5.6 $\mathcal{Z}$-transform MATLAB Laboratory Experiment

Purpose: This experiment presents the frequency domain analysis of discrete-time systems using MATLAB. The impulse, step, sinusoidal, and exponential responses of discrete-time systems will be examined using the system transfer function method based on the $\mathcal{Z}$-transform. In addition, MATLAB will be used to perform the partial fraction expansion and to find the inverse $\mathcal{Z}$-transform. Note that MATLAB *uses only the integral representation of linear discrete-time systems*.

Part 1. Consider the linear discrete-time system represented by the system transfer function

$$H(z) = \frac{z - 0.5}{z\left(z + \frac{1}{2}\right)\left(z - \frac{1}{3}\right)\left(z + \frac{1}{4}\right)}$$

Using MATLAB, find and plot:

(a) The impulse response of the system.

(b) The step response of the system.

(c) The zero-state response due to the input signal $f[k] = \sin[2k]u[k]$.

(d) The zero-state response due to the input signal $f[k] = (-2)^k u[k]$.

Part 2. Consider the systen transfer function

$$H(z) = \frac{z^4 - 3z^3 + 5z^2 + 2z}{2z^7 + 5z^5 - 3z^4 + z^3 - 2z^2 + 3z - 1}$$

(a) Find the factored form of the transfer function using the MATLAB function `[z,p,k]=tf2zp(num,den)`.

(b) The partial fraction expansion of rational functions can be performed using the MATLAB function `residue`. Find the inverse $\mathcal{Z}$-transform of the given transfer function using `residue`, that is, find analytically the system impulse response. Plot the system impulse response.

(c) Repeat appropriately the steps outlined in (b) such that the system step response is obtained.

Part 3. Find and plot the zero-input response of the system whose transfer function is given in Part 2, and whose initial conditions are given by

$$y[-4] = 1,\ y[j] = 0,\ j = -7, -6, -5, -3, -2, -1$$

(*Hint:* Find $I_y^i(z)$ and $\Delta(z)$ as defined in formulas (5.45) and (5.67) respectively. Then, use the MATLAB function `dimpulse` with the coefficients of $I_y^i(z)$ and $\Delta(z)$ representing, respectively, the corresponding numerator and denominator coefficients.)

Submit four plots for Part 1, one plot for Part 2, and one plot for Part 3, and present analytical results obtained in Parts 2–3.

Comment: Instructors can design additional MATLAB laboratory experiments or additional parts of laboratory experiments by selecting from the problem section (Section

5.9) any set of problems that require the use of MATLAB or Simulink. (This comment is in general applicable to any chapter in this textbook.)

SUPPLEMENT:

$$\begin{aligned} I_y^i(z) = &\left(a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \cdots + a_1 z + a_0\right) z^{-(n-1)} y[-1] \\ &+\left(a_{n-2}z^{n-2} + a_{n-3}z^{n-3} + \cdots + a_1 z + a_0\right) z^{-(n-2)} y[-2] \\ &+\left(a_{n-3}z^{n-3} + a_{n-4}z^{n-4} + \cdots + a_1 z + a_0\right) z^{-(n-3)} y[-3] \\ &+ \cdots + (a_1 z + a_0) z^{-1} y[-(n-1)] + a_0 y[-n] \end{aligned} \tag{5.45}$$

$$\Delta(z) = z^n + a_{n-1}z^{n-1} + a_{n-2}z^{n-2} + \cdots + a_1 z + a_0 \tag{5.67}$$