8.7.3 Experiment 3–Response of Discrete Systems

Part 1. Consider a discrete-time linear system represented by its transfer function

$$H(z) = \frac{4z}{z^2 + z + 0.25}$$

(a) Find the impulse response using the MATLAB function `dimpulse`.

(b) Find the step response using the function `dstep` and plot both the state and output responses.

(c) Find the system output response due to a unit step function, $f[k] = u[k]$, and initial conditions specified by $y[-1] = 0,\ y[-2] = 1$. Use the function `dlsim` and specify input at every time instant of interest. Take `k=0:1:20` (defines k at $0, 1, 2, \ldots, 19, 20$). Check analytically that the results obtained agree with the analytical results for $k = 10$.

(d) Obtain the state space form for this system by using the function `tf2ss`. Repeat parts (a), (b), and (c). Use the MATLAB statements

```
[y,x]=dimpulse(A,B,C,D)
[y,x]=dstep(A,B,C,D)
[y,x]=dlsim(A,B,C,D,f,x0)
```

respectively, with f and $x[0]$ as defined in (c). Compare the results obtained.

Part 2. Consider the discrete-time linear system represented by

$$y[k+2] + \frac{5}{6}y[k+1] + \frac{1}{6}y[k] = f[k+1],$$

$$f[k] = (0.8)^k u[k], \qquad y[-1] = 2, \quad y[-2] = 3$$

(a) Find the system state and output responses by using the MATLAB function `dlsim`. (*Hint:* Use `[y,x]=dlsim(A,B,C,D,f,x0)` with `k=0:1:10`.) Note that the initial condition must be found. This can be obtained by playing algebra with the state space and output equations. Compare the obtained simulation results with analytical results.

(b) Find the zeros and poles of this system by using the function `tf2zp`.

(c) Find the system state and output responses due to initial conditions specified in Part 2(a) and with the impulse delta function as an input. Use the `dlsim` function.

(d) Solve the problem in (c) analytically using the $\mathcal{Z}$-transform. Plot results from (c) and compare results.

Part 3. Consider a dynamic system represented in the continuous-time state space form in Section 8.7.2, Experiment 2, Part 3.

(a) Discretize the continuous-time system using the MATLAB function `c2d`. Assume that the sampling period is $T = 1$.

(b) Find the eigenvalues, eigenvectors, and characteristic polynomial of the obtained discrete-time system.

(c) Find the state transition matrix at time instant $k = 5$.

(d) Find the unit impulse response and plot output variables.

(e) Find the unit step response and plot the corresponding output variables.

(f) Assume that the initial system condition is $\mathbf{x}(0) = [-1 \quad 0 \quad 1 \quad -0.5]^T$. Find the system state and output responses due to an input given by $f[k] = \sin[k], \ \ 0 \leq k \leq 1000$. Take `k=0:10:1000`. Use the `dlsim` function. Compare the obtained discrete-time results with the continuous-time results for the same system studied in Section 8.7.2, Experiment 2.

(g) Find the system transfer functions. Note that you have one input and two outputs, which implies two transfer functions. The matrices **C** and **D** are not changed due to discretization. (*Hint:* Use the function `ss2tf`.)

SUPPLEMENT:

Continuous-time state space form from Section 8.7.2

$$\mathbf{A} = \begin{bmatrix} -0.01357 & -32.2 & -46.3 & 0 \\ 0.00012 & 0 & 1.214 & 0 \\ -0.0001212 & 0 & -1.214 & 1 \\ 0.00057 & 0 & -9.1 & -0.6696 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -0.433 \\ 0.1394 \\ -0.1394 \\ -0.1577 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \mathbf{0}^{2\times 1}$$