STATIC OPTIMIZATION IN EUCLIDEAN SPACES (Nonlinear Programming

(A) UNCONSTRAINED OPTIMIZATION

$\min f(x)$ subject to $x \in S \subset E^n$, f = scalar function

RESULT 1

$f(x)$ = convex function, S = convex set

$\Rightarrow$

(1) $\nabla f(x^*)^T (x - x^*) \geq 0 \quad \forall x \in S$

represents the necessary and sufficient condition for x^* to solve the original optimization problem.

$$\nabla f(x) = \text{gradient vector} = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

RESULT 2

$f(x)$ is any function (scalar) over E^n, $x \in E^n$

Necessary condition for a local minimum:

(2) $\nabla f(x^*) = 0$ gradient equal to zero

or

(3) $H(x^*) \geq 0$ Hessian (matrix) positive semidefinite

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & & & \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

Note that the necessary condition (2) requires that $f(x): E_n \to E_1$ is differentiable, and that the necessary condition (3) require that $f(x)$ is twice differentiable

SUFFICIENT CONDITION

$f(x)$ is twice differentiable at x^* and

(4) $\nabla f(x^*) = 0$ and $H(x^*) > 0$ positive definite

(B) INEQUALITY CONSTRAINTS

$f: E^n \to E^1$ and $g_i: E^n \to E^1$

$$\boxed{\min f(x) \quad \text{subjet to} \quad x \in X \subset E^n \quad \text{and} \quad g_i(x) \le 0,\ i=1,2,..,m}$$

RESULT 3 Kuhn-Tucker Necessary conditions

$$\left.\begin{aligned} \nabla f(x^*) + \nabla g(x^*)\, u &= 0 \\ u^T g(x^*) &= 0 \\ u &\ge 0 \end{aligned}\right\} \qquad \text{note } g = \begin{bmatrix} g_1 \\ g_2 \\ g_m \end{bmatrix} \quad \nabla g(x^*) = (\ )^{n \times m} \text{ matrix}$$

u = vector of Lagrange multipliers

If $f(x)$ and $g(x)$ are convex (more precisely $g_i(x)$ convex for every i) then the above conditions are also sufficient.

The above can be written as

$$\nabla f(x^*) + \sum_{i=1}^{m} u_i \nabla g_i(x^*) = 0$$

$$u_i g_i(x^*) = 0, \quad i = 1,2,..,m$$

$$u_i \ge 0, \quad i = 1,2,..,m$$

(C) INEQUALITY and EQUALITY CONSTRAINTS

Minimize $f(x)$

subject to $g_i(x) \le 0, \quad i = 1, 2, \dots, m$

$h_i(x) = 0, \quad i = 1, 2, \dots, \ell$

$x \in X \subset E^n$

RESULT 4 Kuhn-Tucker Necessary Conditions

There are exist scalars u_i and v_i such that

$$\nabla f(x^*) + \nabla g(x^*) u + \nabla h(x^*) v = 0$$

$$u^T g(x^*) = 0$$

$$u \ge 0$$

or

$$\nabla f(x^*) + \sum_{i=1}^{m} u_i \nabla g_i(x^*) + \sum_{i=1}^{\ell} v_i \nabla h_i(x^*) = 0$$

$$u_i g_i(x^*) = 0$$

$$u_i \ge 0$$

u_i, v_i are Lagrange multipliers.

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