7.9 MATLAB Experiment on Continuous-Time Systems

Purpose: In this experiment, we analyze time responses of a real physical system using MATLAB. We study impulse, step, and sinusoidal responses of the yaw rate dynamics under the influence of the rudder for a commercial aircraft, and draw some useful conclusions about the aircraft's dynamic behavior. In addition, by examining the location of the aircraft eigenvalues and poles we make conclusions about its internal and BIBO stability. By performing this experiment, students will realize how simple and easy it is to analyze higher-order linear continuous-time dynamic systems using MATLAB.

A mathematical model that describes the lateral dynamics of a commercial aircraft is given by a fourth-order differential equation [16]. Using the numerical data from [16], the corresponding differential equation is given by

$$\begin{aligned} &y^{(4)}(t) + 0.6363y^{(3)}(t) + 0.9396y^{(2)}(t) + 0.5123y^{(1)}(t) + 0.0037y(t) \\ &= -0.475f^{(3)}(t) - 0.248f^{(2)}(t) - 0.1189f^{(1)}(t) - 0.0564f(t) \end{aligned}$$

where $y(t)$ is the yaw rate and $f(t)$ stands for the changes in the rudder.

Part 1. Using the MATLAB function `impulse`, find the impulse response, that is, observe the yaw rate changes due to an impulse delta signal disturbance acting on the rudder, $f(t) = \delta(t)$. Plot the impulse time response in the range $t \in [0, 200]$ seconds. Find the impulse response analytically by using the methodology from Section 7.4. Plot the obtained analytical result using MATLAB, and compare it with the obtained simulation result (using the MATLAB function `impulse`).

Part 2. Find the step response using the MATLAB function `step`. Plot the step response during the first 200 seconds. Comment on the physical meaning of the obtained results. Do you expect that the aircraft moves to the right when the rudder moves to the left and vice versa? Check the obtained steady state value for the yaw rate by using formulas (4.39–40).

Part 3. Assume that the rudder is under wind disturbances that can be approximated by a sinusoidal function, $f(t) = \sin(t)$ for the first 200 seconds. Find and plot the aircraft yaw rate dynamics during that time interval. Estimate the maximal yaw rate change due to a sinusoidal disturbance whose maximal magnitude is equal to 1. (*Hint:* Use the MATLAB function `lsim(num,den,f,t)` with `t=0:0.1:200` and `f=sin(t)`.)

Part 4. Examine the aircraft's internal stability by finding its eigenvalues. Use the MATLAB function `roots`. Find the aircraft's transfer function zeros and poles and check whether common factors of the transfer function numerator and denominator can be cancelled. Comment on the aircraft's bounded-input BIBO stability.

Comment: *Examining system stability should be the first task* in any analysis of a linear system. If the system is found to be unstable, the system should not be subjected to any input; instead, stabilization techniques should be first applied (for example, by introducing a feedback loop as presented in Section 4.4), and then the system can be tested for various input signals. In this experiment, we examine the system stability at the end to be consistent with the order of presentation in chapter.

SUPPLEMENT:

$$H(0) = K\frac{(-z_1)(-z_2)\cdots(-z_m)}{(-p_1)(-p_2)\cdots(-p_n)} = \frac{b_0}{a_0} \tag{4.39}$$

For $f(t) = au(t) \leftrightarrow F(s) = a/s$, we have

$$y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \lim_{s\to 0}\left\{sH(s)\frac{a}{s}\right\} = H(0)a \tag{4.40}$$