

7.10 MATLAB Experiment on Discrete-Time Systems

Purpose: In this experiment, we analyze time responses of a higher-order linear discrete-time system using MATLAB. We study system impulse, step, and sinusoidal responses. In addition, system internal and BIBO stability are examined. By performing this experiment, the students will realize how simple and easy is to analyze higher-order linear discrete-time dynamic systems using MATLAB.

Consider the linear discrete-time system

$$y[k+5] + 2.2833y[k+4] + 1.2500y[k+3] + 0.0625y[k+2] - 0.0833y[k+1] - 0.0125y[k] = f[k+2] + 1.6f[k+1] + 0.15f[k]$$

Part 1. Examine the system's internal stability by finding its eigenvalues. Use the MATLAB function `roots`. Find the system's transfer function zeros and poles and check whether common factors of the transfer function numerator and denominator can be cancelled. Comment on the system's BIBO stability.

Part 2. Using the MATLAB function `dimpulse` find the system impulse response. Plot the system impulse response for $k=0:1:14$ and $k=0:1:40$. Comment on the differences of the two plots, and explain the reason for the time behavior of the system impulse response. Find the impulse response analytically using the methodology from Section 7.3. Plot the obtained analytical result using MATLAB and compare it with the obtained simulation result (using the MATLAB function `dimpulse`). Form the reduced-order system by cancelling the zero at -1.5 and the pole at -1.4999 , and find the impulse response of the reduced order system for $k=0:1:40$.

Part 3. Find the step response using the MATLAB function `dstep`. Plot the step response $k=0:1:14$ and $k=0:1:40$. Comment on the differences of the two plots and explain the reason for the time behavior of the system step response. Find the step response of the reduced order system for $k=0:1:40$. Check the obtained steady state value for the step response of the reduced-order system using formulas (5.49–50).

Part 4. Find and plot the system sinusoidal response. (*Hint:* Use the MATLAB function `dl Sims(num, den, f)` with $f = \sin(k)$ for $k=0:1:14$ and $k=0:1:40$.) Find and plot the sinusoidal reduced order system response for $k=0:1:40$.

SUPPLEMENT:

$$H(1) = \frac{b_m + b_{m-1} + \cdots + b_1 + b_0}{1 + a_{n-1} + a_{n-2} + \cdots + a_1 + a_0} = \frac{\sum_{i=0}^m b_i}{1 + \sum_{j=0}^{n-1} a_j} \quad (5.49)$$

For $F(z) = az/(z-1) \leftrightarrow f[k] = au[k]$ we have

$$\begin{aligned} y_{ss} &= \lim_{k \rightarrow \infty} y[k] = \lim_{z \rightarrow 1} \left\{ \frac{z-1}{z} Y(z) \right\} = \lim_{z \rightarrow 1} \left\{ \frac{z-1}{z} H(z) \frac{az}{z-1} \right\} \\ &= \lim_{z \rightarrow 1} \{ H(z)a \} = H(1)a \end{aligned} \quad (5.50)$$