6.6 MATLAB Laboratory Experiments on Convolution

Purpose: In this section we design two experiments dealing with continuous- and discrete-time convolutions and their applications to linear continuous- and discrete-time dynamic systems. The purpose of the first experiment is to present the convolution operator, and to demonstrate some of its properties in both continuous- and discrete-time domains. By writing and modifying the corresponding MATLAB programs, students will master every step of the convolution process. In the second experiment, the convolution method will be used to determine the zero-state responses of both continuous- and discrete-time linear dynamic systems by using the famous formula that states that *the response of a linear system at rest due to an arbitrary input is the convolution of that input with the system impulse response*.

6.6.1. Convolution of Signals

In this experiment, students must verify the commutativity and distributivity properties of continuous- and discrete-time convolutions.

Part 1. Continuous-Time Signals

The convolution of two signals is defined by

$$f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau) d\tau, \quad -\infty < t < \infty \tag{6.30}$$

Consider the continuous-time signals

$$f_1(t) = p_2(t), \quad f_2(t) = \Delta_2(t), \quad f_3(t) = [u(t) - u(t - 5)]$$

Write a MATLAB program to perform and verify the continuous-time convolution commutativity and associativity properties:

$$f_1(t) * f_2(t) = f_2(t) * f_1(t)$$
$$f_1(t) * (f_2(t) + f_3(t)) = f_1(t) * f_2(t) + f_1(t) * f_3(t)$$

While writing the program you must discretize the convolution integral given in (6.30); that is the integral in (6.30) must be approximated by a finite sum of the form

$$f_1(kT) * f_2(kT) \approx T \sum_{i=k_0}^{k} f_1(iT) f_2[(k - i)T] \tag{6.31}$$

Take $T = 0.1$, plot the results obtained, and comment on the durations of the signals obtained through the convolution procedure. Observe that the commutativity and distributivity properties hold for the approximate continuous-time convolution, and conclude that when $T \to 0$ these properties also hold for the convolution integral (6.30).

Part 2. Discrete-Time Signals

In the discrete-time domain, the convolution of two signals is defined by

$$f_1[k] * f_2[k] = \sum_{m=-\infty}^{m=\infty} f_1[m] f_2[k - m], \quad -\infty < k < \infty \quad (6.32)$$

Consider the discrete-time signals

$$f_1[k] = \operatorname{sinc}[k]\{u[k + 5] - u[k - 5]\}$$
$$f_2[k] = 1 - p_2[k], \ 0 \le k \le 5$$
$$f_3[k] = u[k] - u[k - 5]$$

Write a MATLAB program to perform the operations defined in (6.17–18). Plot the results obtained and comment on the convolution signal durations.

Verify the results by using the MATLAB function `conv`.

Hint: Use the MATLAB program given in Section 6.5 and write its modifications. The program can be used for calculating both the continuous-time and discrete-time convolutions. Note that you must plot the convolution results obtained in the proper time intervals. Use `plot(t,fapr)` with `t=t1+t2:T:T1+T2`, where $[t_i, T_i], i = 1, 2$, define the duration intervals of the signals being convolved.

SUPPLEMENT:

1) *Commutativity*

$$f_1[k] * f_2[k] = f_2[k] * f_1[k] \quad (6.17)$$

2) *Distributivity*

$$f_1[k] * \{f_2[k] + f_3[k]\} = f_1[k] * f_2[k] + f_1[k] * f_3[k] \quad (6.18)$$