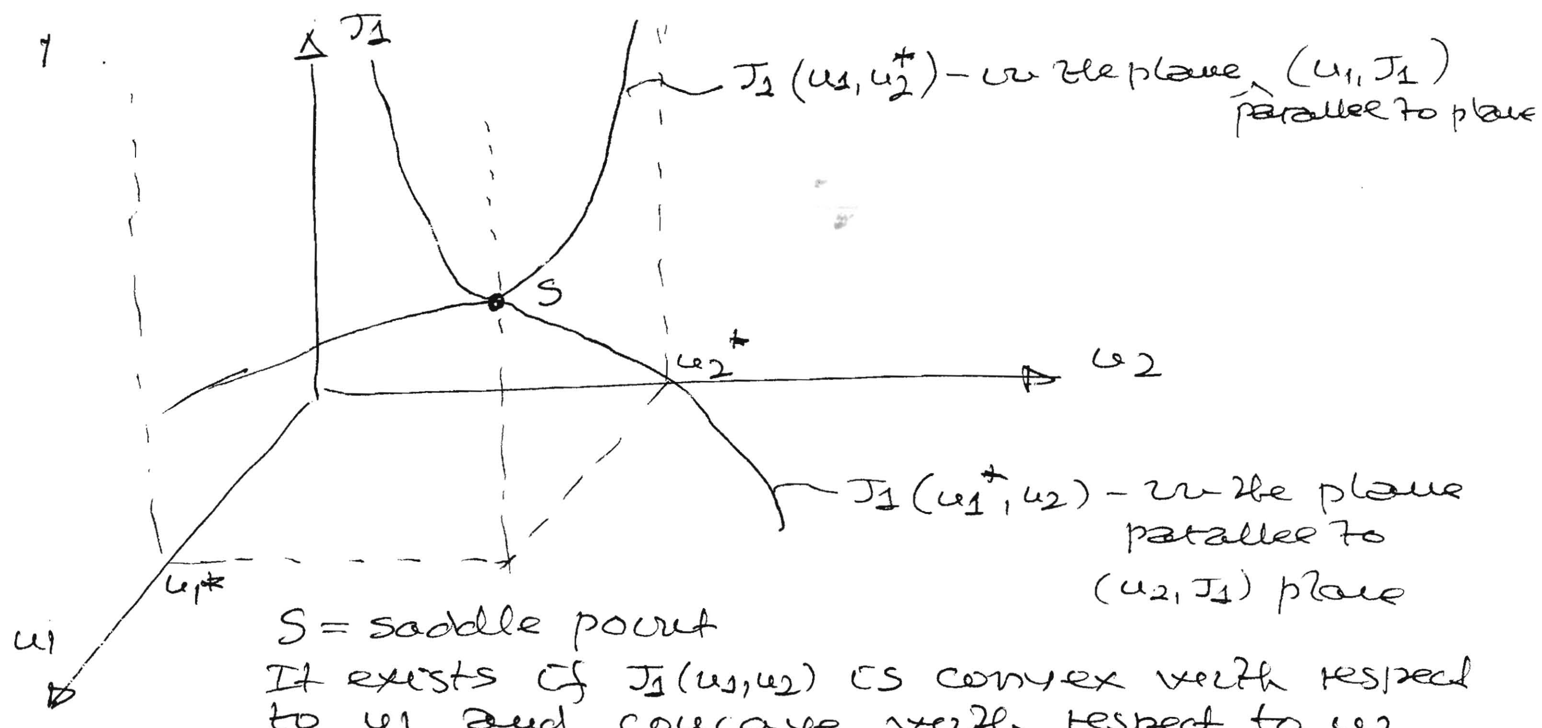
$$\implies J_1(u_1^*, u_2^*) \ge J_1(u_1^*, u_2)$$

we get from (1)

$$\boxed{J_1(u_1^*, u_2) \le J_1(u_1^*, u_2^*) \le J_1(u_1, u_2^*)} \qquad (2)$$

which represents the saddle point condition.

Apparently in this zero-sum game the player 1 is minimizer and the player 2 is the maximizer.



S = saddle point

It exists if $J_1(u_1, u_2)$ is convex with respect to u_1 and concave with respect to u_2