8.7.2 Experiment 2—Response of Continuous Systems

Part 1. Consider a continuous-time linear system represented by its transfer function

$$H(s) = \frac{s+5}{s^2+5s+6}$$

(a) Find and plot the impulse response. Use the MATLAB function `impulse`.

(b) Find and plot the step response using the function `step`.

(c) Find the zero-state system output response due to an input given by $f(t) = e^{-3t}, \quad t \geq 0$. Note that you must use the function `lsim` and specify input at every time instant of interest. That can be obtained by `t=0:0.1:5` (defines t at $0, 0.1, 0.2, ..., 4.9, 5$), `f=exp(-3*t)`, and `y=lsim(num,den,f,t)`. Check that the results obtained agree with analytical results at $t = 1$.

(d) Obtain the state space form for this system by using the function `tf2ss`. Repeat parts (a), (b), and (c) for the corresponding state space representation. Use the following MATLAB instructions

```
[y,x]=impulse(A,B,C,D)
[y,x]=step(A,B,C,D)
[y,x]=lsim(A,B,C,D,f,t)
```

respectively, with f and t as defined in (c). Compare the results obtained.

Part 2. Consider the continuous-time linear system represented by

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = \frac{df(t)}{dt} + f(t),$$
$$f(t) = e^{-4t}u(t), \quad t \geq 0, \qquad y(0^-) = 2, \quad \dot{y}(0^-) = 1$$

(a) Find the complete system state and output responses by using the MATLAB function `lsim`. Compare the simulation results obtained with analytical results. (*Hint:* Use `[y,x]=lsim(A,B,C,D,f,t,x0)` with t = 0:0.1:5.) Note that the initial condition for the state vector, `x0`, must be found. This can be obtained by playing algebra with the state and output equations and setting $t = 0$.

(b) Find the zeros and poles of this system using the function `tf2zp`.

(c) Find the system state and output responses due to initial conditions specified in Part 2(a) and the impulse delta function as an input. Since you are unable to specify the system input in time (the delta function has no time structure), you cannot use the `lsim` function. Instead use either the MATLAB function `initial` (zero-input response) or the MATLAB program given at the end of this part. The required response is obtained analytically as follows

$$\mathbf{x}(t) = e^{\mathbf{A}t}\left(\mathbf{x}(0^-) + \mathbf{B}\right)$$

where $\mathbf{A}$ and $\mathbf{B}$ stand for the system and input matrices in the state space. Thus, the new initial condition is given by $\mathbf{x}(0^-) + \mathbf{B}$.
(*Hint:* To find and plot the system state response for the given matrix $\mathbf{A}$ and the corresponding initial conditions, the MATLAB program attached can be used.)

```
t=0:0.1:5;
for i=1:1:51;
x(:,i)=expm(A*t(i))*(x0+B);
end
plot(t,x(1,:))
plot(t,x(2,:))
```

(d) Justify the answer obtained in (c). Solve the same problem analytically using the Laplace transform. Plot results from (c) and compare with these results. Can you draw any conclusion for this "nonstandard" problem from the point of view of the system initial conditions at $t = 0^+$? (The standard problem requires that for the impulse response all initial conditions are set to zero.)

Part 3. Consider the dynamic system [17] represented in the state space form by

$$\mathbf{A} = \begin{bmatrix} -0.01357 & -32.2 & -46.3 & 0 \\ 0.00012 & 0 & 1.214 & 0 \\ -0.0001212 & 0 & -1.214 & 1 \\ 0.00057 & 0 & -9.1 & -0.6696 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -0.433 \\ 0.1394 \\ -0.1394 \\ -0.1577 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D} = \mathbf{0}^{2\times 1}$$

This is a real mathematical model of an F-8 aircraft [18]. Using MATLAB, determine the following quantities.

(a) The eigenvalues, eigenvectors, and characteristic polynomial. Take `p=poly(A)` and verify that `roots(p)` also produces the eigenvalues of matrix $\mathbf{A}$.

(b) The state transition matrix at the time instant t = 1. Use the function `expm`.

(c) The unit impulse response and plot output variables. Hint: Use `impulse(A,B,C,D)`.

(d) The unit step response. Plot the corresponding output variables.

(e) Let the initial system condition be $\mathbf{x}(0) = \begin{bmatrix} -1 & 1 & 0.5 & 1 \end{bmatrix}^T$. Find the system state and output responses due to an input given by $f(t) = \sin(t),\ 0 < t < 1000$. (*Hint:* Take `t=0:10:1000` and find the corresponding values for $f(t)$ by using the function `sin` in the form `f=sin(t)`. Then use the `lsim` function.)

(f) Find the system transfer functions. Note that you have one input and two outputs which implies two transfer functions. (*Hint:* Use the function `ss2tf`.)

(g) Find the inverse of the state transition matrix $\left(e^{\mathbf{A}t}\right)^{-1} = e^{-\mathbf{A}t}$ at t = 2.

Part 4. Consider a linear continuous-time dynamic system represented by its transfer function

$$H(s) = \frac{(s+1)(s+3)(s+5)(s+7)}{s(s+2)(s+4)(s+6)(s+8)(s+10)}$$

(a) Input the system zeros and poles as column vectors. Note that in this case the static gain k = 1. Use the function `zp2ss(z,p,k)` to get the state space matrices.

(b) Find the eigenvalues and eigenvectors of matrix $\mathbf{A}$.

(c) Verify that the transformation $\mathbf{x} = \mathbf{P}\tilde{\mathbf{x}}$, where $\mathbf{P}$ is the matrix whose columns are the eigenvectors of matrix $\mathbf{A}$, produces in the new coordinates the diagonal system matrix $\mathbf{\Lambda} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$ with diagonal elements equal to the eigenvalues of matrix $\mathbf{A}$.

(d) Find the remaining state space matrices in the new coordinates. Find the transfer function in the new coordinates and compare it with the original transfer function.

(e) Compare the unit step responses of the original and transformed systems.