GAME THEORY

1928 von Neumann had published a paper on game theory. That year marks the beginning of mathematical game theory.

1921 in statistics, Borel introduced the concept of game theory

1944 von Neumann and Morgenstern published the book "Theory of Games and Economic Behaviour"

1957 Luce and Raiffa (Columbia Univ. statistics group) published the book "Games and Decisions", Wiley

Hence, the foundation of game theory was established within mathematics, statistics, and economy.

Another source comes from engineering applications

1965 Isaacs published "Differential Games"
This book was actually written 1951–1955, but forbidden for publication due to potential military applications.
The central theme are zero-sum differential games
min max (distance between two moving objects)

970 The third source to games is the optimal control theory rapidly evolving by the end of sixties and during the 1970's.
These are mostly dynamic games
1) cooperative games (Pareto strategies)
2) conflict games
   2a) simultaneous decision making (Nash strategies)
   2b) sequential decision making (Stackelberg strategies)
3) zero-sum differential games.

70's, 80's, 90's Development of game theory in
mathematics, statistics
economy
politics, sociology, ecology, biology, hydrology
engineering
military

1990's Important for EE students and researchers applications of games to networking, communications, and signal processing.
Comeback of zero-sum games to control (since H_∞-optimization (optimization of system transfer function in the frequency domain = zero-sum differential game problem

1994 John Nash Jr. got the Nobel Price for economics together with John Harsanyi and Reinhard Selton – both game theorists in economics.

Journals:

1) Game Theory (Math - too much theoretical and mathematical)

2) JOTA (Math & Eng - friendly to grad. student
Journal Optimization Theory and Application

3) Games and Economic Behavior (Economy)

4) IEEE Transactions on Control
5) Automatica

control journal (readable for all EE grad-student

6) Recently IEEE Trans. on Communications
" " " Networking
IEEE J. Selected Areas in Communications
IEEE Trans. on Signal Processing, Journal of ACM

Games and Related Course at Rutgers

16:220:546 Topics in Game Theory (Economics)
(Prof. E. Friedman and R. McLean

no graduate course on games in Mathematics and Statistics Departments.

16:198:521 Linear Programing (Comp. Sci. Dept.)
16:540:510 Deterministic Models in Industrial Eng (Linear Programming)

Ind Eng

STATIC OPTIMIZATION

16:540:615 Nonlinear Programming
16:198:524 Nonlinear Programming Algorithms
16:332:510 Synthesis of Optimum Control Systems
(Dynamic Programming - Dynamic Optimizatio

Need Linear and Nonlinear Programming for STATIC GAMES
Need Dynamic Optimization for DYNAMIC GAMES

CLASSIC GAMES

STATIC GAMES

1) EXTENSIVE FORM
2) NORMAL FORM
3) MATRIX FORM

MODERN GAMES

4) IN EUCLIDEAN SPACES (finite dimensional E^n)
   a) zero-sum games (antagonistic games)
   b) conflict games (Nash and Stackelberg)
   c) cooperative games (Pareto)

DYNAMIC GAMES

5) GAMES IN FUNCTIONAL SPACES
   a) differential games (zero-sum games)
   b) conflict games
   c) cooperative games
   d) coalition games
   e) team theory

All of the above can be deterministic and stochastic

Schools with strong programs in games
Univ. of Illinois (Eng.)
Princeton Univ. (Math – in the past)
Columbia Univ (Stats. – in the past)

These days Lazar teaches a course in EE Dept at Columbia "Resource Allocation and Networking Games". His former student Dr. Korilis teaches the similar course ~~at~~ NYU in the Computer Sci. Dept.

EXTENSIVE FORM (DESCRIPTION)

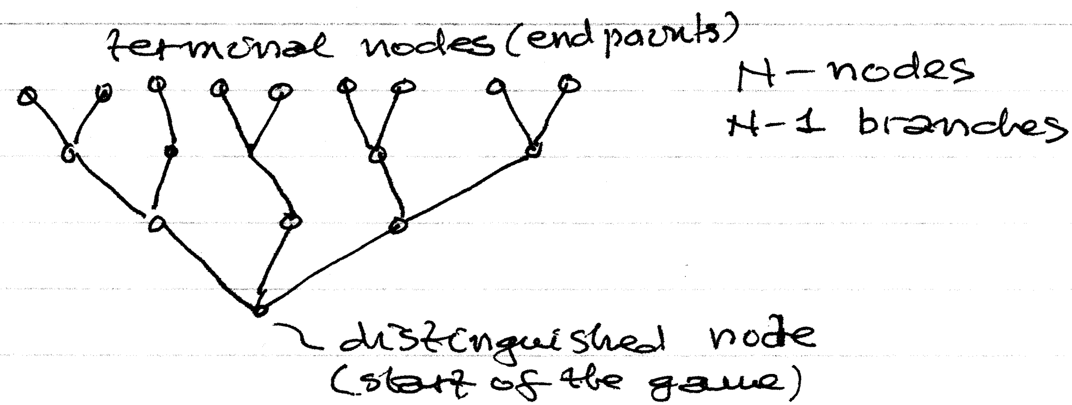
This approach originated in the work of Von Neumann. It is used to describe games with finite number of strategies (moves). These games are called FINITE GAMES. They can be deterministic (with full information) or stochastic (with partial information).

DETERMINISTIC FINITE GAMES:

1) The order of strategies (moves) is precisely defined.
2) The game has a finite number of moves.
3) At every move the number of available alternatives is finite.
4) Every player (decision maker) has complete information about the game.

Such games are represented by GAME TREES (graph)

A <u>connected graph</u> is composed of a finite number of nodes connected by branches. A connected graph without closed loop of branches is called a <u>tree.</u>



Deterministic Finite Game is described by

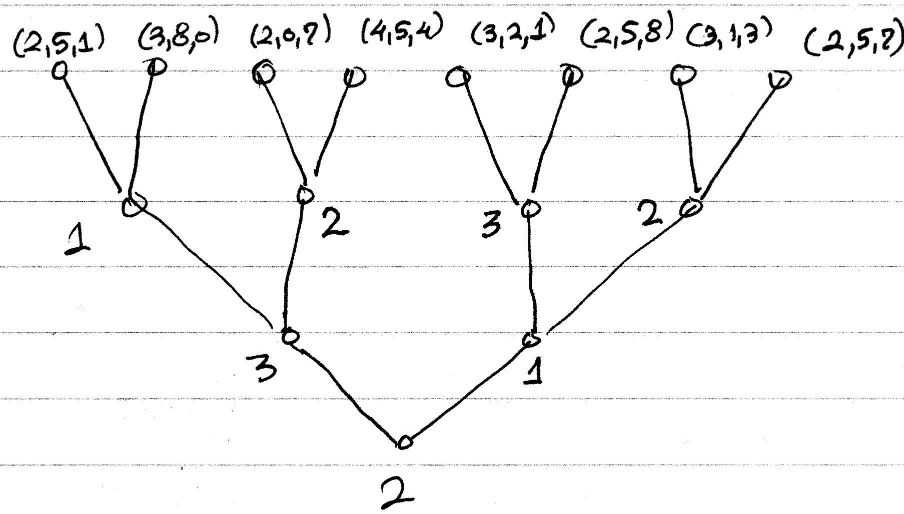
1) A finite tree with a distinguished node.

2) Utility (loss) function assigned to every terminal node

$$x_j(x_1^j, x_2^j, \ldots, x_n^j), \quad j = 1, 2, \ldots k = \text{number of terminal nodes}$$

loss of player n at node j

3) Partition of internal nodes into $S_1, S_2, \ldots, S_n$ subsets indicating who is supposed to make a move at the given node

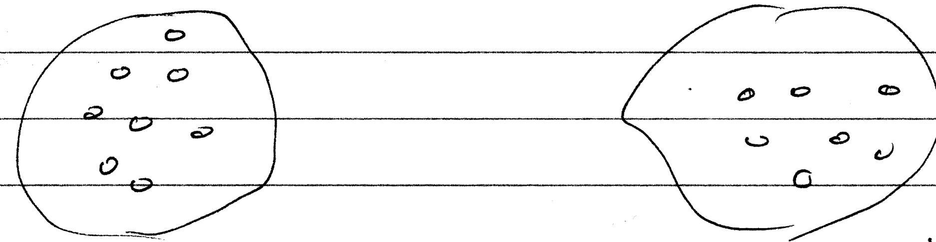


In the case of stochastic games we can do partitioning into $S_0, S_1, S_2, \ldots, S_n$.
S_0 nodes determine their moves randomly.

<u>A strategy</u> of player i determines only one branch at each node at which the player i is supposed to make a move.

Example:

2 piles of stones, with respectively n and m stones



n-stones $\qquad$ m-stones

Game Rule: Take either the same number of stones from each pile or an arbitrary number of stones from one of the piles.

The winner is the player who clears the last stone.

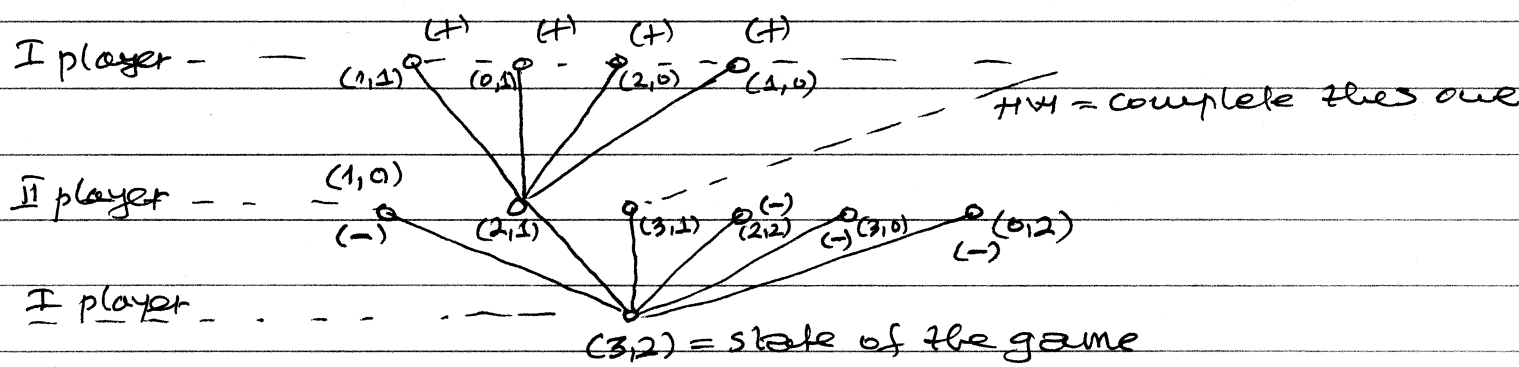
The players take turns.

The maximal number of moves is $n+m$

The players have complete information about the game

$\Rightarrow$ finite deterministic game.

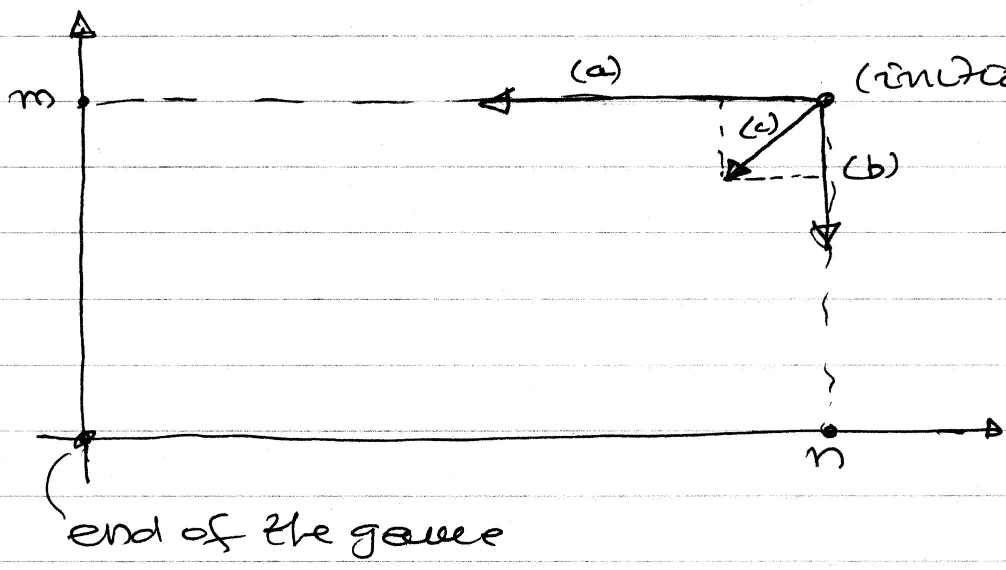
Take case $n=3$ and $m=2$ and draw the tree



(−) = loss for player I, (+) = win for player I

Looks simple. But, what about $n=1119$, $m=235$ or $n=25$, $m=36$ to make the game simpler. Who is going to win this game and what is the winning strategy?

Available strategies



(a), (b), (c) rules of the game

Apparently $(i,0)$, $(0,i)$, (i,i) the player who has to make a move wins.

Also $(1,2)$ the player who makes the move is going to loose. Also, using the tree can be shown that $(3,5) \Rightarrow$ the player who has to make the move is going to loose.

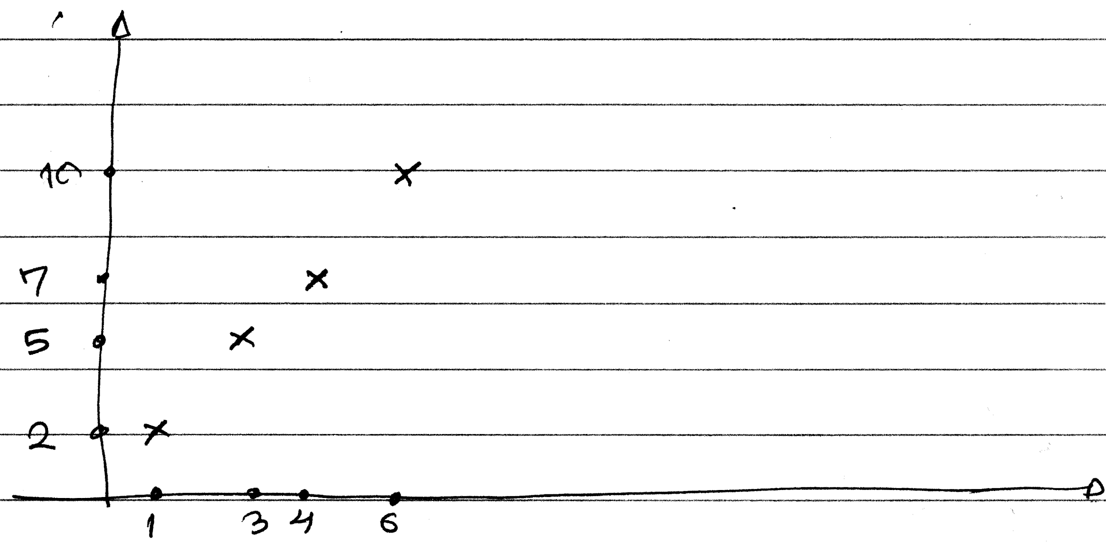
Solution: (tough to find)

$$L = \left(\left[\frac{1+\sqrt{5}}{2} k \right], \left[\frac{3+\sqrt{5}}{2} k \right] \right), \; k=1,2,\ldots$$

[ ] denotes an integer part operation

determines a discrete "surface" on which the player who has to make the move looses the game

"Loosung surface"



$L = \{(1,2), (3,5), (4,7), (6,10), (8,13), (9,15), (11,18), (12,20)$
$(14,23), (16,26), (17,28), (19,29), (21,34), (22,36), \ldots \}$

$= \{ [1.618k], [2.618k] \}$

Note that $\left[\frac{3+\sqrt{5}}{2} k\right] - \left[\frac{1+\sqrt{5}}{2} k\right] = k$

Hence, if the state of the game
$(n,m) \in L$
the player who has to make the move looses

If $(n,m) \notin L$ then the player who has to make the move can bring it to L and assures his own win.

(Ex) $n=25$, $m=36$ $\Rightarrow$ take 3 stones from the first pile
$\Rightarrow (22,36) \Rightarrow L$

$n = 1119$, $m = 235$

$k=90 \Rightarrow \left[\frac{3+\sqrt{5}}{2} k\right] = [235.623] = 235$ and $\left[\frac{1+\sqrt{5}}{2} \cdot 90\right] = 145$

$(145, 235) \Rightarrow L$

$\Rightarrow 1119 - 145 = 974$ hence take 974 stones from the first pile