

Sliding Mode Control with Industrial Applications

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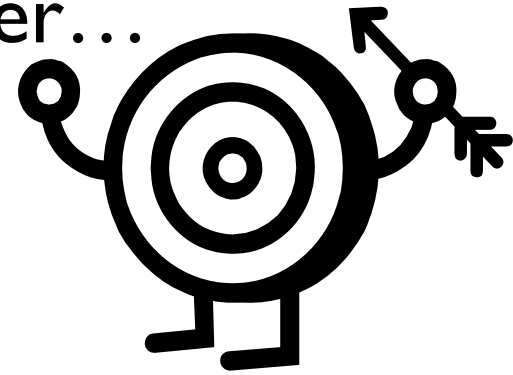
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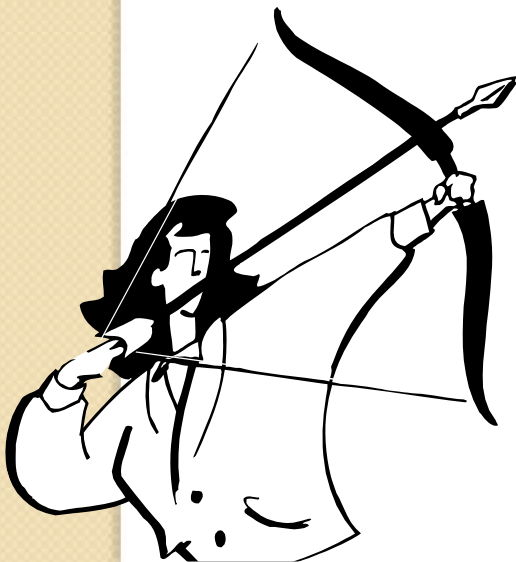
Rutgers, The State University of New Jersey, U. S. A.

Prelude

A Control Engineer as an Archer...

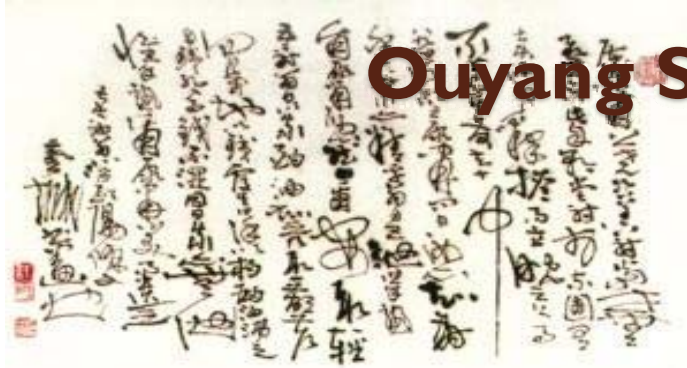


to drive the state to the target.



The Old Oil-Peddler

Ouyang Show(1007~1072A.D.)



賣油翁

宋 歐陽修

陳康肅公堯咨善射，當世無雙，公亦以此自矜。嘗射於家圃，有賣油翁釋擔而立，睨之，久而不去。見其發矢十中八九，但微領之。

康肅問曰：「汝亦知射乎？吾射不亦精乎？」翁曰：「無他，但手熟爾。」康肅忿然曰：「爾安敢輕吾射！」翁曰：「以我酌油知之。」乃取一葫蘆置於地，以錢覆其口，徐以杓酌油瀝之，自錢孔入，而錢不濕。因曰：「我亦無他，惟手熟爾。」康肅笑而遣之。



Princeton/Central Jersey Section of IEEE
- Circuits and Systems Chapter Meeting

<http://baike.baidu.com/> 2008/11/17

Sliding Mode: an illustration



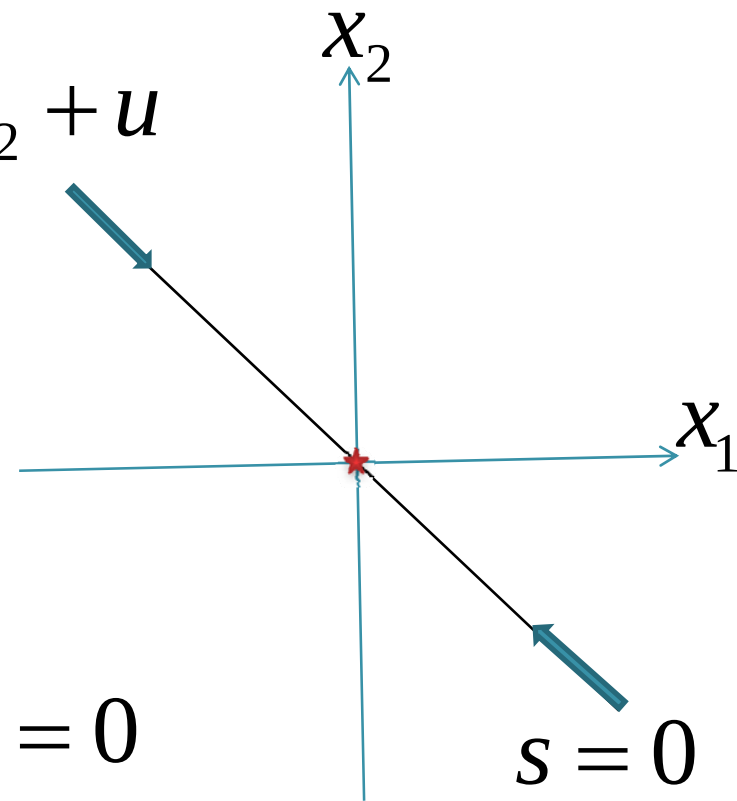
A “funnel-like” domain

System:
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2x_1 - 3x_2 + u \end{cases}$$

Target: $(x_1, x_2) = (0, 0)$

Funnel: $x_2 = -x_1$

New target: $s = x_1 + x_2 = 0$



A Physical Example

Coulomb Friction

$$m \frac{dv}{dt} = -K \operatorname{sgn}(v) + f_a$$

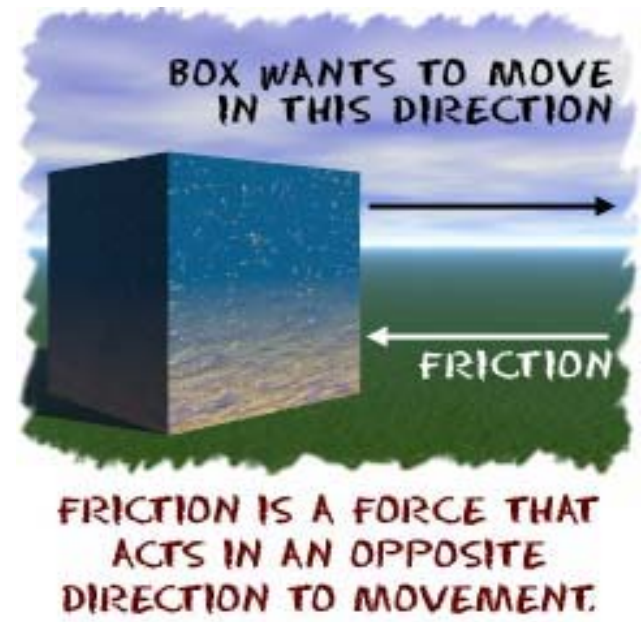
f_a : applied force

For example, $K = 3$, $f_a = -2$

$$v > 0 \Rightarrow m \frac{dv}{dt} = -3 - 2 < 0 \Rightarrow v \downarrow$$

$$v < 0 \Rightarrow m \frac{dv}{dt} = 3 - 2 > 0 \Rightarrow v \uparrow$$

sliding mode: $v \equiv 0$ (if $|f_a| < K$).



<http://www.physics4kids.com>

Contents (1/2)

- Introduction to Sliding Mode
 - Variable Structure Systems
 - Invariance Condition (Matching Condition)
- Variable Structure Systems (VSS) Design
 - Sliding Surface Design
 - Discontinuous Control
- Discrete-Time Sliding Mode
 - $O(T)$ v.s. $O(T^2)$ Boundary Layer
 - Switching v.s. Continuous Control

Contents (2/2)

- Minimum-Time Torque Control (SRM)
- Temperature Control (Plastic Extrusion P.)
- Rod-less Pneumatic Cylinder Servo
- Wireless Network Power Control

- Singular Perturbations
- Distributed Parameter Systems

A “funnel-like” domain (cont.)

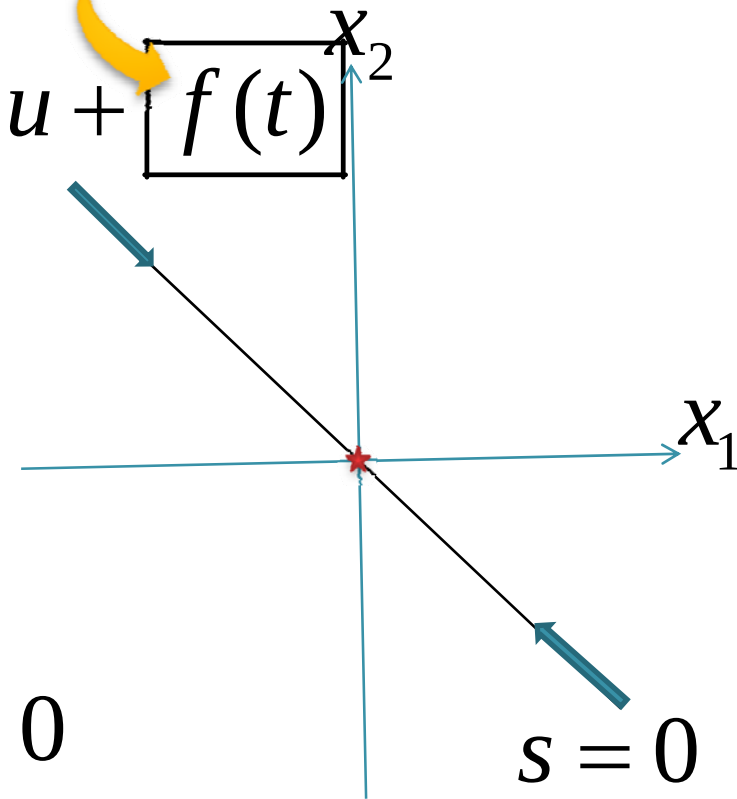
System:
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disturbance

Target: $(x_1, x_2) = (0, 0)$

Funnel: $x_2 = -x_1$

New target: $s = x_1 + x_2 = 0$



A “funnel-like” domain (cont.)

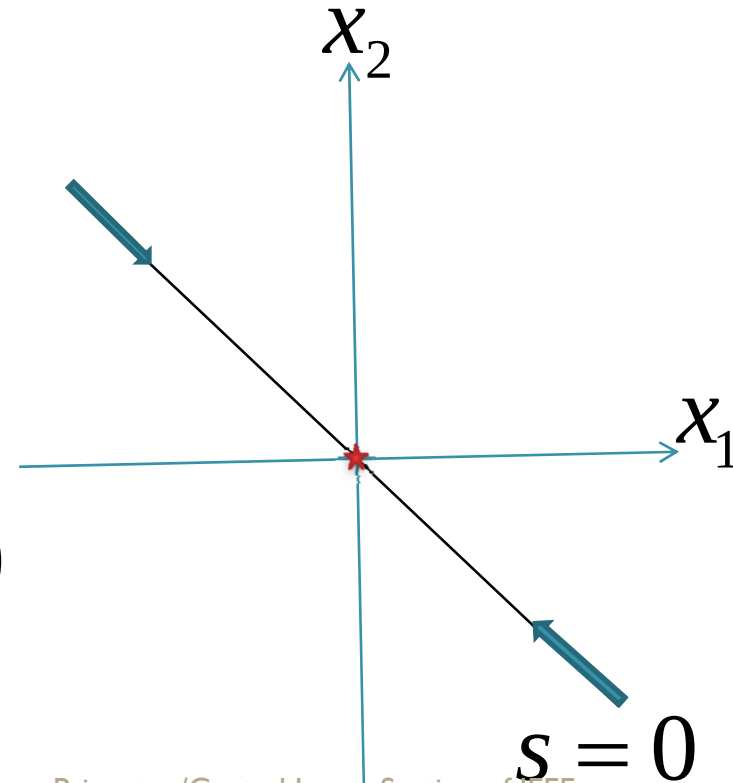
System:
$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2x_1 - 3x_2 + u + \boxed{(\Delta_1 x_1 + \Delta_2 x_2)} \end{cases}$$

Parameter variations

Target: $(x_1, x_2) = (0, 0)$

Funnel: $x_2 = -x_1$

New target: $s = x_1 + x_2 = 0$

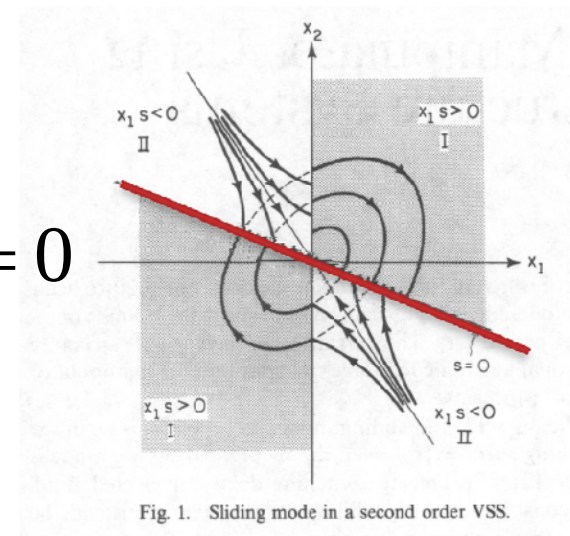


Introduction to Sliding mode

- Variable Structure Systems

$$\text{System : } \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_2 - bu \end{cases} \quad (\text{K. D. Young, 1978})$$

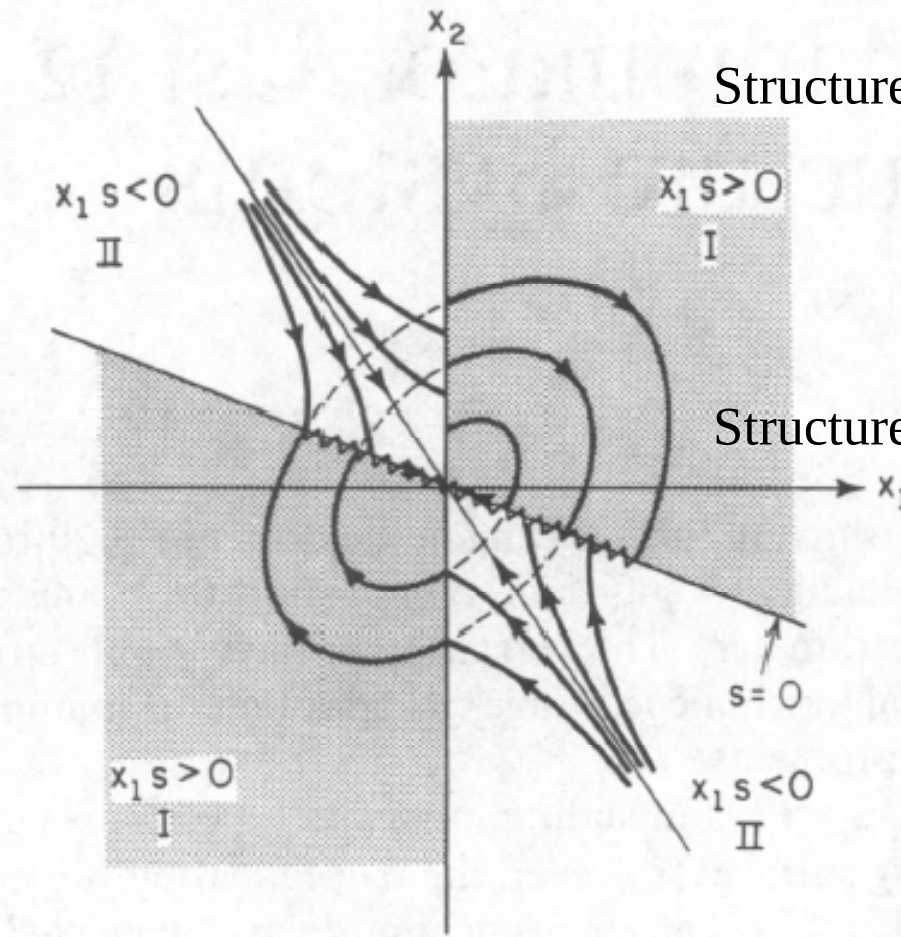
Switching line : $s = cx_1 + x_2 = 0$
(Sliding Surface)



$$\text{Control : } u = \begin{cases} \alpha x_1, & \text{if } x_1 s > 0 \text{ (region I)} \\ -\alpha x_1, & \text{if } x_1 s < 0 \text{ (region II)} \end{cases}$$

Introduction to Sliding mode

- Variable Structure Systems



$$\text{Structure I: } \dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ -b\alpha & a \end{bmatrix} \underline{x}$$

$s^2 - as + b\alpha = 0$: spiral out

$$\text{Structure II: } \dot{\underline{x}} = \begin{bmatrix} 0 & 1 \\ b\alpha & a \end{bmatrix} \underline{x}$$

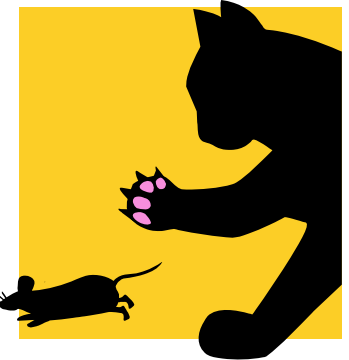
$s^2 - as - b\alpha = 0$: saddle point

Fig. 1. Sliding mode in a second order VSS.

Introduction to Sliding mode

- Invariance Condition

Q: Can the control law possibly reject the disturbance out of the system?



$$\dot{\underline{x}} = A\underline{x} + \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 2 & -1 \end{bmatrix} \underline{u} + \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} f(t)$$

$$= A\underline{x} + \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} u_1 + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} u_2 + \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} f(t)$$

Introduction to Sliding mode

- Invariance Condition

Control subspace: $span \left\{ \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right\}$

Disturbance subspace: $span \left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \right\}$

* The disturbance can be rejected by the control law if (and only if) **the control subspace covers the disturbance subspace.**

Introduction to Sliding mode

- Invariance Condition

In this example,
$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = 2 \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

If $d(t)$ is known, then the control law

$$u_1 = v_1 - 2f(t)$$

$$u_2 = v_2 - f(t)$$

Will lead to

$$\dot{x} = Ax + \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} v_1 + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} v_2 + \left(-2 \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \right) f(t) + \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} f(t) = Ax + \begin{bmatrix} 0 & 2 \\ -1 & 1 \\ 2 & -1 \end{bmatrix} \underline{v}$$

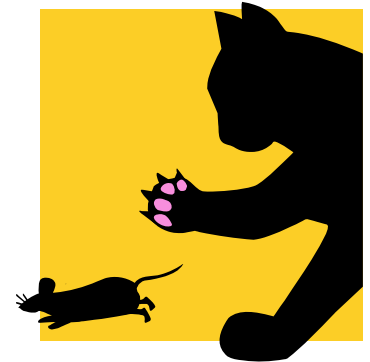
Introduction to Sliding mode

- Invariance Condition

(Drazenovic, 1969):

The system with disturbance

$$\dot{x} = Ax + Bu + Ef(t)$$



is invariant of $f(t)$ in sliding mode $s \equiv 0$ iff

$$\text{rank} [B \mid E] = \text{rank} [B].$$

In the previous example,

$$\text{rank} \left[\begin{array}{cc|c} 0 & 2 & 2 \\ -1 & 1 & -1 \\ 2 & -1 & 3 \end{array} \right] = \text{rank} \left[\begin{array}{cc} 0 & 2 \\ -1 & 1 \\ 2 & -1 \end{array} \right] = 2.$$

A “funnel-like” domain (revisit)

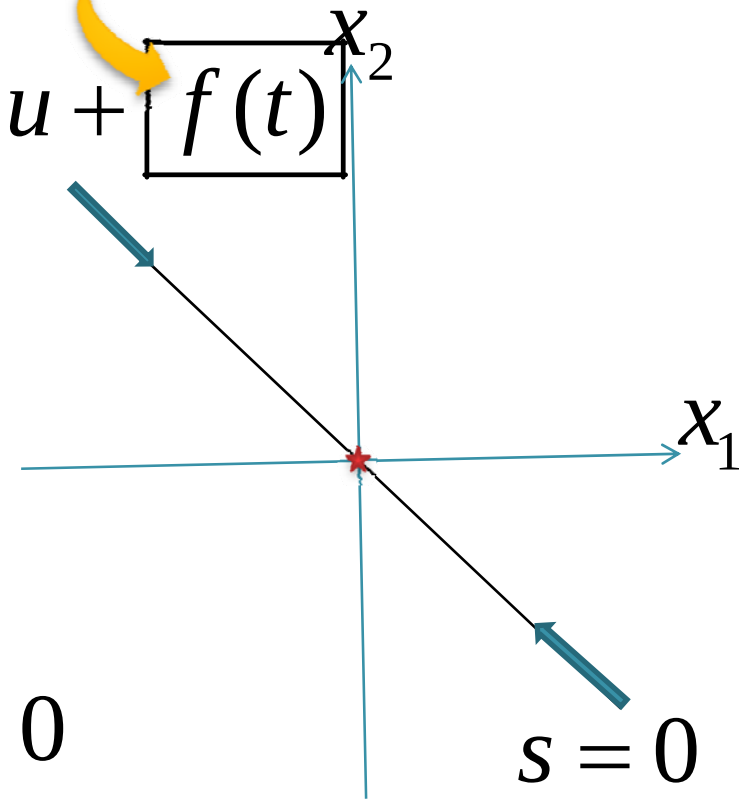
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disturbance

Target: $(x_1, x_2) = (0, 0)$

Funnel: $x_2 = -x_1$

New target: $s = x_1 + x_2 = 0$



A “funnel-like” domain (revisit)

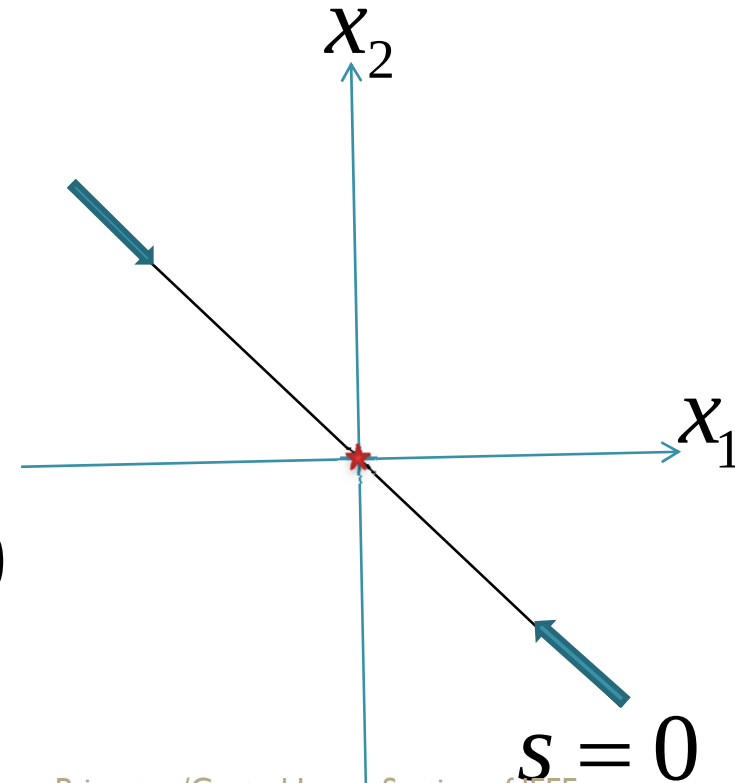
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Parameter variations

Target: $(x_1, x_2) = (0, 0)$

Funnel: $x_2 = -x_1$

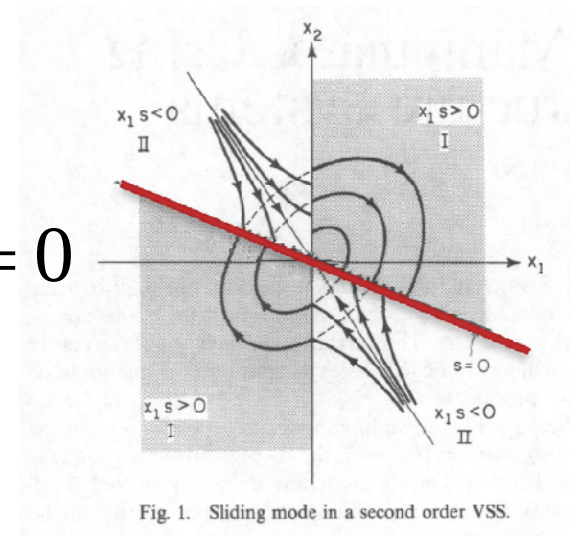
New target: $s = x_1 + x_2 = 0$



VSS Design

$$\text{System : } \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_2 - bu \end{cases} \quad (\text{K. D. Young, 1978})$$

Switching line : $s = cx_1 + x_2 = 0$
(Sliding Surface)



$$\text{Control : } u = \begin{cases} \alpha x_1, & \text{if } x_1 s > 0 \text{ (region I)} \\ -\alpha x_1, & \text{if } x_1 s < 0 \text{ (region II)} \end{cases}$$

VSS Design

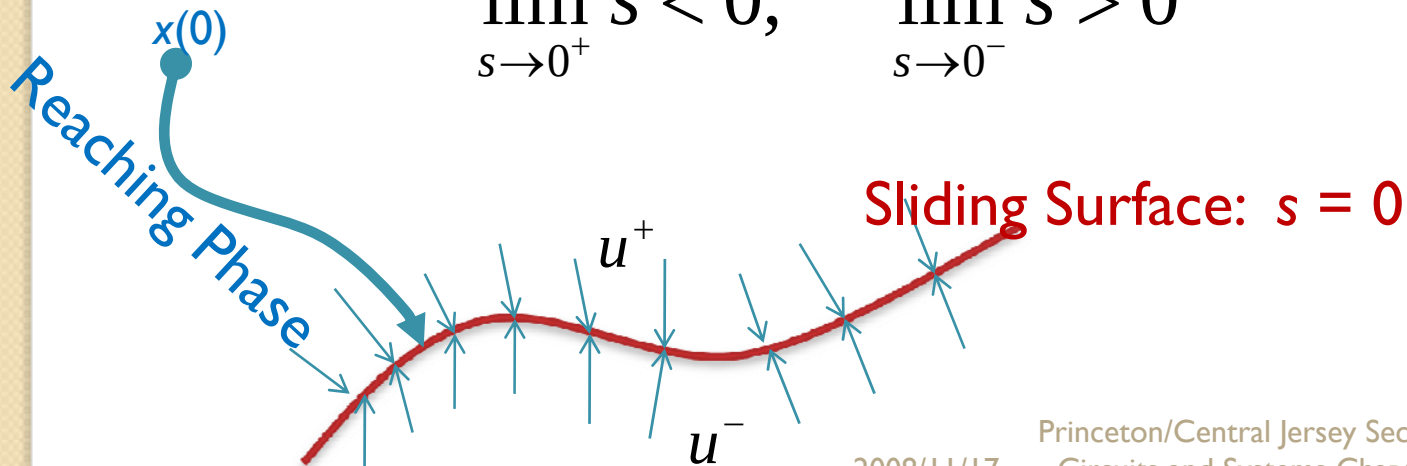
Existence of sliding mode

i. Reaching condition (sufficient, global)

$$\dot{s} < -\sigma \operatorname{sgn}(s), \quad \sigma > 0$$

ii. Sliding condition (sufficient, local)

$$\lim_{s \rightarrow 0^+} \dot{s} < 0, \quad \lim_{s \rightarrow 0^-} \dot{s} > 0$$



VSS Design

Finite time reaching: $\dot{s} < -\sigma \operatorname{sgn}(s)$, $\sigma > 0$

Lyapunov function

$$V = s^2$$

$$\frac{dV}{dt} = 2s\dot{s} = -2\sigma|s| < 0$$

$$\therefore s(x) \rightarrow 0$$

Reaching time $T < \frac{\sqrt{V(0)}}{\sigma} = \frac{|s(0)|}{\sigma}$

VSS Design

A heuristic example

$$(D^3 + a_3 D^2 + a_2 D + a_1)y(t) = (b_3 D^2 + b_2 D + b_1)(u + f(t)), \quad D = \frac{d}{dt}$$

$$\dot{\underline{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$

$$y = [b_1 \quad b_2 \quad b_3] \underline{x}$$

(i) **Sliding surface** $s(\underline{x}) = x_3 + c_2 x_2 + c_1 x_1 = 0$

Sliding mode dynamics:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \end{cases}, \text{ with } x_3 = -c_1 x_1 - c_2 x_2.$$

VSS Design

$$(ii) \quad \frac{ds(x)}{dt} = \frac{\partial s}{\partial x} \frac{dx}{dt}$$

$$\dot{s} = \dot{x}_3 + c_2 \dot{x}_2 + c_1 \dot{x}_1$$

$$= -a_1 x_1 - a_2 x_2 - a_3 x_3 + u + f(t) + c_2 x_3 + c_1 x_2$$

(iii) Discontinuous control

$$u = a_1 x_1 + (a_2 - c_1) x_2 + (a_3 - c_2) x_3 - (f_{\max} + \sigma) \operatorname{sgn}(s)$$

- Reaching condition $\dot{s} < -\sigma \operatorname{sgn}(s)$, $\sigma > 0$
satisfied if $|f(t)| < f_{\max}$.

VSS Design

Two steps in VSS design

$$\dot{\underline{x}} = \underline{f}(\underline{x}, u, t)$$

1. Choose sliding surface

$$s(\underline{x}) = 0$$

2. Design discontinuous control to render sliding mode

$$\dot{\underline{x}} = \begin{cases} \underline{f}(\underline{x}, u^+, t) = \underline{f}^+, & s > 0 \\ \underline{f}(\underline{x}, u^-, t) = \underline{f}^-, & s < 0 \end{cases}$$

VSS Design

- Sliding Surface

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + Df(t)$$

Eigenspace Approach v.s. Lyapunov Approach

Nonsingular transformation

$$TB = \begin{bmatrix} 0_{(n-m) \times m} \\ B_2 \end{bmatrix}, \quad T\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u + \begin{bmatrix} 0 \\ D_2 \end{bmatrix} f(t)$$

Sliding surface

$$x_2 = -Kx_1, \text{ or } s(\underline{x}) = [K \quad I_{m \times m}]T\underline{x}$$

Stabilizing feedback

$$\dot{\underline{x}} = A\underline{x} - BK\underline{x} + Df(t)$$

Lyapunov function

$$PA_s + A_s^T P = -Q$$

$$V(\underline{x}) = \frac{1}{2} \underline{x}^T P \underline{x}$$

Sliding surface

$$s(\underline{x}) = B^T P \underline{x}$$

VSS Design

- Discontinuous Control

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} + Df(t)$$

sliding surface: $s(\underline{x}) = C\underline{x} = 0$

$$\dot{s} = CA\underline{x} + CB\underline{u} + CDf(t)$$

Signum function
(Single-input case)

$$u = -(CB)^{-1}CAx - (CB)^{-1}(\sigma + d_{\max})\text{sgn}(s)$$

v.s.

Unit Control
(Multi-input case)

$$u = -(CB)^{-1}CAx - (CB)^{-1}(d_{\max} + \sigma) \cdot \frac{s}{\|s\|}$$

Discrete-Time Sliding Mode

Continuous-time

$$\dot{x} = Ax + Bu + Df(t)$$

$$\text{Sliding surface: } s(t) = Cx(t) = 0$$

Sampled-data

$$x_{k+1} = \Phi x_k + \Gamma u_k + d_k$$

$$\text{Sliding surface: } s_k = Cx_k = 0$$

$$\Phi = e^{AT}, \quad \Gamma = \int_0^T e^{A\tau} d\tau B,$$

$$d_k = \int_{kT}^{(k+1)T} e^{A(t-\tau)} Df(\tau) d\tau = \int_0^T e^{A\lambda} Df((k+1)T - \lambda) d\lambda$$

Discrete-Time Sliding Mode

$$x_{k+1} = \Phi x_k + \Gamma u_k + d_k$$
$$\Gamma = \int_0^T e^{A\lambda} d\lambda B, \quad d_k = \int_0^T e^{A\lambda} Df((k+1)T - \lambda) d\lambda$$

Issues on disturbance rejection:

1. Matching condition fails $d_k \notin \text{range}(\Gamma)$

However, $d_k = \Gamma f(kT) + O(T^2)$

2. Non-causal disturbances

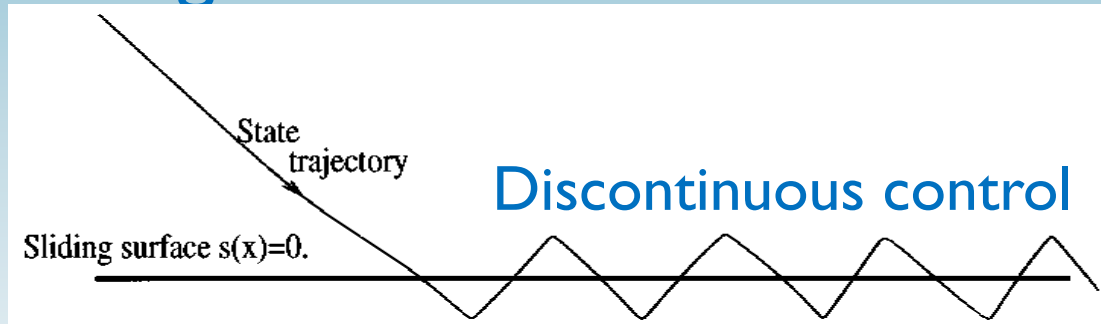
However, $d_k = d_{k-1} + O(T^2)$

$$d_{k-1} = x_k - \Phi x_{k-1} - \Gamma u_{k-1}$$

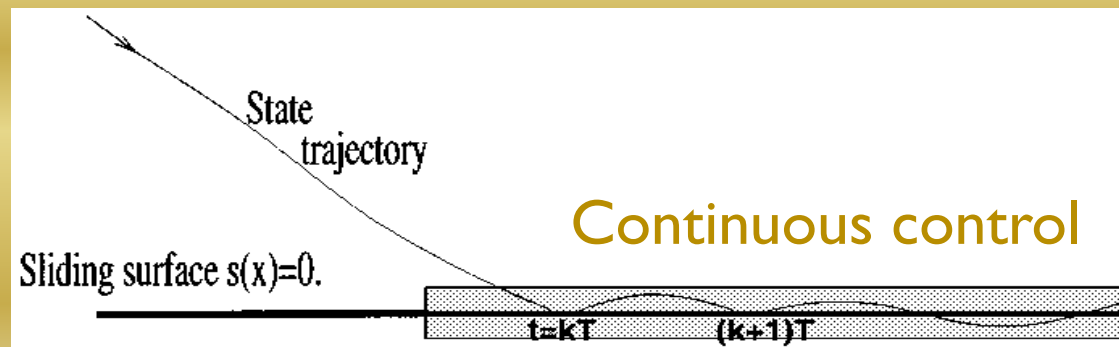
Ultimate achievable accuracy: $O(T^2)$

Discrete-Time Sliding Mode

Quasi-sliding mode $s_k = O(T)$, $s(t) = O(T)$



Discrete-time sliding mode $s_k = 0$, $s(t) = O(T^2)$



Discrete-Time Sliding Mode

Ukin's discrete-time equivalent control

(Continuous control law)

$$s_{k+1} = C\Phi x_k + C\Gamma u_k + Cd_k = 0$$

$$u_k^{eq} = -(C\Gamma)^{-1}C(\Phi x_k + d_k)$$

Discrete-time $O(T^2)$ sliding mode control

(Modification of u_k^{eq})

$$u_k = -(C\Gamma)^{-1}C(\Phi x_k + d_{k-1})$$

$$\Rightarrow s_{k+1} = C(d_k - d_{k-1}) = O(T^2)$$

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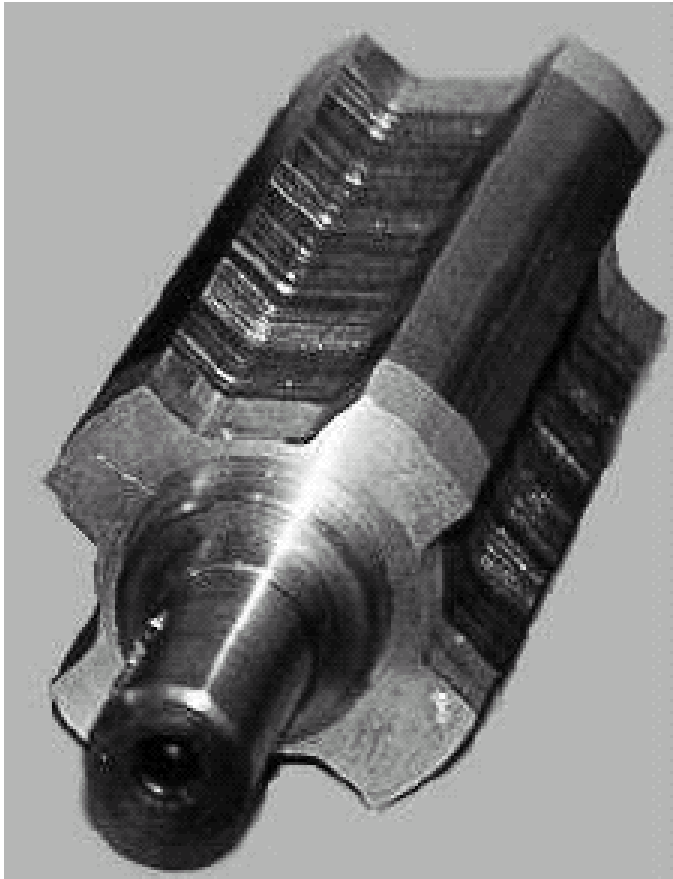
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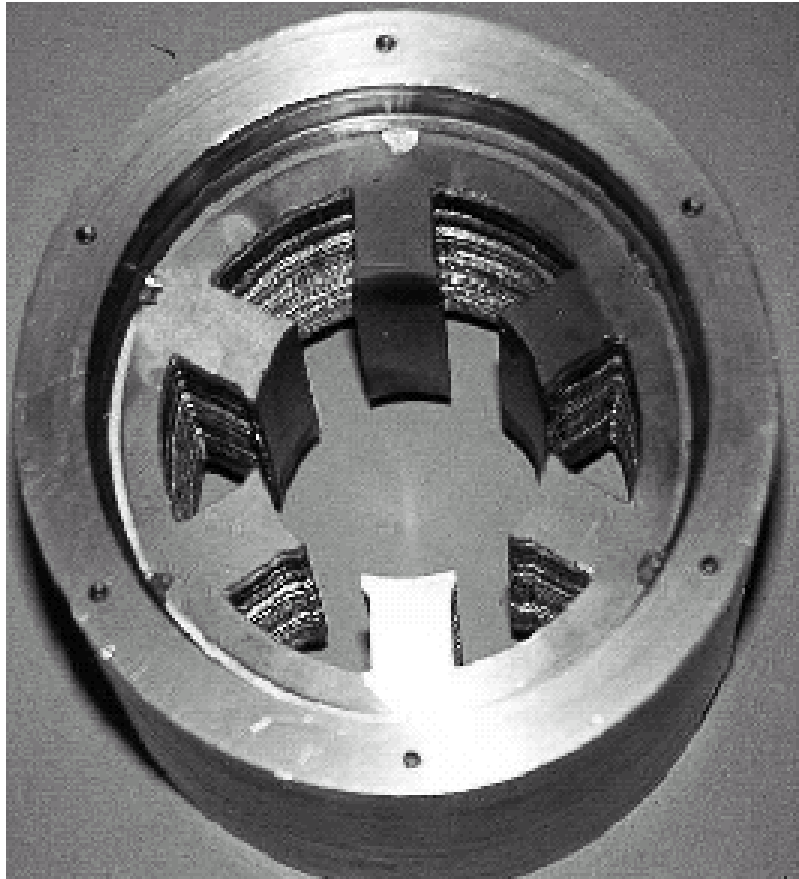
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Switched Reluctance Motors

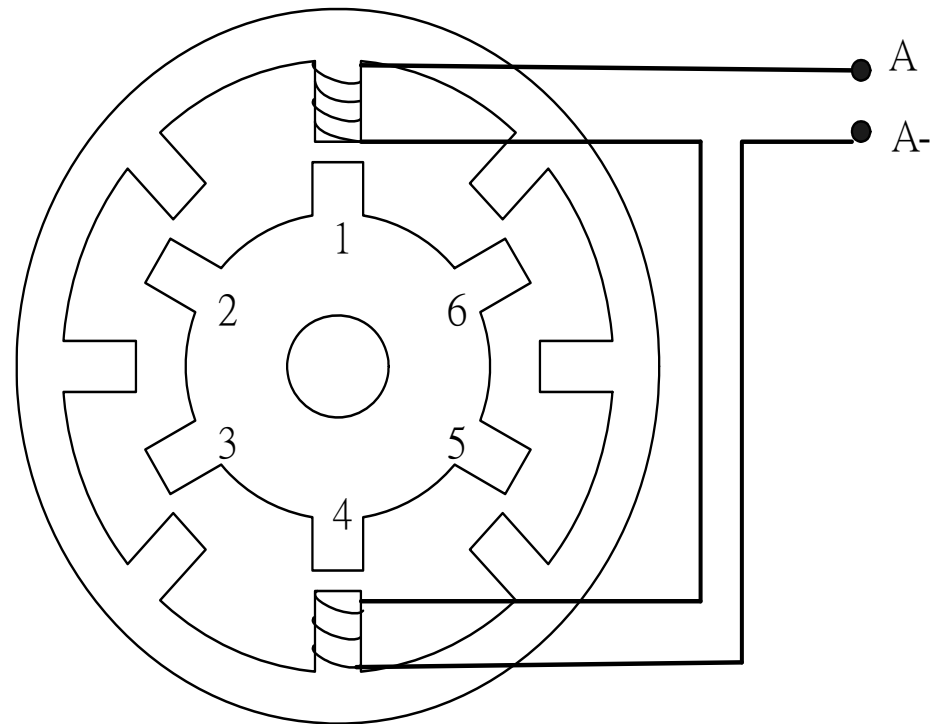


Rotor



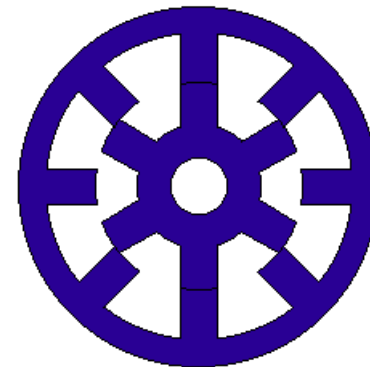
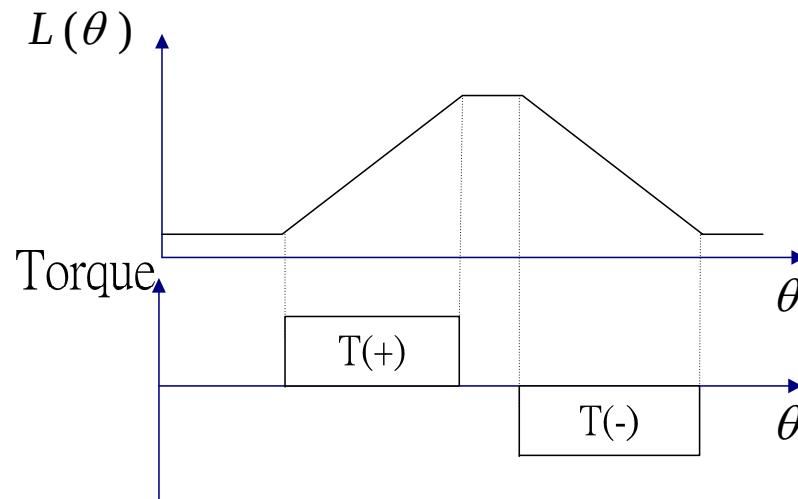
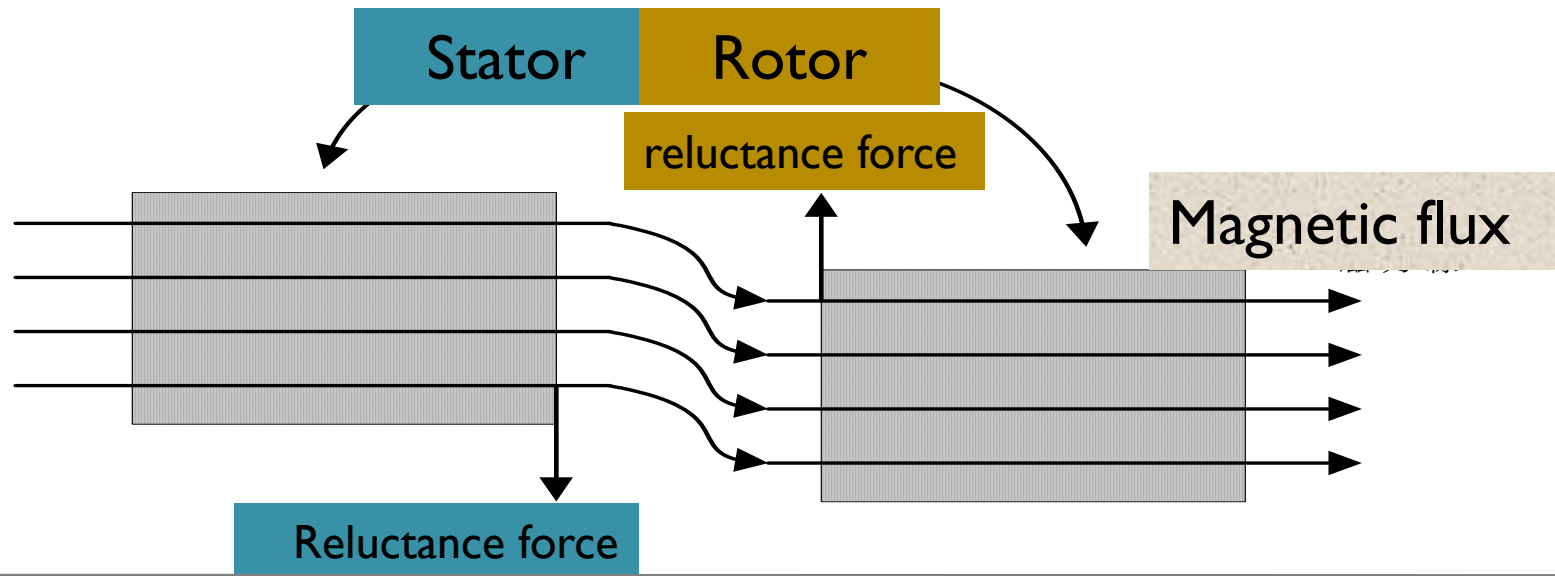
Stator

Switched Reluctance Motors

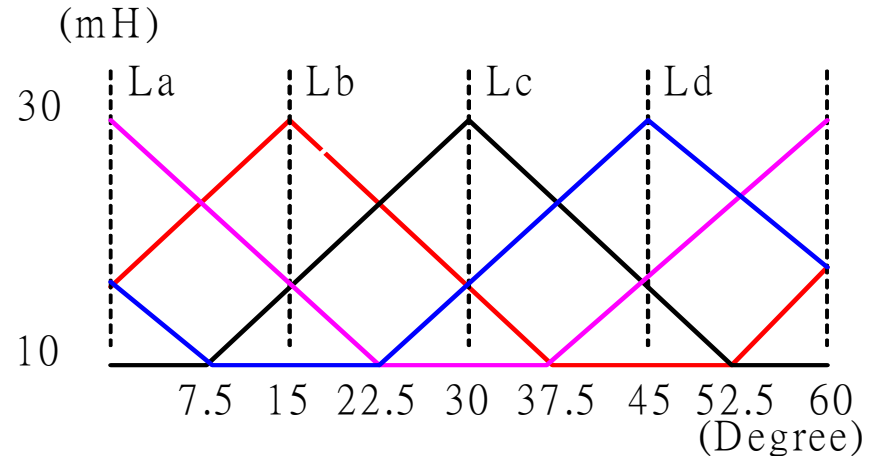
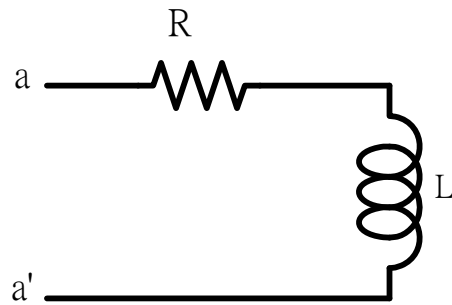


4-phase, 8/6-pole SRM

Switched Reluctance Motors



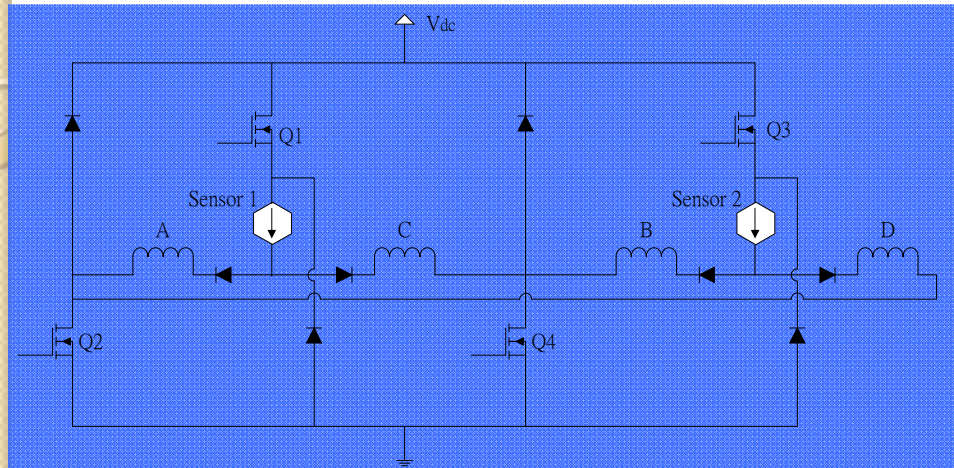
Switched Reluctance Motors



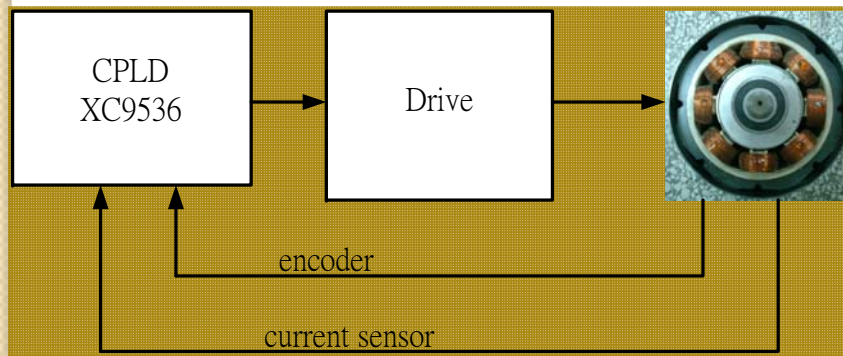
$$V_j = L_j(\theta) \frac{di_j}{dt} + \frac{dL_j(\theta)}{dt} i_j + R_j i_j, \quad j = A, B, C, D$$

$$\text{Torque} \propto i^2 : T_j = \frac{1}{2} \frac{dL_j(\theta)}{d\theta} i_j^2$$

Minimum-Time Torque Control



Drive Circuit



Control System Setup

System dynamics:

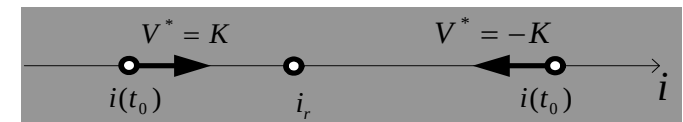
$$\frac{di}{dt} = -A(\theta, \omega)i + B(\theta)V(t)$$

Sliding surface:

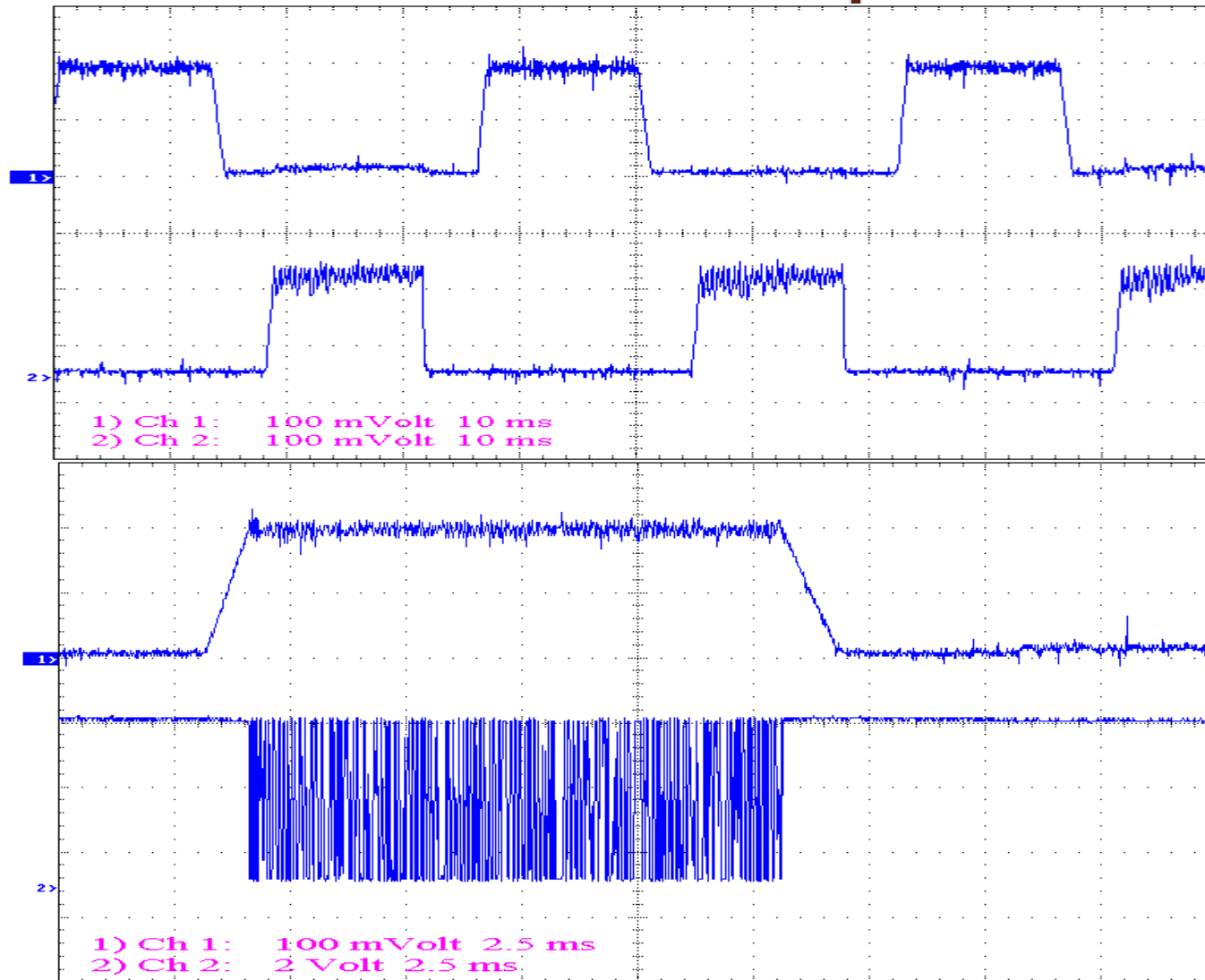
$$s = i - i_r = 0$$

Minimum-Time VSC:

$$V(t) = \begin{cases} -V_{dc}, & \text{if } i(t) > i_r \\ V_{dc}, & \text{if } i(t) < i_r \end{cases}$$

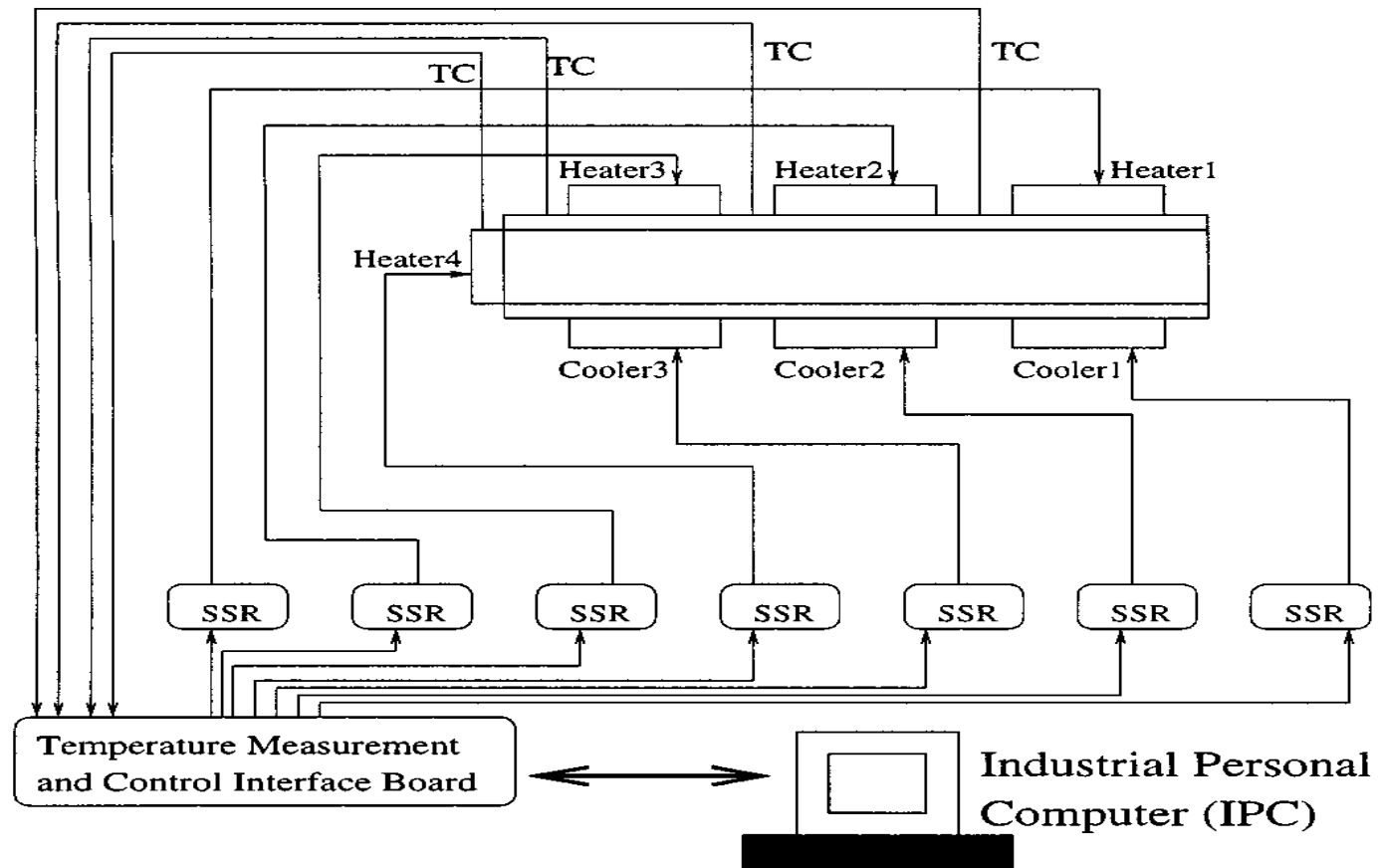


Minimum-Time Torque Control



Temperature Control

- A Plastic Extrusion Process



System setup: ITRI (Industrial Technology Research Institute), Taiwan, R.O.C.

Temperature Control

- A Plastic Extrusion Process

Dynamics of Channel temperature:

$$\frac{dy}{dt} = -\frac{hA}{\rho cV} y + \frac{hA}{\rho cV} y_{\infty} + \frac{1}{\rho cV} w$$

Rate of change of energy input:

$$\frac{dw}{dt} = a(w, u, t) = \begin{cases} a^+(w, u, t), & u \geq 0 \\ a^-(w, u, t), & u < 0 \end{cases}$$

Single-channel dynamics:

$$\dot{y} = -my + v$$

$$\dot{v} = \frac{a(w, u, t)}{\rho cV} + \xi$$

Temperature Control

- A Plastic Extrusion Process

Single-channel dynamics:

$$\dot{y} = -my + v$$

$$\dot{v} = \frac{a(w, u, t)}{\rho c V} + \xi$$

Sliding surface (PI control):

$$\dot{v} = -K_p \dot{e} - K_I e \quad \text{or} \quad V(s) = (K_p + \frac{K_I}{s})(y_{desired}(s) - Y(s))$$

Output feedback sliding mode control

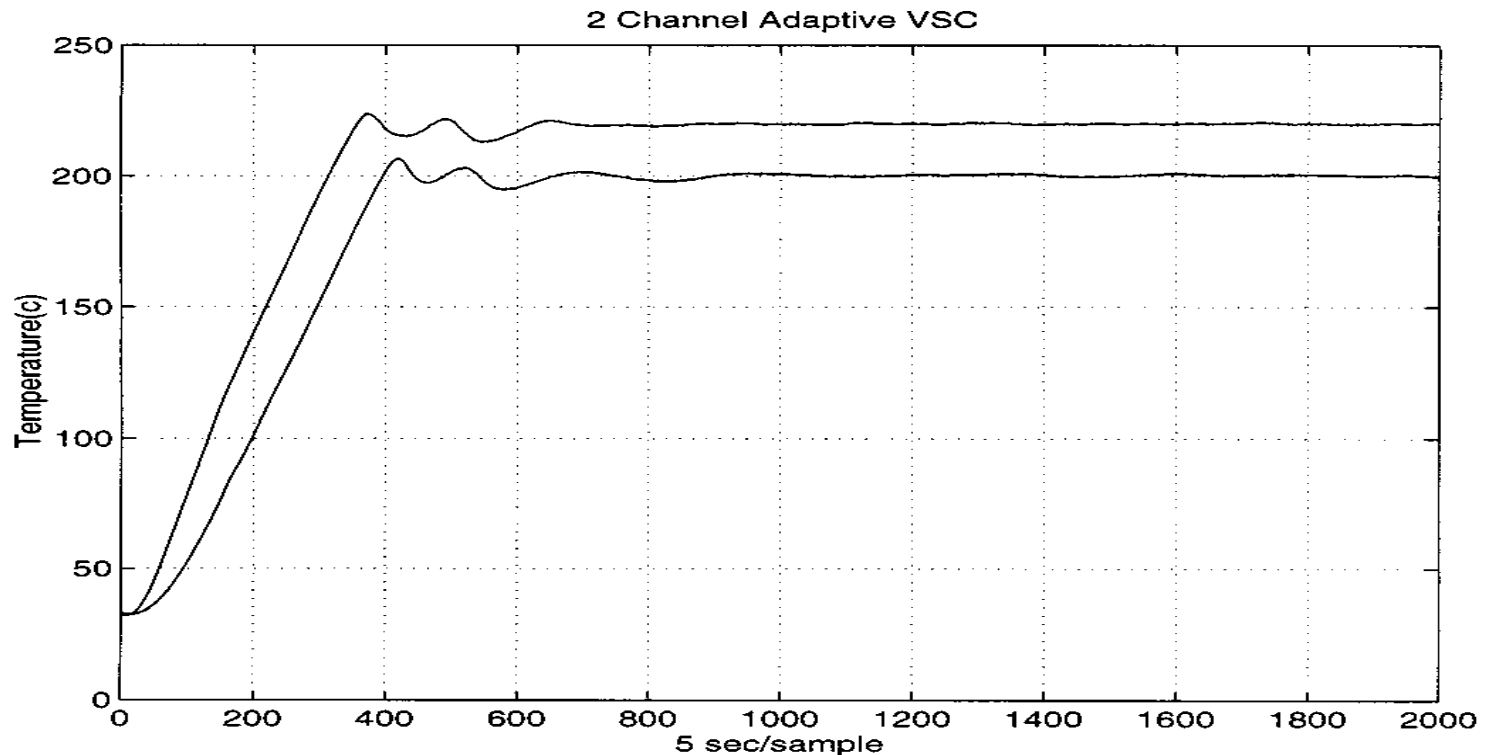
$$s = \ddot{e} + (K_p + m)\dot{e} + K_I e$$

$$s_k \approx a_1 s_{k-1} + b_0 e_k + b_1 e_{k-1}$$

Temperature Control

- A Plastic Extrusion Process

Switching control law: $u_k = \begin{cases} \text{heat, } s_k < 0 \\ \text{water, } s_k > 0 \end{cases}$



Rod-less Pneumatic Cylinder Servo

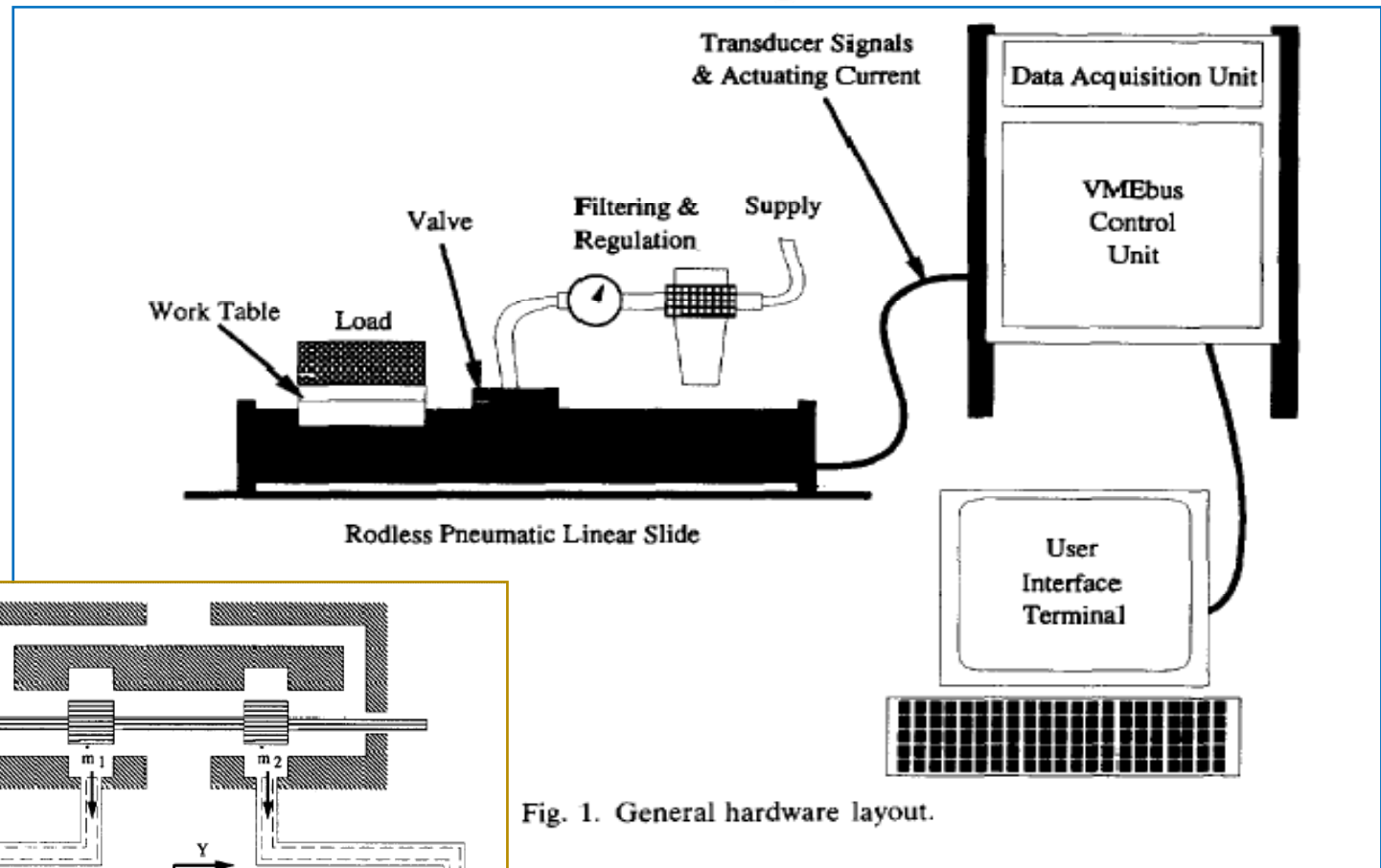


Fig. 1. General hardware layout.

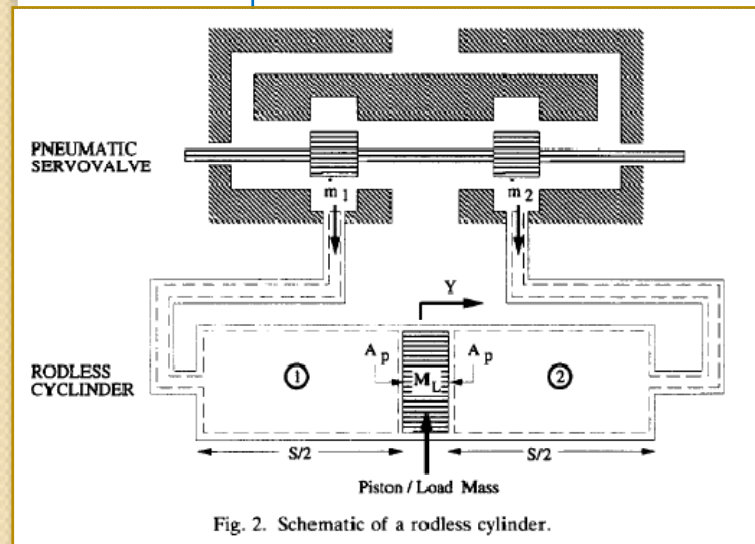


Fig. 2. Schematic of a rodless cylinder.

Rod-less Pneumatic Cylinder Servo

Equation of motion:

$$M_L \ddot{Y} = -\mu_u \dot{Y} - \mu_c \operatorname{sgn} \dot{Y} + A_T \Delta P$$

Pressure dynamics:

$$\Delta \dot{P} = \psi_1(P_1, P_2, Y, X) X + \psi_2(P_1, P_2, Y, \dot{Y}, X)$$

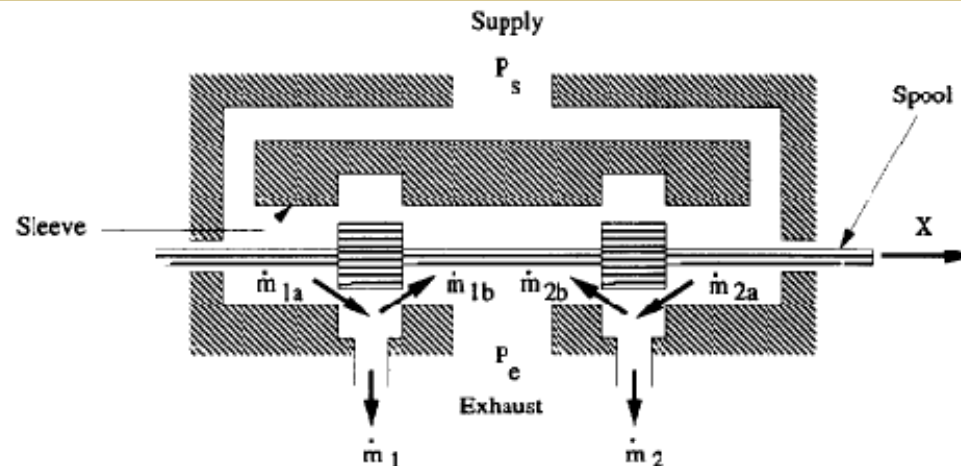


Fig. 3. Schematic of a four-way servovalve. Princeton/Central Jersey Section of IEEE
2008/11/17 - Circuits and Systems Chapter Meeting

Rod-less Pneumatic Cylinder Servo

Sliding surface:

$$\sigma = \Delta P - \frac{M_L}{A_T} \left[\ddot{Y}_d + k_1(\dot{Y}_d - \dot{Y}) + k_2(Y_d - Y) \right] + \frac{1}{A_T} \left[\mu_u \dot{Y} + \mu_c \operatorname{sgn} \dot{Y} \right]$$

Variable Structure Control: $X = -X_v \operatorname{sgn}(\sigma)$

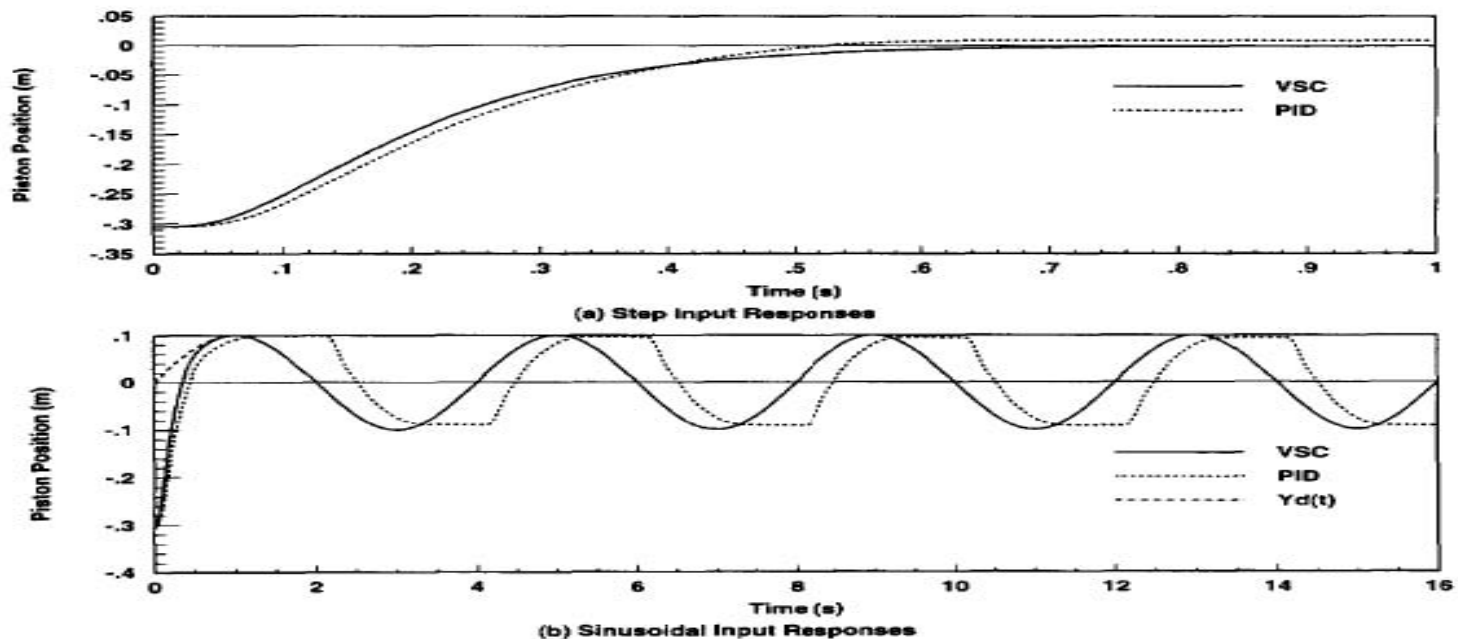
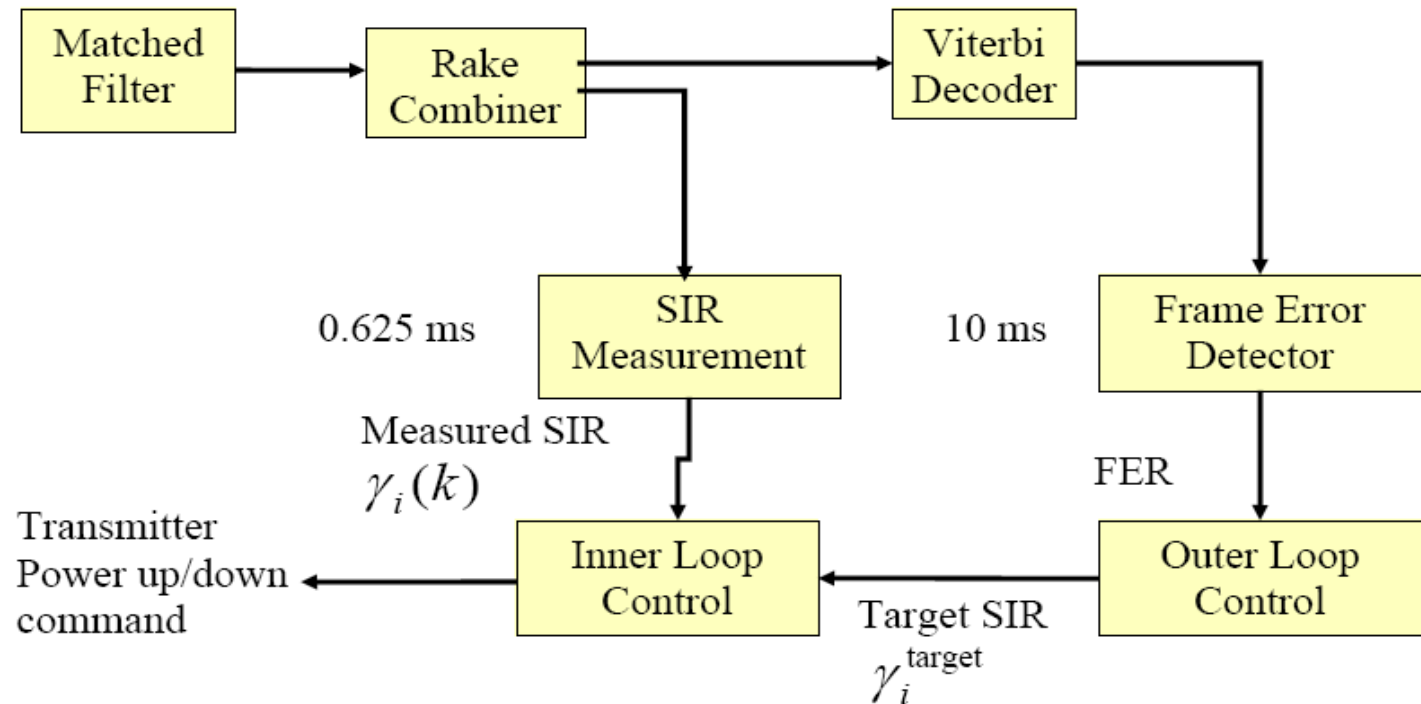


Fig. 8. Comparison of variable-structure control and classic PID control.

Wireless Network Power Control



Base Station Behavior

Zoran Gajic, Plenary Lecture 2007 CACS International Automatic Control Conference
National Chung Hsing University, Taichung, Taiwan, November 9-11, 2007

Wireless Network Power Control

Signal-to-interference ratio (SIR) for the i th user

$$\bar{\gamma}_i(t) = \frac{\bar{g}_{ii}(t)\bar{p}_i(t)}{\bar{I}_i(t)} = \frac{\bar{g}_{ii}(t)\bar{p}_i(t)}{\sum_{j \neq i} \bar{g}_{ij}(t)\bar{p}_j(t) + \bar{v}_i(t)}, i = 1, 2, \dots, N.$$

Control objective: $\bar{\gamma}_i(t) \rightarrow \bar{\gamma}_i^{tar}$

Logarithmic scale representation

$$\gamma_i = g_{ii} + p_i - I_i(p_{-i})$$

System dynamics:

$$\dot{p}_i(t) = u_i(t), \quad i = 1, 2, \dots, N$$

$$\gamma_i(t) = p_i(t) + f_i(t)$$

Wireless Network Power Control

Sliding surface $s_i(t) = \gamma_i(t) - \gamma_i^{tar} = 0$

Discrete-time representation

$$s_i(k+1) = s_i(k) + Tu_i(k) + d(k) + v(k)$$

Discrete-time SMC (continuous)

$$u_i(k) = -\frac{1}{T}s_i(k) - \frac{1}{T}(d(k-1) + v(k-1))$$

Stability analysis

$$\begin{bmatrix} s_i(k+1) \\ u_i(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ -\frac{1}{T} & -1 \end{bmatrix} \begin{bmatrix} s_i(k) \\ u_i(k) \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix} \frac{1}{T} (d(k) + v(k))$$

$$\det \begin{bmatrix} z-1 & -T \\ \frac{1}{T} & z+1 \end{bmatrix} = z^2 = 0$$

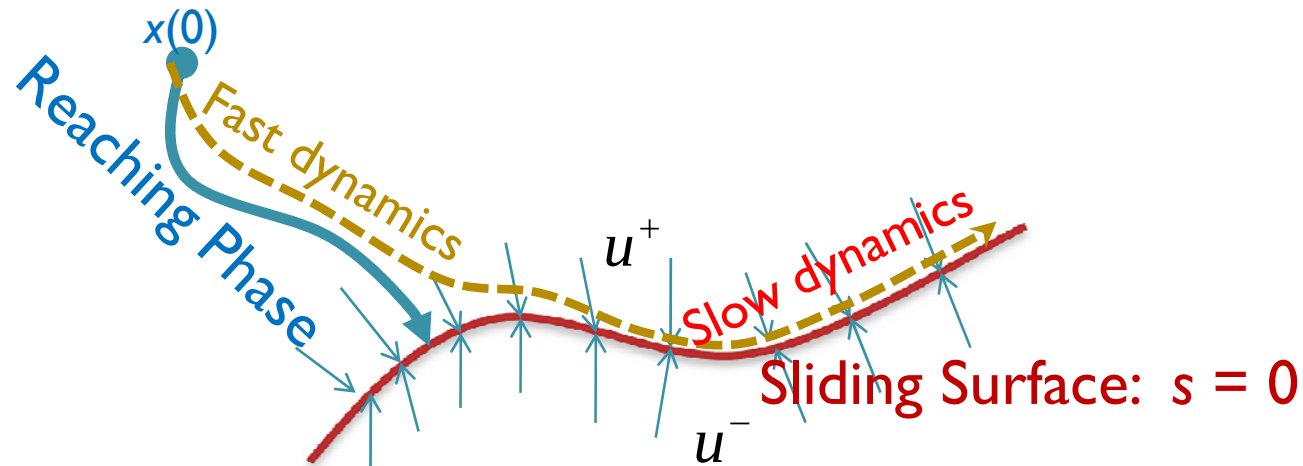
Contents (2/2)

- Minimum-Time Torque Control (SRM)
- Temperature Control (Plastic Extrusion P.)
- Rod-less Pneumatic Cylinder Servo
- Wireless Network Power Control

- Singular Perturbations
- Distributed Parameter Systems

Extended Research Problems

- Singular Perturbations



Singularly Perturbed Systems	VSS
Quasi-steady state (slow mode)	Sliding mode
Fast mode	Reaching phase
Continuous control	Discontinuous control
Infinite-time reaching (boundary layer)	Finite-time reaching

Extended Research Problems

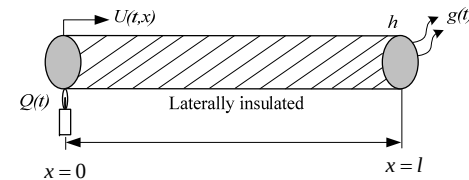
- Distributed Parameter Systems

Heat Conduction System :

$$U_t(x,t) = \alpha U_{xx}(x,t) + \beta U(x,t)$$

Boundary conditions $U(0,t) = 0$

$$U(l,t) = Q(t) + f(t)$$



Kernel function

$$k(x,y)$$

$$U(x,t)$$

$$w(x,t)$$

$$w_t = \alpha w_{xx} - cw$$

$$w(0,t) = 0$$

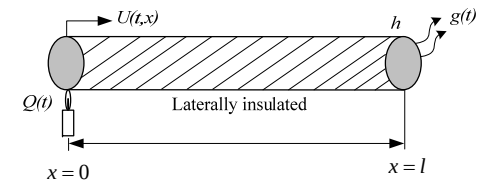
$$w(l,t) = Q(t) + f_w(t)$$

Extended Research Problems

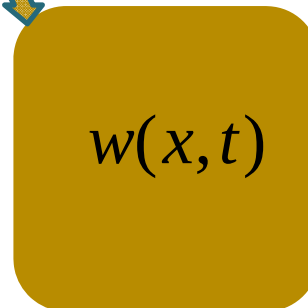
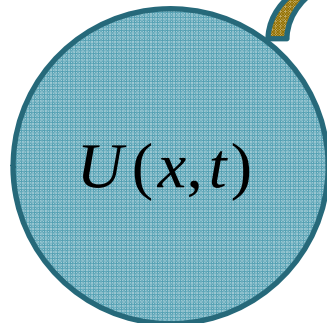
- Distributed Parameter Systems

Sliding surface:

$$S(t) = \omega_x(l, t) = 0$$



Kernel function : $k(x, y) = -\lambda y \frac{I_1\left(\sqrt{\lambda(x^2 - y^2)}\right)}{\sqrt{\lambda(x^2 - y^2)}}$



$$w_t = \alpha w_{xx} - cw$$

$$w(0, t) = 0$$

$$w(l, t) = Q(t) + f_w(t)$$

Extended Research Problems

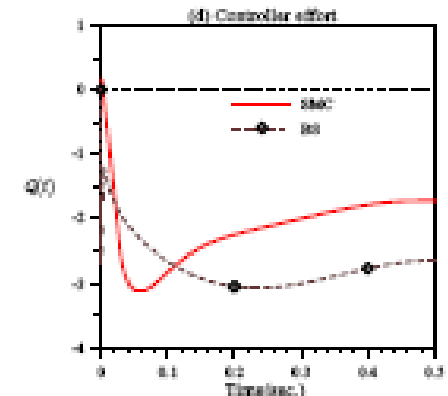
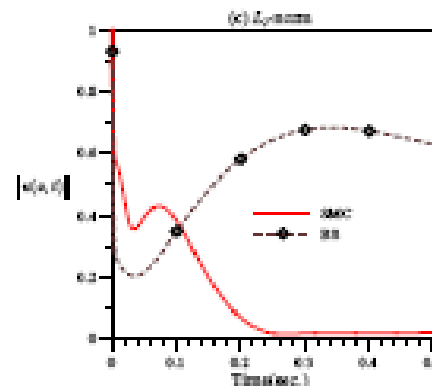
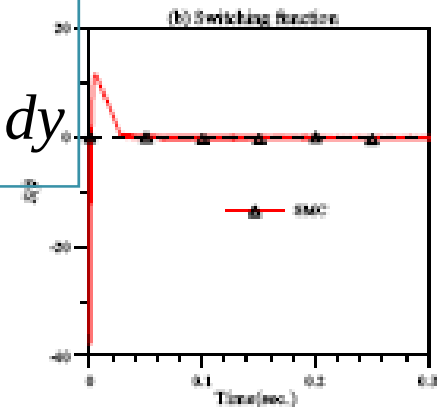
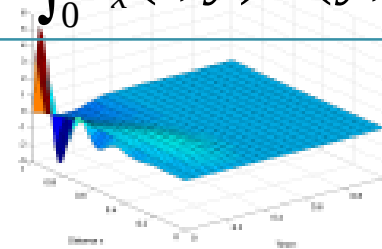
- Distributed Parameter Systems

Sliding surface: $S(t) = w_x(l, t) = 0$

$$S(t) = U_x(l, t) - k(l, l)U(l, t) - \int_0^l k_x(l, y)U(y, t)dy$$

Sliding Mode Control:

$$Q(t) = -K_m \int_0^t \text{sgn}(S(\tau))d\tau$$



Conclusions

- Merits of VSS
 - Robustness (against disturbances and system parameter variations)
 - Reduced-order system design
 - Simple control structure
 - Fits switching power electronics control

Conclusions

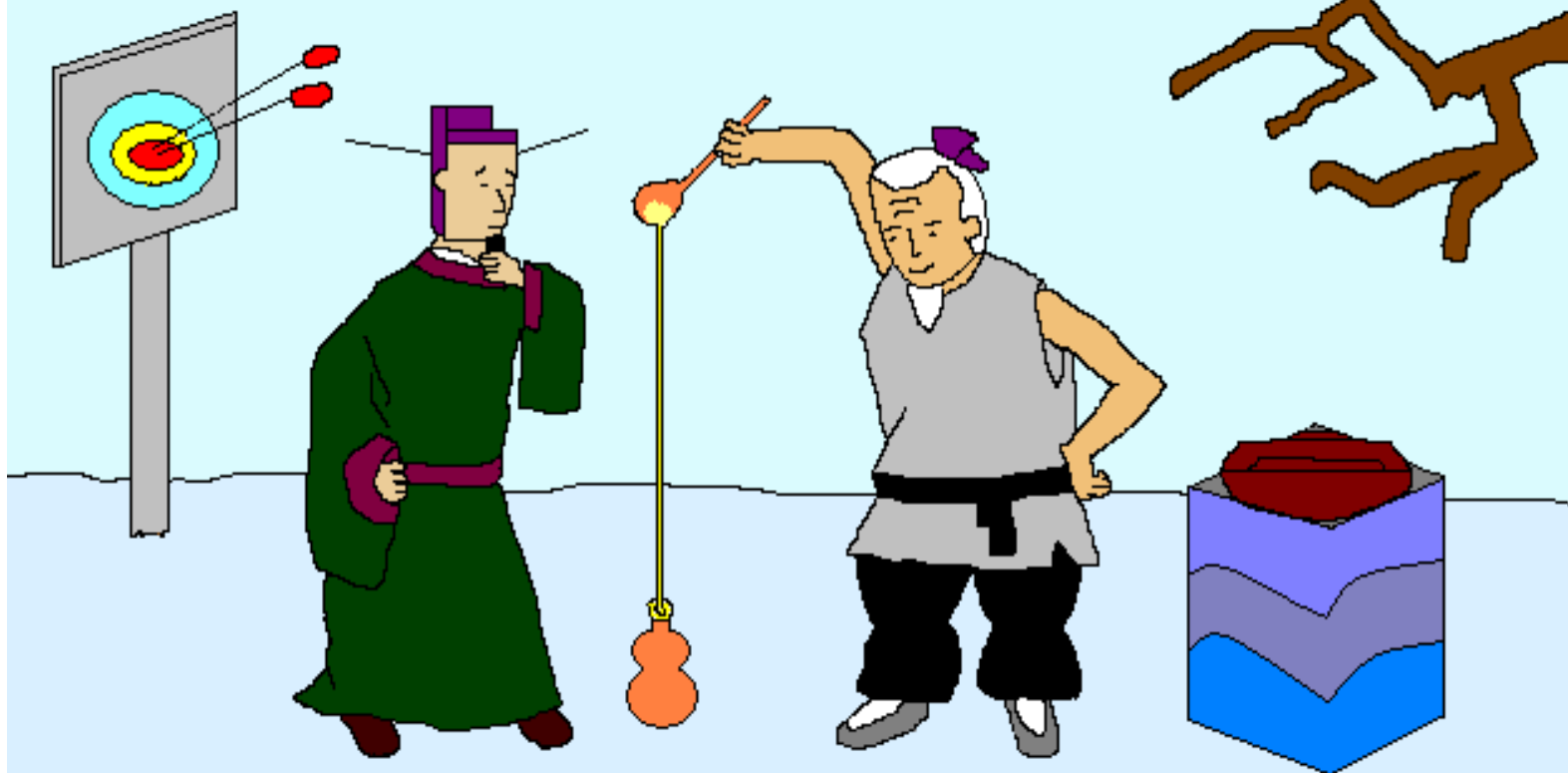
- Chattering Reduction (Elimination)
 - Discrete-Time Sliding Mode
 - Filtering methods
- Interesting Problems
 - Output feedback
 - Singular Perturbations
 - Infinite-dimensional systems
 - Systems with delays (e.g. wireless network)



Thank You!

陈康康公尧咨善射，当世无双，公亦以此自矜。尝射千家圃，有卖油翁释担而立，睨之，久而不去。见其发矢十中八九，但微颔之。康康问曰：“汝亦知射乎？康康问曰：“汝亦知射乎？吾射不亦精乎？”翁曰：“无他，但手熟尔。”康康忿然曰：“尔安敢轻吾射！”翁曰：“以我酌油知之。”乃取一葫芦置千曲，以钱覆其口，徐以杓酌油沥之，只钱孔入，而钱不湿。因曰：“我亦无他，为手熟尔。”

《卖油翁》



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