12.7 Laboratory Experiment on Control Systems

Problem: A Pitch Controller for a BOEING Aircraft

The linearized equations governing the motion of a commercial aircraft, obtained by using the linearization methodology presented in Section 8.6, are given [10] by

$$\frac{d\alpha(t)}{dt} = -0.313\alpha(t) + 56.7q(t) + 0.232\delta_e(t)$$
$$\frac{dq(t)}{dt} = -0.0139\alpha(t) - 0.426q(t) + 0.0203\delta_e(t)$$
$$\frac{d\theta(t)}{dt} = 56.7q(t)$$

where $\theta(t)$ represents the pitch angle. The corresponding open-loop transfer function obtained from (12.59) is given by

$$\frac{\Theta(s)}{\Delta(s)} = \frac{1.151s + 0.1774}{s^3 + 0.739s^2 + 0.921s} = \frac{1.151(s + 0.1541)}{s(s^2 + 0.739s + 0.921)} = 1.151H(s)$$

In this experiment we design an autopilot that controls the pitch angle $\theta(t)$ of this aircraft. The autopilot is obtained by forming a closed-loop system with unity feedback and a controller of the form $KG_c(s)$. For simplicity, we assume that $K = 1.151K'$, so that the loop transfer function is $KG_c(s)H(s)$.

Part 1. Find the steady state unit step and unit ramp errors of the original closed-loop system ($K = 1, G_c(s) = 1$, $M(s) = H(s)/(1 + H(s))$). The unity feedback closed-loop transfer function can also be obtained using the MATLAB statement `[cnum,cden]=feedback(num,den,1,1,-1)`. Plot the closed-loop system step and ramp responses and observe (check) the corresponding steady state errors. (*Hint:* In order to find the ramp response, use the MATLAB function `y=lsim(cnum,cden,t,t)` with `t=0:0.1:30`.)

Part 2. Find the value for the static gain K such that the steady state error due to the unit ramp is reduced to $e_{\rm ss}^{\rm ramp} \leq 0.1$. For the obtained value of K, plot the corresponding closed-loop system step and ramp responses and notice the steady state unit ramp error improvement. Also observe the transient step response worsen due to an increased value of the static gain K. (*Hint:* Use the same time interval as in Part 1.)

Part 3. Plot Bode diagrams with the value for the static gain K obtained in Part 2. Use `bode(K*num,den);` and `margin(K*num,den)` to find the phase and gain stability margins, and observe that the phase margin is quite pure. Design a PD controller of the form $G_c(s) = G_{\rm PD}(s) = 1 + K_d s$ to improve the phase stability margin such that the controlled system has a phase stability margin close to 50° (see Figure 12.24 and note that unity feedback is used by the design requirement). Find the closed-loop step response of the system controlled by the PD controller, and compare it to the closed-loop step response of the uncontrolled system whose static gain K is found in Part 2.

Comment on the transient response improvement of the controlled system. (*Hint:* Try several values for the derivative constant in the range $0 < K_d < 10$.)

Part 4. Use a PI controller of the form $G_c(s) = G_{\text{PI}}(s) = K_p + K_i/s$. Though the PI controller introduces a negative phase shift, you still can improve the system phase stability margin by placing the new gain crossover frequency in the range of low frequencies. Find the closed-loop step response of the system controlled by the PI controller and compare it to the closed-loop step response of the system controlled by the PD controller. Observe that the PI controller reduces the system frequency bandwidth (ω_{cg} gets smaller) so that the system step response with the PI controller is slower, that is, it has longer rise time. Which controller do you prefer? Explain.

Part 5. Use a PID controller of the form $G_c(s) = G_{\text{PID}}(s) = 1 + K_d s + K_i/s$ and tune the values for K_d and K_i such that satisfactory results are obtained for both the transient response parameters and the steady state errors. Try also to achieve satisfactory stability margins, that is, $Pm \in [30°, 60°]$. Compare the results obtained with the corresponding results obtained in Parts 3 and 4. Which among the three controllers considered for this particular aircraft control problem do you prefer?

SUPPLEMENT:

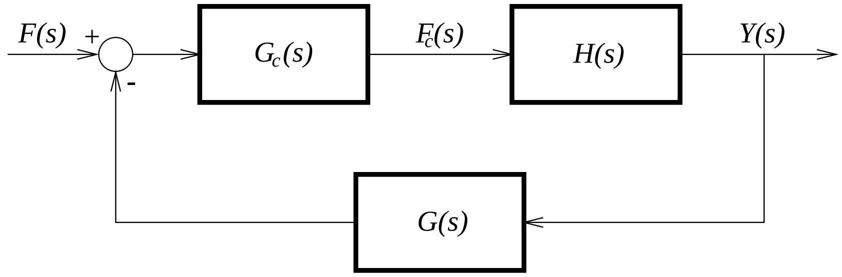

FIGURE 12.24: Feedback system controlled by a dynamic controller