# Chaos and bifurcation in Power Electronics

## Medical Instruments Implications

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## Number of Variables →

| Nonlinearity ↓ | $n = 1$ | $n = 2$ | $n \geq 3$ | $n >> 1$ | Continuum |
|---|---|---|---|---|---|
| | *Growth, Decay, or Equilibrium* | *Oscillations* | | Collective Phenomena | Waves and patterns |
| Linear | Exponential Growth<br>RC circuit<br>Radioactive decay | Linear oscillator<br>Mass and Spring<br>RLC circuit<br>2-body problem<br>(Kepler, Newton) | Civil engineering, Structures<br>Electrical Engineering | Coupled harmonic oscillators<br>Solid-state physics<br>Molecular dynamics<br>Equilibrium statistical mechanics | Elasticity<br>Wave equations<br>Electromagnetism (Maxwell)<br>Quantum mechanics<br>Heat and diffusion<br>Acoustics<br>Viscous fluids |
| | | | | *The frontier* | |
| | | | *Chaos* | | *Spatio-temporal complexity* |
| Nonlinear | Fixed points<br>Bifurcations<br>Overdamped systems, relaxational dynamics<br>Logistic equation for single species | Pendulum<br>Anharmonic oscillators<br>Limit cycles<br>Biological oscillators (neurons, heart cells)<br>Predator-Prey cycles<br>Nonlinear electronics (van der Pol,Josephson) | Strange Attractors<br>3-body problem<br>Chemical Kinetics (Feigenbaum)<br>Fractals (Mandrelbrot)<br>Forced nonlinear oscillators<br><br>Practical uses of chaos<br>Quantum chaos? | Coupled nonlinear oscillators<br>Lasers, nonlinear optics<br>Nonequilibrium statistical mechanics<br>Nonlinear solid-state physics (semiconductors)<br>Josephson arrays<br>Heart cell synchronization<br>Neural networks<br>Immune system<br>Ecosystems<br>Economics | Nonlinear Waves (Shocks, Solitons)<br>Plasmas<br>Earthquakes<br>General relativity (Einstein)<br>Quantum field theory<br>Reaction-diffusion, biological and chemical waves<br>Fibrillation<br>Epilepsy<br>Turbulent fluids (Navier-Stokes)<br>Life |

Poincaré in 1899 first glimpsed the possibility of chaos

Dynamics, nonlinear oscillators

*applied in radio*

*radar*

*PLL's*

*Lasers*

New Mathematical Techniques developed by :

- *Van der Pol*
- *Andronov*
- *Littlewood*
- *Cartwright*
- *Levinson*
- *Smale*
- *Kawakami*

# Sources of nonlinearities in Power Electronics

1. Switching devices(intrinsically nonlinear).
2. Reactive and/or nonlinear energy storing components( inductors and capacitors).
3. Electrical Machines and drives.

## Power switching devices

- Diode
- Thyristor (SCR)
- Bipolar junction transistor (BJT)
- Power MOSFET
- IGBT

*1.) Iteration (feedback) + nonlinear elements → Bifurcation → Chaos*

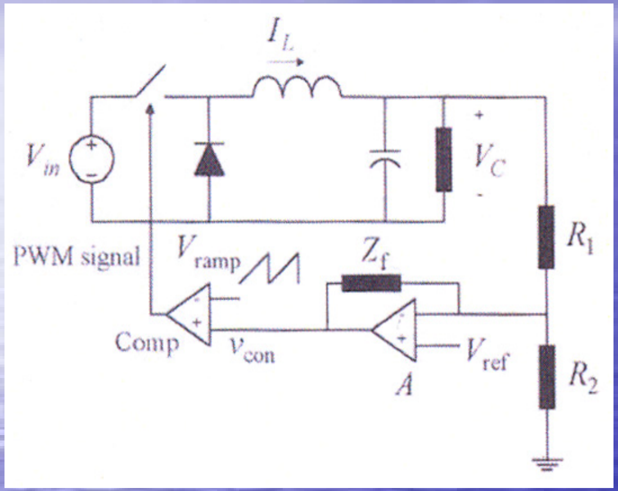
*2.) Universality of Chaos*

*3.) (Strange) Attractors.*

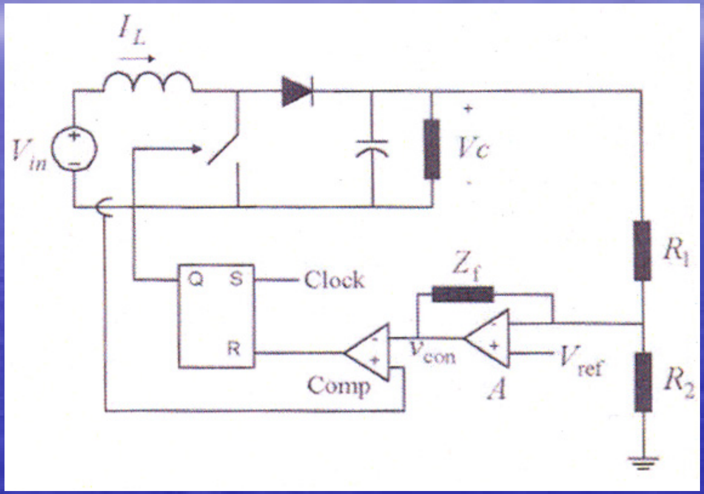
*Attractor : Loose definition ;A set to which all neighboring trajectories converge.*

*Strange attractor; an attractor that exhibits sensitive dependence on initial conditions.*

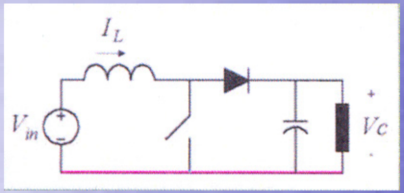
DC/DC Buck converter (Vc<Vin)

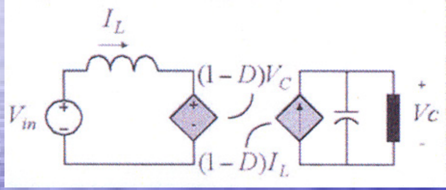


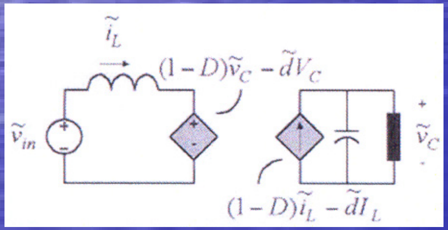

DC/DC Boost converter (Vc>Vin)



Classical Analysis of a boost converters is performed by using a linear model shown below [3]

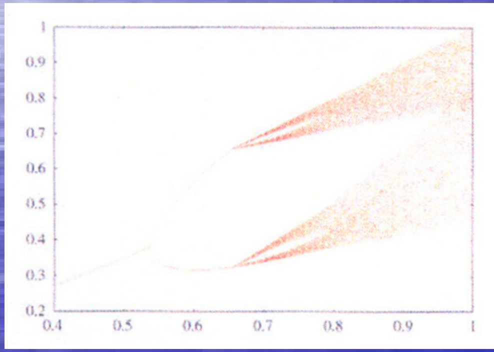


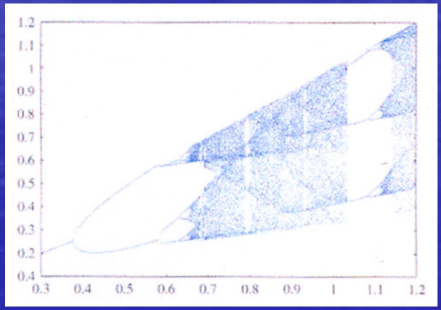



Period doubling route to chaos in DC/DC converters
Computer simulation [3]

-Computer generated bifurcation diagrams from a current-mode boost converter showing two different scenarios depending upon output capacitance size.
Period doubling bifurcation from a current-mode boost converter with relatively small output capacitance ,output current level being the bifurcation parameter.




***Feigenbaum's Number***

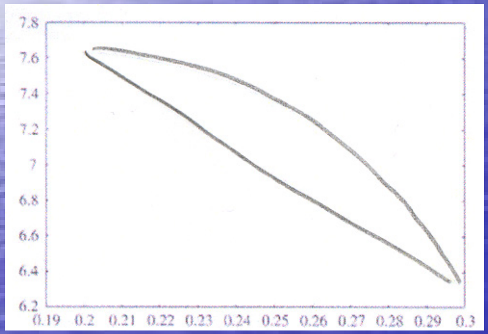
$$\delta = \frac{A_n - A_{n-1}}{A_{n+1} - A_n}$$

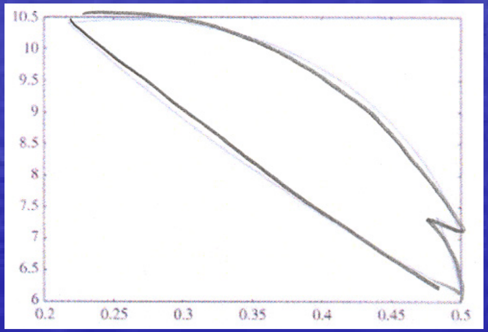
δ=4.66920160910299067185320382046620161725818557747576863274565134300413433021131473713868974402394801381716598485518981513440862714202793252231244298889089085994493546323671341153248171421994745564436582379320200956105833057545861765222207038541064674949428498145339172620056875566595233987560382563722 5

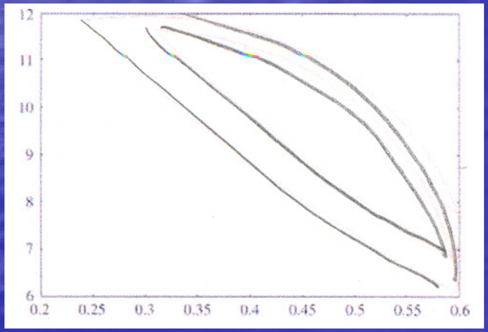
Typical attractors from current programmed DC/DC Converters. Upper left: period -1, upper right period- 2 Lower left period- 4: lower right : chaos

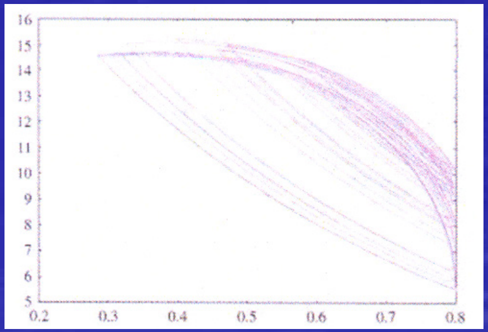
-border collision bifurcation [2] examples with output current as bifurcation parameter.

V(v) versus i(A) phase portrait



# Phase portraits,fixed points, existence , uniqueness and topological consequences

General form of a vector field on a phase plane

$dx1/dt=f1(x1,x2)$
$dx2/dt=f2(x1,x2)$
By flowing along the vector field,a phase point traces out a solution x(t), corresponding to a trajectory winding through the phase plane

- fixed points $f(x^*)=0$ correspond to steady states or equilibrium
- closed orbits $x(t+T)=x(t)$

Numerical computation of phase portraits (Runge Kutta)
- Poincare-Bendixson theorem : If a trajectory is confined to a closed, bounded region and there are no fixed points in the region, then the trajectory must eventually approach a closed orbit.

## Computer methods to determine stability and bifurcation phenomena

- Methods applicable to discrete –time systems obtained from continuous-time systems by suitable sampling(Poincaré section).

  NOTE: There are no general methods to construct a Poincaré map.
- Nonlinear Systems and Stability of Periodic Solutions
  - Obtain the periodic solution (Newton-Raphson)
  - Compute the Jacobian matrix of the Poincaré map at the periodic solution .
  - Evaluate the eigenvalues of the Jacobian.
- Nonlinear Systems and Bifurcation of Periodic Solutions

  Kawakami method to calculate the bifurcation values.

  Example : Calculate values of a Class E amplifier

Techniques of Numerical Investigation in Power Systems

1. Simulation of nonlinear switching systems give rise to some distinctive problems :

-Numerical integration of ODE’s assume that

a) The solution x (t) is smooth and

b) By choosing the integration step small , few terms are sufficient for required accuracy.

Both these assumptions are routinely violated in power electronic circuits.

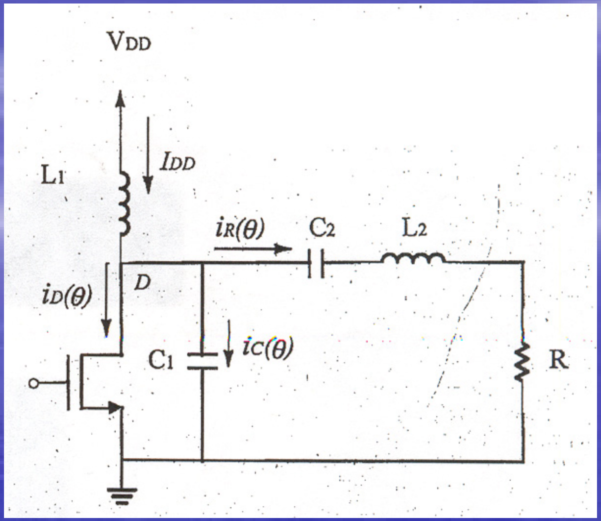
2. Problems arising from varying topology. This requires “a priori” knowledge of circuit operation or use of non-ideal switches in simulation (PSPICE).

3. Problems arising from Incompatible Boundary Conditions

In certain circuits x (t) is itself discontinuous. This can happen at the closing of a switch across a capacitor. If the cap has an initial voltage $v \neq 0$, then an infinite current flows at the switching instant, dissipating energy $\frac{1}{2}Cv^2$. To reduce such loss, a major Class of power converters (S,DE,F,E/F) has been proposed such that when the switch closes there is no voltage across it.(ZVS switching).

Microtrend Systems has developed a software package to determine circuit values such that ZVS switching is achieved even in the startup and transient conditions.

Consider a dynamic circuit (Class E amplifier ) described by a differential Equation




$\dot{x} = f(t, x, \lambda)$ (1)

Where $t \in R$, $x \in R^n$, $\lambda \in R^m$

t = time

n = n dimensional state

m = m dimensional system parameter

We consider

$f: R \times R^n \times R^m \rightarrow R^n (t, x, \lambda) \rightarrow f(t, x, \lambda)$

Which is assumed a $C^\infty$ mapping and periodic with $t_T$:

$f(t+t_T, x, \lambda) = f(t, x, \lambda)$

We also assume that (1) has a solution

$x(t) = \varphi(t, x_0, \lambda)$ defined on

$-\infty < t < +\infty$ with every initial condition

$X \in R^n$ and every $\lambda \in R^m$: $x(0) = \varphi(0, x_0, \lambda) = x_0$

A diffeomorphism T from space $R^n$ into itself (Poincaré mapping)

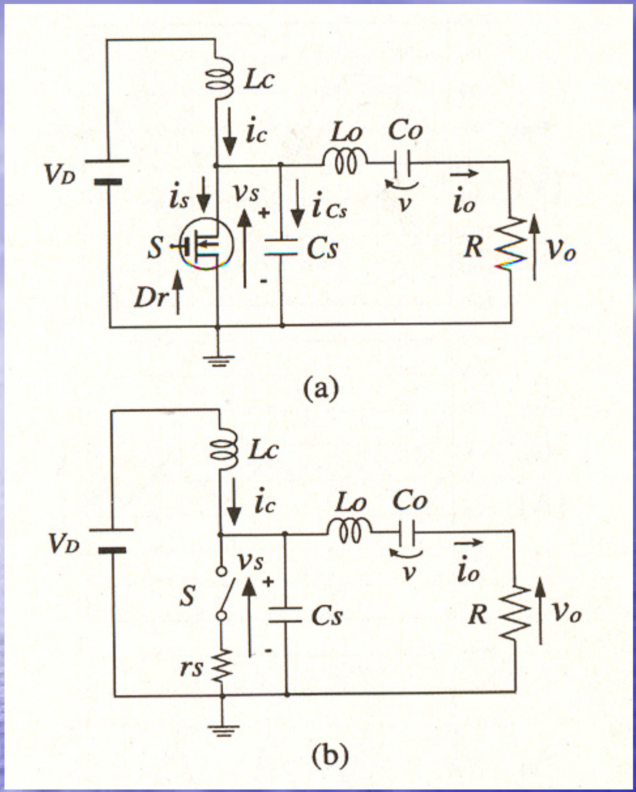
$T: R^n \rightarrow R^n \quad x_0 \rightarrow T(x_0, \lambda) = \varphi(t_T, x_0, \lambda)$.

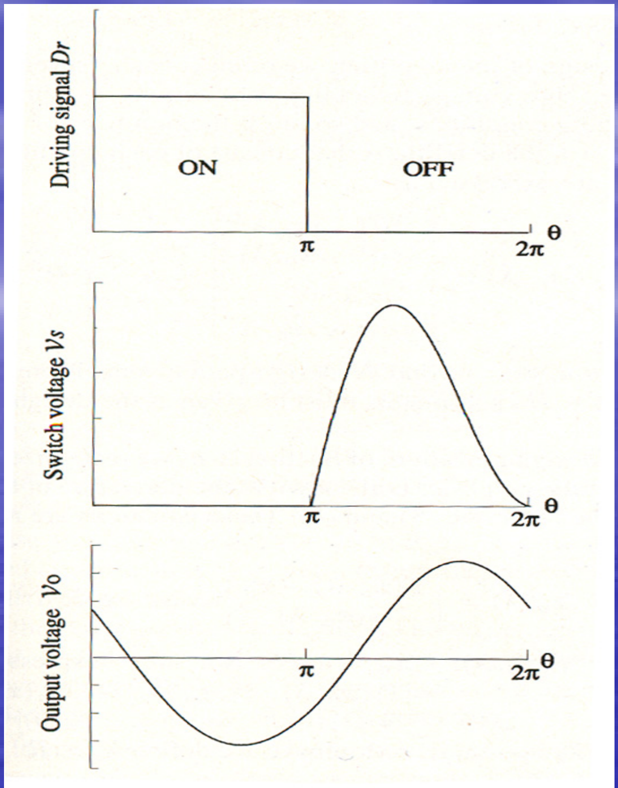
If a solution $x(t) = \varphi(t, p_0, \lambda)$ is periodic

The point $p_0 \in R^n$ is a fixed point of T

$T(p_0, \lambda) = p_0 \qquad (2)$

If $p_0 = x_0$, (2) correspond to a transient condition




Class E amplifier and typical waveforms

Logistic map and Power Electronics.

From Logistic map

$F(x) = \mu x(1-x)$

*Consider iterative function*

$X_{n+1} = F(X_n, \mu) \longleftrightarrow (V_{n+1} = F(V_n, d))$.

*Mechanism of Period doubling.*

$$\frac{F''(x)}{F'(x)} - \left[\frac{3\ F''(x)}{2\ F'(x)}\right]^2 < 0$$

*Schwarzian (Sf)(x)*

*(Sf)(x)<0 - a necessary condition for period doubling to occur*

Modeling a DC/DC converter as a first order iterative map [3]

$$V_{n+1} = \alpha V_n + \frac{\beta d_n^2 V_{in} (V_{in} - V_n)}{V_n} \quad \text{buck converter}$$

$$V_{n+1} = \alpha V_n + \frac{C d_n^2 V_{in}^2}{V_n - V_{in}} \quad \text{boost converter}$$

$$\alpha = 1 - \frac{T}{CR} + \frac{T^2}{2c^2R^2}$$

$$d_n = H(D+K(V_n - V_{ref}))$$

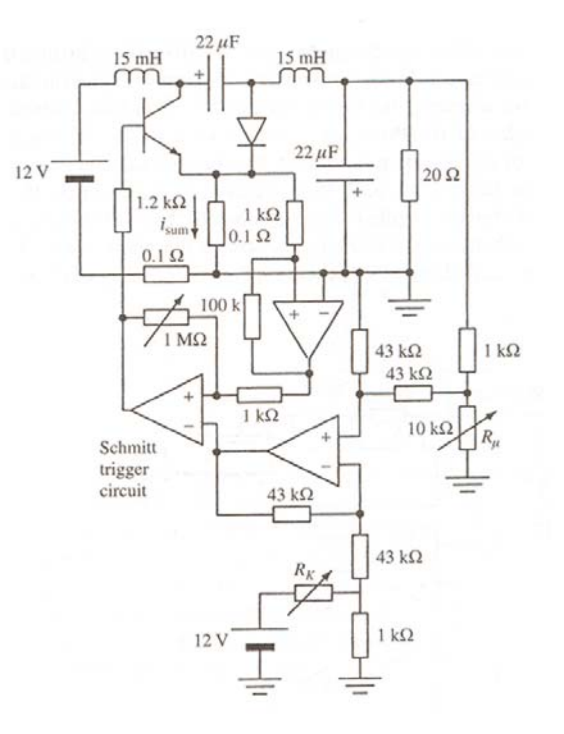
$$H(x) = \begin{cases} 0 & x < 0 \\ 1 & x > 1 \\ x & \text{Otherwise} \end{cases}$$

$$\beta = \frac{T2}{2LC}$$

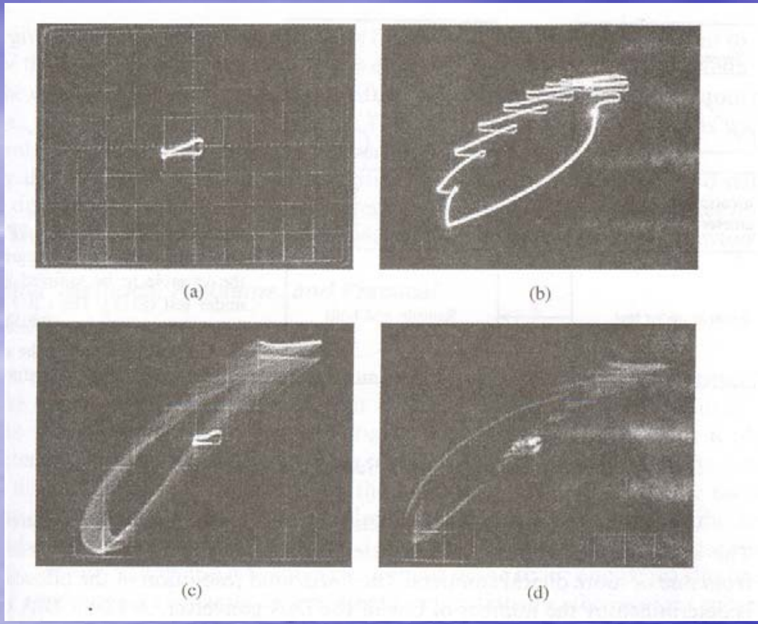
Experimental investigation of nonlinear phenomena in Power Electronics [3]

Focus on the following aspects :

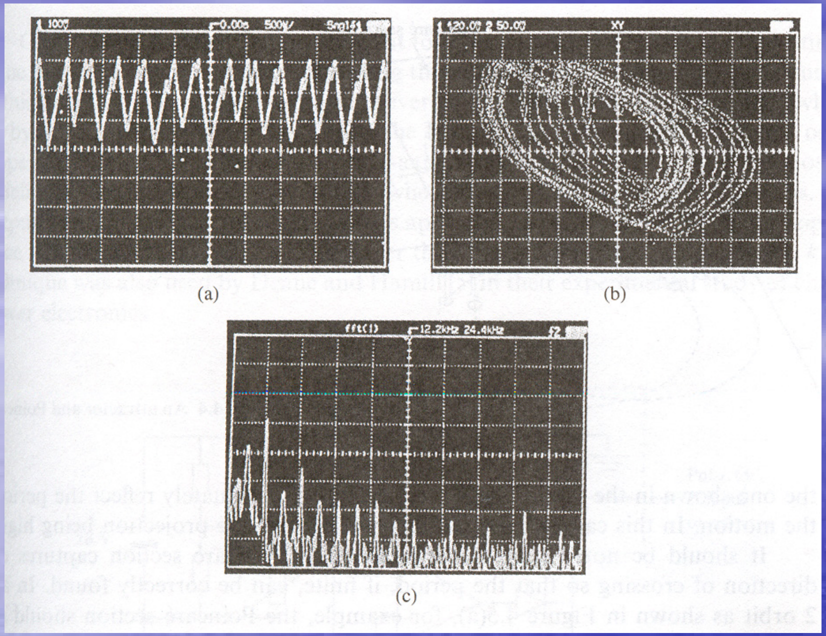
1. Displaying time domain waveforms
2. Phase portraits
3. Frequency spectra
4. Poincaré section
5. Bifurcation diagram



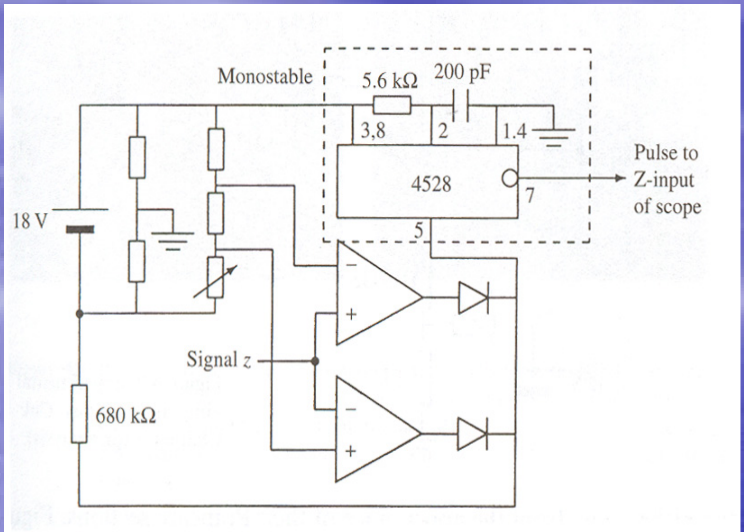

Experimental circuit of free running autonomous Ćuk converter



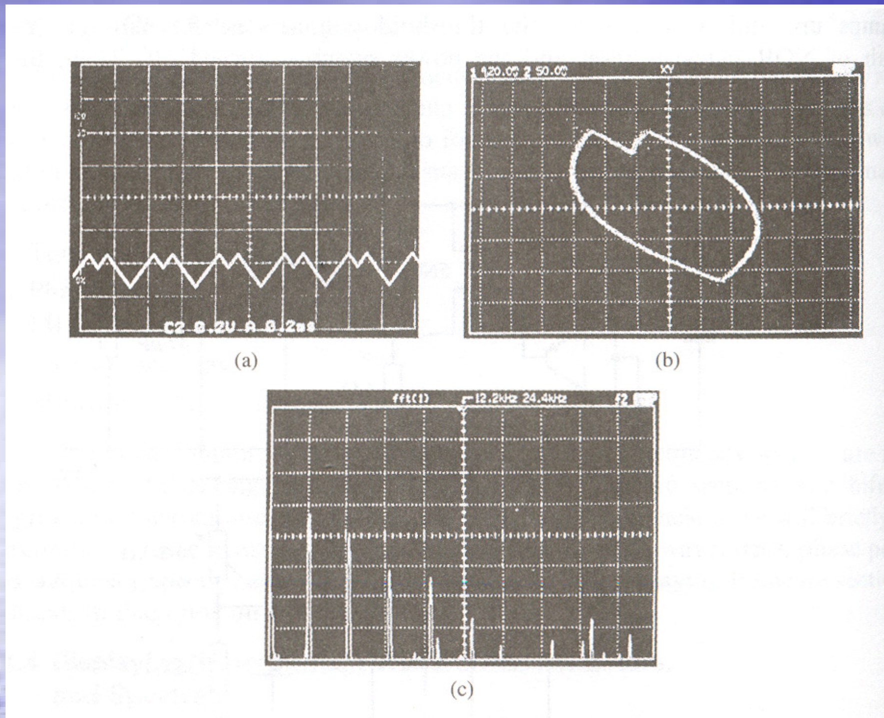

Phase portraits from autonomous Ćuk converter Showing (a) fixed point: (b) limit cycle (c) quasi-periodic orbit, (d) chaotic orbit. The Poincaré section are highlighted in (b),(c) and (d). The output voltage across the 20Ω load is used as input to the Poincaré section detector circuit




Experimental waveform, phase portrait and frequency spectrum for Ćuk converter operating under current-mode control showing chaotic operation. (a) Inductor current,(b)phase portrait of inductor current against a capacitor voltage,
(c) FFT of inductor current

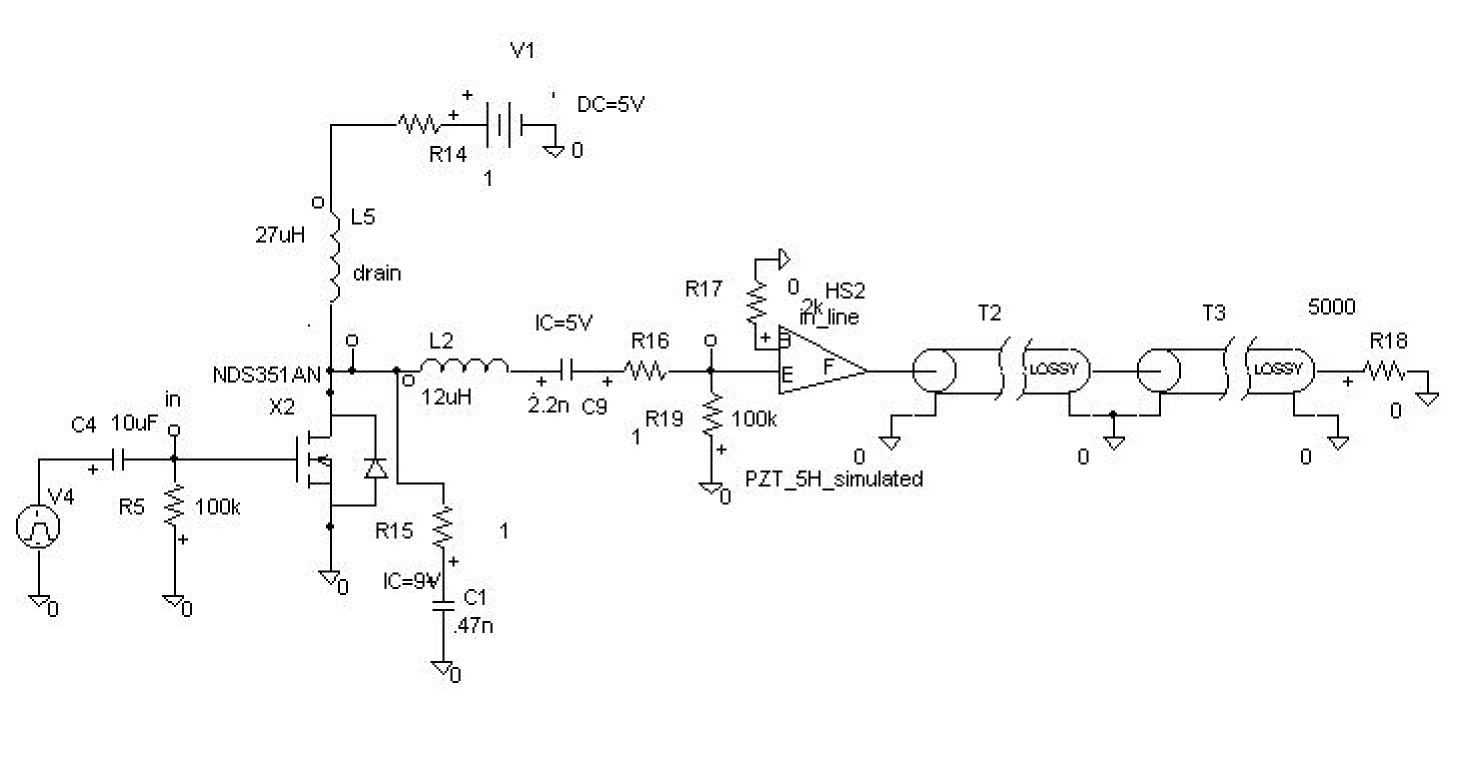



Circuit for detecting intersection of attractor and Poincaré section.

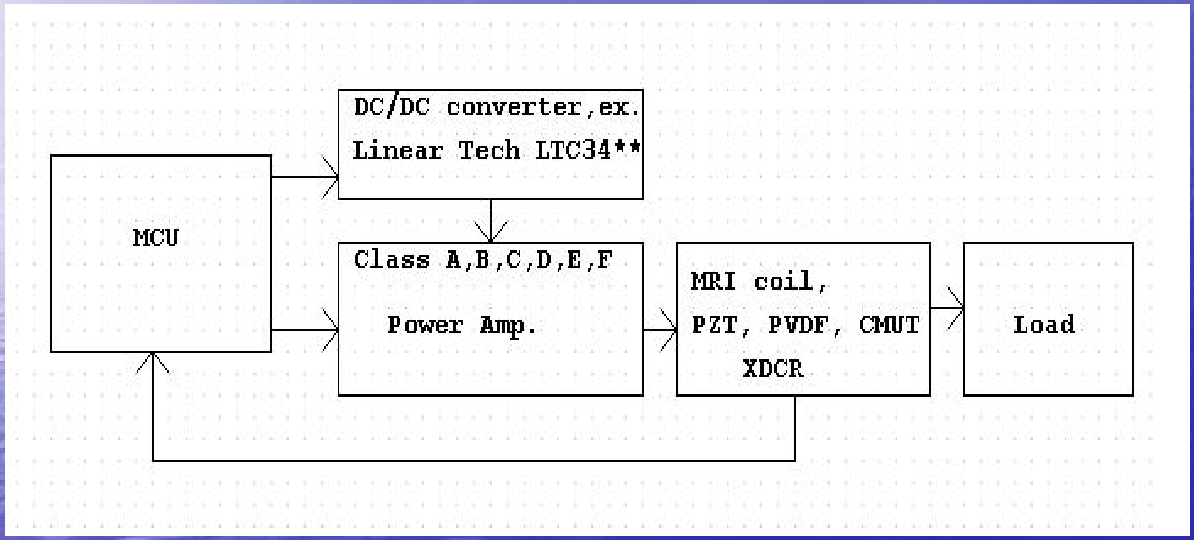



Experimental waveform, phase portrait and frequency spectrum for Ćuk converter operating under current-mode control showing Period-2 operation.(a) Inductor current, (b) phase portrait of inductor current against a capacitor voltage
(c) FFT of inductor current [2] .

# Class E/F Power Amp followed by the KLM model of a loaded PZT5-H transducer



# MCU based typical instrument



Some questions :

-Does a Class E,F  DC/AC inverter have a chaotic region ?

-What is the electrical/acoustical signal profile during chaotic behavior ?

-What are the physiological effects, if any, of the

energy delivered during chaotic behavior.

-Is there a formalism describing the general conditions of operation and transition to chaotic behavior ?

-Can such a system be brought back into a stable state?

-Conjecture ?.  In non HIFU applications ,

namely therapy and imaging,the effect of chaotic

operation  is lower efficiency of energy transfer  .

-Based on the design values of a DC/AC Class E inverter

is a chaotic behavior predictable [6]

References :

[1] " Symbolic Analysis of Switching Sytems:Application to Bifurcation Analysis of DC/DC Switching Converters "
D.Dai, Chi K.Tse , IEEE Tran. on CAS vol.52.no 8. ,August 2005

[2] Nonlinear Phenomena in Power Electronics
S. Banerjee, G. Varghese Eds. New York: IEEE Press,2000

[3] "Complex behavior of switching power converters"
C.K.Tse,M.di Bernardo in Proc.IEEE,vol.90,no5.May,2002

[4] "Practical Numerical Algorithms for Chaotic Systems
" T.S. Parker,and L.O. Chua New York ,
Springer Verlag 1989

[5] "New attempt in tissue characterization: decreased chaos in myocardial echo in patients with dilated cardiomyopathy
" T.Masuyama,K.Yamamoto et.al. Ultrasound in Med. Biology
Jan.2002

[6] " Prediction of chaotic behavior "T.Oguchi, H.Nijmeijer
IEEE Trans. on CAS. Vol.52 no.11 2005.

# Useful links:

http://www-chaos.umd.edu/index.html
http://www.student.math.uwaterloo.ca/~pmat370/JavaLinks.html
http://www.maths.strath.ac.uk/research/postgrad/industrial.html#9
http://www.apmaths.uwo.ca/~bfraser/nll/version1/bifurcation.html
http://www.apmaths.uwo.ca/~bfraser/nll/version1/links.html
http://www.cevis.uni-bremen.de/fractals/nsfpe/Chaos_Lab/biffamily.html
http://ewh.ieee.org/soc/icss/advancesincircuitsandsystems_november_2005.htm