Distributed Cooperative Spectrum Sensing based on Weighted Average Consensus

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Abstract— In this paper, we study the distributed spectrum sensing in cognitive radio networks. Using weighted average consensus algorithm, we develop a weighted soft measurement combining scheme without the centralized fusion center. After the measurement by the energy detector, each secondary user (SU) exchanges their own measurement statistics with its local neighbors, and chooses the information exchanging rate according to the estimated average signal-to-noise ratio (SNR). We prove the convergence of the consensus iteration, and each SU will hold the global decision statistics from the weighted soft measurement combining throughout the network. The proposed scheme is robust with respect to temporary communication link failures. Simulation results show our method has a better performance than the existing average consensus-based approach.

I. INTRODUCTION

Cognitive Radio (CR) [1] aims to improve the spectrum utilization by allowing unlicensed secondary user (SU) to operate in the 'white spaces' of the licensed spectrum bands. One of the key techniques in cognitive radio is spectrum sensing, which enables the secondary users to detect the absence or presence of the primary user (PU) in the spectrum. Recent research progress shows cooperative spectrum sensing is an effective and promising methodology to improve the spectrum sensing performance under practical shadowing, fading and time-varying wireless channels [2]. In this paper, we consider the weighted average consensus-based approach for distributed cooperative spectrum sensing.

The main advantage of cooperative spectrum sensing is to enhance the sensing performance by exploiting the observation diversity of the spatially located SUs. By cooperation, CR users can share their sensing information for making a combined decision which is more accurate than the individual decisions [2]. Among many works on cooperative spectrum sensing, most studies focused on developing centralized cooperation approaches [3]. There are two stages in the cooperative sensing. In the first stage, each SU makes the measurement using appropriate detecting techniques. In the second stage, SUs report their measurement information to the centralized fusion center, then a global decision is made at the fusion center according to certain measurement combining method.

In the sensing stage, energy detection is widely applied because it requires lower design complexity and no priori knowledge of primary users, compared to other techniques,

such as matched filter detection or cyclostationary detection [4]. During fusion, different measurement combining methods have been considered including hard bit combining [5], soft gain combining [6][7]. However, centralized cooperative spectrum sensing requires the entire received data be gathered at one place. In large SU networks, it is less scalable, less fault tolerant, and has redundant reporting cost. To overcome these disadvantages, bio-inspired consensus-based scheme is introduced to spectrum sensing in [8] for the decentralized measurement combining for the cooperative SUs. However, average consensus only ensures the average soft combining of the global measurements, which is less effective under the practical Rayleigh fading channels.

The main contribution of this paper is to propose a weighted soft measurement combining scheme without the centralized fusion center. After the measurement by the energy detector, SUs exchange their decision statistics with their local neighbors, and weight the information exchanging rate according to their own estimated average signal-to-noise ration (SNR) values. We rigorously prove that after the consensus is reached, each SU holds the global decision statistics from the weighted soft combining throughout the network. The convergence of the algorithm will not be affected by the communication link switching or temporary link failures. The simulation results show the significant improvement of the sensing performance compared to the existing consensus-based approach, especially under the low SNR Rayleigh fading channels.

The rest of this paper is organized as follows: In section II, we introduce the fundamentals about the energy detector and the consensus related SU network model. We describe our proposed weighted average consensus-based spectrum sensing scheme in Section III. In section IV, we present simulation results and make comparison with the existing approach. We conclude this study in Section V. The related theoretic proofs are given in the Appendix.

II. SPECTRUM SENSING AND SECONDARY USERS NETWORK MODELING

In the consensus-based spectrum sensing scenario, a distributed network is established by the SU group. The related models on the sensing stage and information fusion stage are presented in this section.

A. Sensing and Measurement Stage

In this paper, we mainly consider the energy detector under the Rayleigh fading channel model [9]. For each SU, the

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received signal $y(t)$ is modeled as

$$y(t) = \begin{cases} n(t), & H_0 \\ hx(t) + n(t), & H_1, \end{cases} \quad (1)$$

where h represents gain of the channel, $x(t)$ is the signal from PU , $n(t)$ is the additive white Gaussian noise.

Following [9][10], under Rayleigh fading, the channel gain h is random and the resulting SNR γ follows an exponential distribution. The output Y of each energy detector has the following distributions under hypothesis:

$$Y \sim \begin{cases} \chi^2_{2TW}, & H_0 \\ \chi^2_{2TW-2} + Y_e, & H_1, \end{cases} \quad (2)$$

where χ^2_{2TW} and χ^2_{2TW-2} denote random quantities under central chi-square distributions with mean $2TW$ and $2TW - 2$, respectively. Y_e has an exponential distribution with parameter $2(\bar{\gamma} + 1)$, $\bar{\gamma}$ represents the average SNR of the fading channel. The hypothesis H_0 and H_1 denote the absence and presence of the primary user , respectively. TW is the preset time-bandwidth product of the energy detector which can be set to any integer, we denote $TW = m$ for simplicity.

In the consensus-based scenario, we assume n SUs cooperate with each other. The ith SU detects the energy and obtain the measurement $Y_i \in \mathbb{R}^+$. Since the distributions under H_0 and H_1 hold the mean $2m$ and $2m + 2\bar{\gamma}$, respectively, each SU can simply estimates the average SNR from recent measurements as [7]

$$\bar{\gamma}_i = \frac{1}{2l} \sum_{j=k-l}^{k} (Y_{i,j} - 2m), \quad (3)$$

where $Y_{i,j}$ is the jth measurement of the ith secondary user, l is the estimation window.

B. The SU Network Model and Consensus Notions

In the information fusion stage, SUs communicate with their local neighbors through the SU network and adopt the consensus iteration to obtain the global measurement statistics. For convenience, we assign an index set $\mathcal{I} = \{1, 2, ..., n\}$ for the SU network formed by n SUs.

To model the consensus algorithm, we adopt the standard undirected graph model for the bidirectional SU communication network. The SU network is represented by an undirected graph $\mathcal{G} = (\mathcal{E}, \mathcal{V})$, where $\mathcal{V} = \{v_i | i \in \mathcal{I}\}$ is a finite nonempty set of nodes. We refer the ith node as the ith SU. The two names, SU and node, will be used alternatively. The edge set $\mathcal{E} = \{e_{ij} = (v_i, v_j) | i, j \in \mathcal{I}\}$. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j : e_{ij} \in \mathcal{E}\}$. A path in $\mathcal{G}$ consists of a sequence of nodes $v_1, v_2, ..., v_l, l \geq 2$, satisfying $(e_{m,m+1}) \in \mathcal{E}, \forall 1 \leq m \leq l - 1$. The graph $\mathcal{G}$ is *connected* if any two distinct nodes in $\mathcal{G}$ are connected by a path. When considering the directed graph (i.e. digraph), we refer to v_i and v_j as the tail and head of a directed edge $e_{ij} = (v_i, v_j)$, which represents the unidirectional communication link between two neighboring SUs. A digraph is called *strongly connected* if it is possible to reach any node starting from any other node following the edge directions.

In the case of the time-varying communication links, we model the SU network by $\mathcal{G}(k) = (\mathcal{E}(k), \mathcal{V})$, where $\mathcal{E}(t)$ is the set of active edges at time k. Let $\mathcal{N}_i(k) = \{j \in \mathcal{V} | \{i, j \in \mathcal{E}(k)\}\}$, and $d_i(k) = \mathcal{N}_i(k)$ denote the degree (number of neighbors) of node i at time k.

Let $\mathcal{G}_i = (\mathcal{E}_i, \mathcal{V}), i = 1, \ldots, r$, denote a finite collection of graphs with common vertex set $\mathcal{V}$. Their union is a graph $\mathcal{G}$ with the same vertex set and an edge set that is the union of the $\mathcal{E}_i's$, i.e., $\mathcal{G} = \bigcup_{i=1}^{r} \mathcal{G}_i = (\bigcup_{i=1}^{r} \mathcal{E}_i, \mathcal{V})$. The set of undirected graphs $\{\mathcal{G}_1, \ldots, \mathcal{G}_r\}$ is called *jointly connected* if their union is a connected graph. The set of digraphs $\{\mathcal{G}_1, \ldots, \mathcal{G}_r\}$ is called *jointly connected* if their union is a strongly connected digraph.

In the consensus-based spectrum sensing context, for the n SUs modeled by the graph $\mathcal{G}$, the ith SU is assigned a state variable $x_i, i \in \mathcal{I}$. The ith SU uses x_i for representing its measurement statistics of the energy detection. By reaching consensus, we mean the individual state x_i asymptotically converge to a common value x^*, i.e.,

$$x_i(k) \rightarrow x^* \text{ as } k \rightarrow \infty, \forall i \in \mathcal{I}, \quad (4)$$

where k is the discrete time step, $k = 0, 1, 2, ...$, and $x_i(k)$ is updated based on the previous states of node i and its neighbors.

III. SPECTRUM SENSING BASED ON WEIGHTED AVERAGE CONSENSUS

In this section, we present consensus-based distributed scheme to achieve the weighted measurement combining through local interactions among secondary users, in stead of processing the measurements in the centralized fusion center.

A. Weighted Average Consensus Algorithm

Following the notation of consensus [11], we denote the value of the ith agent's measurement Y_i as x_i, and the proposed weighted average consensus-based combining scheme is shown as

$$x_i(k+1) = x_i(k) + \frac{\epsilon}{\sigma_i} \sum_{j \in N_i} (x_j(k) - x_i(k)), i \in \mathcal{I} \quad (5)$$

where ϵ is the iteration step size satisfying the maximum node degree constraint [11], and $\sigma_i \geq 1$ is the weighting ratio.

Remark 1: An important feature of the algorithm is the choice of the weighting factor σ_i. If the ith SU has higher confidence about its own measurement, it sets a larger σ_i, which makes the iteration (5) rely less on the local information exchange. On the contrary, an SU with lower measurement confidence will set a smaller σ_i and relies more on the information from the network in order to improve the overall performance. Therefore, the statistics value across the whole SU network will be dominated by the SUs with higher confidence.

Next, we show that if the communication topologies formed by the SU network are jointly connected, all the

SUs' decision statistics will reach consensus. The final convergence value is:

$$x_i(k) \to x^* = \frac{\sum_{i=1}^{n} \sigma_i x_i(0)}{\sum_{i=1}^{n} \sigma_i} \text{ as } k \to \infty, \forall i \in \mathcal{I}. \quad (6)$$

B. Weighted Measurement Combining

In cooperative spectrum sensing, weighted combining is a natural scheme to improve the detecting performance [6][7]. The global decision statistic Y_g is from the weighted sum of the n local measurements

$$Y_g = \sum_{i=1}^{n} \omega_i Y_i, \quad (7)$$

where Y_i is the non-quantized output from the energy detector of the ith secondary user, and ω_i is the related weight. Particularly, when $\omega_i = 1/n, \forall i \in \mathcal{I}$, Y_g is the average of the global measurements.

In [8], an average consensus-based approach is proposed to ensure the distributed measurement combining. In detail, the fusion algorithm is shown as

$$x_i(k+1) = x_i(k) + \epsilon \sum_{j \in N_i} (x_j(k) - x_i(k)), i \in \mathcal{I} \quad (8)$$

where ϵ satisfies the maximum node degree constraint [11]. After the algorithm converges, the average measurement combining is achieved as

$$x_i(k) \to x^* = \frac{\sum_{i=1}^{n} x_i(0)}{n} \text{ as } k \to \infty, \forall i \in \mathcal{I}. \quad (9)$$

We refer this case as *equal gain combining* (EGC). Apparently, the EGC approach does not consider the practical channel conditions and the channel fading will affect its performance.

In contrast, the proposed fusion algorithm (5) aims to take each SU's channel condition into consideration. To account for the Rayleigh fading effects, a simple method is setting the weight ω_i directly from the estimated average SNR as [7]

$$\omega_i = \frac{\bar{\gamma}_i}{\sum_{i=1}^{N} \bar{\gamma}_i}. \quad (10)$$

Accordingly, if we choose the ratio σ_i in (5) as

$$\sigma_i = \begin{cases} \bar{\gamma}_i, & \text{if } \bar{\gamma}_i \geq 1 \\ 1, & \text{if } \bar{\gamma}_i < 1, \end{cases} \quad (11)$$

where $\bar{\gamma}_i$ is obtained from (3), and the saturation is to ensure $\sigma_i \geq 1$ for all i, then the decision value (6) of the consensus algorithm equals to

$$x_i \to x^* = \frac{\sum_{i=1}^{n} \bar{\gamma}_i x_i(0)}{\sum_{i=1}^{n} \bar{\gamma}_i}. \quad (12)$$

The sensing performance according to the weighted combining (12) is intended to outperform the performance of the EGC approach under Rayleigh fading channels. We refer our proposed method as *weighted gain combining* (WGC).

Finally, by comparing the decision value x^* with a predefined threshold λ, every SU locally obtains the global decision as:

$$\text{Decision } H = \begin{cases} 1, & x_i = x^* > \lambda \\ 0, & \text{otherwise.} \end{cases} \quad (13)$$

C. The Main Theorem

For convenience, we re-write the algorithm (5) in the following compact form:

$$x(k+1) = Px(k), \quad (14)$$

where $x = [x_1, \ldots, x_n]^T$, and P is the so-called Perron matrix [11], defined as

$$P = I - \epsilon \Sigma^{-1} L, \quad (15)$$

where $\Sigma = \text{diag}\{\sigma_1, \ldots, \sigma_n\}$, $L \in \mathbb{R}^{n \times n}$ is the Laplacian matrix of the communication graph $\mathcal{G}$ formed by the secondary user nodes, chosen as

$$l_{ij} = \begin{cases} d_i, & \text{if } i = j, \\ -1, & \text{if } i \neq j, \quad j \in \mathcal{N}_i, \\ 0, & \text{otherwise,} \end{cases} \quad (16)$$

where $d_i = |\mathcal{N}_i|$ is the degree of node i. The maximum node degree constraint [11] means

$$0 < \epsilon < (\max_i |\mathcal{N}_i|)^{-1} \triangleq \frac{1}{\Delta}, i \in \mathcal{I} \quad (17)$$

where Δ is called the maximum node degree of the network.

For a network of n secondary users, there are a finite number, say a total of r, of possible communication graphs. We denote the set of all possible graphs by $\{G_1, \ldots, G_r\}$, and the set of corresponding Perron matrices by $\{P_1, \ldots, P_r\}$. The weighted average consensus algorithm is given by

$$x(k+1) = P_{i(k)} x(k), \quad (18)$$

where the indices $i(k)$ are integers and satisfy $1 \leq i(k) \leq r$ for all $k > 0$.

Theorem 1: For the iteration process (18), if the step size ϵ satisfies maximum node degree constraint, and the elements of matrix $\Sigma = \text{diag}\{\sigma_1, \ldots, \sigma_n\}$ satisfy $\sigma_i \geq 1, \forall i \in \mathcal{I}$, and the collection of bidirectional communication graphs that occur infinitely often are jointly connected, then the iteration converges as

$$\lim_{k \to \infty} P_{i(k)}^k x(0) = \frac{\sum_{i=1}^{n} \sigma_i x_i(0)}{\sum_{i=1}^{n} \sigma_i} \mathbf{1}. \quad (19)$$

That is, the algorithm (5) converges to:

$$\lim_{k \to \infty} x_i(k) = \frac{\sum_{i=1}^{n} \sigma_i x_i(0)}{\sum_{i=1}^{n} \sigma_i}, \forall i \in \mathcal{I}. \quad (20)$$

The proof is given in Appendix A.

Remark 2: Theorem 1 requires weak long-term connectivity which contains both deterministic and stochastic time-varying graph sequences, and the convergence rate in general may not exist. If we further assume each link has an independent probability to fail, then follow the work of [12],

the algorithm will converge with probability 1. The variance of total deviation from weighted average is given as

$$E(\| x(k) - \frac{\sum_{i=1}^{n} \sigma_i x_i(0)}{\sum_{i=1}^{n} \sigma_i} \mathbf{1} \|^2) \to 0 \text{ as } k \to \infty, \quad (21)$$

and the convergence rate is exponential.

Remark 3: Theorem 1 encompasses the average consensus in [13] as a special case when $\Sigma = I$ and $P_{i(k)}$ are symmetric matrices. Apparently, fixed communication topology is a special case when $P_{i(k)} = P, \forall k \geq 0$, where the convergence rate is also exponential given by the Perron-Frobenius Theorem [14].

With the maximum node degree weights setup, we can further extend the results to unidirectional communication links assuming the communication topologies are *balanced graphs*, that is, in every iteration step, the number of neighbors each SU receives information from is the same as the number of neighbors to which it sends out information.

Corollary 1: For the iteration process (18), if the communication topology of the SU network is a balanced graph in every iteration step, and the collection of unidirectional communication graphs that occur infinitely often are jointly connected, then under the same conditions of Theorem 1, the iteration converges to

$$\lim_{k \to \infty} P_{i(k)}^k x(0) = \frac{\sum_{i=1}^{n} \sigma_i x_i(0)}{\sum_{i=1}^{n} \sigma_i} \mathbf{1}. \quad (22)$$

The proof is given in Appendix B.

Remark 4: In the iteration, maintaining node degree balanced is a distributed constraint for each SU, without centralized information.

IV. SIMULATIONS AND DISCUSSIONS

We conducted simulations to evaluate the effectiveness of our scheme. For the performance evaluation, we adopt the Monte Carlo method since closed form solutions for detection probability and false alarm do not exist generally.

A. Network Setup

To compare with the existing consensus-based scheme, we set up a similar scenario as in [8]. In particular, 10 SUs cooperate with each other and form a communication graph as shown in Fig. 1. All SUs are assumed to be static and experiencing i.i.d. Rayleigh fading without spacial correlation, and they are running the synchronized clock during the sensing process and the consensus iteration.

In the sensing stage, we directly generate the output Y_i of every SU's energy detector individually under the hypothesis H_1 in (2), with $TW = 6$ at the selected center frequency and bandwidth of interest. Each SU estimates the average SNR $\bar{\gamma}_i$ from (3) with the estimation window $l = 2$. Each SU then sets $x_i(0) = Y_i$ and starts the measurement fusion using the proposed scheme with the step size $\epsilon = 0.19$ in Eqn. (5). The final decision is made after the consensus result x^* is reached.

Fig. 2 shows the convergence of the proposed algorithm iteration under fixed communication graph. We found that
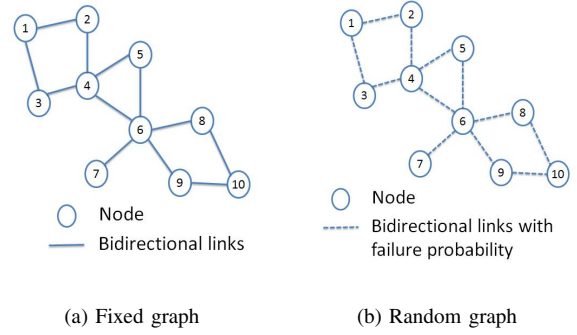

(a) Fixed graph   (b) Random graph

Fig. 1. Communication network of 10 SUs [8].

within 30 steps the difference of x_i among the SU network are less than 1dB, which indicates the consensus has been reached on the global decision statistics. Fig. 3 shows the convergence of the algorithm under random graph process with independent link failures. We set the failure probability of each link to 0.4. We observed that the consensus is achieved within 40 iteration steps, which is slightly slower than the fixed graph case.
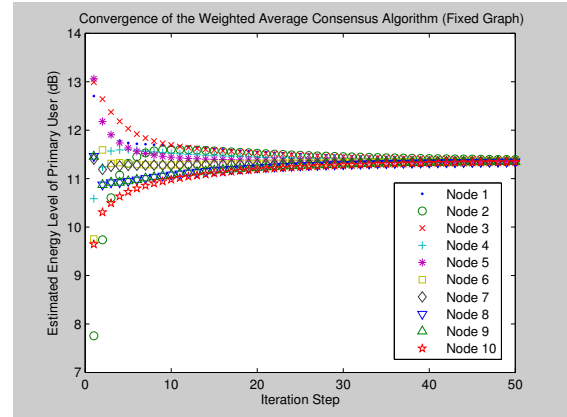

Fig. 2. Convergence of the network with a 10-node fixed graph. $\epsilon = 0.19$

Under the bidirectional communication link condition, for fixed topology, shown in Fig. 1(a), we denote our results as Weighted Gain Combining (WGC), and denote the results in [8] as Equal Gain Combining (EGC). When the communication links have independent probability of 40% to fail, as shown in Fig. 1(b), we denote simulation results as WGC_s and EGC_s, respectively.

B. Metrics

For comparison, we mainly consider P_d (probability of detection), P_f (probability of false alarm), and average SNR γ as metrics. A high P_d will result in high P_f, which increases the interference to primary users. On the other hand, a low P_f will result in low P_d and lead to low spectrum utilization. The average SNR in the measuring channel reflects the channel condition. It is challenging to
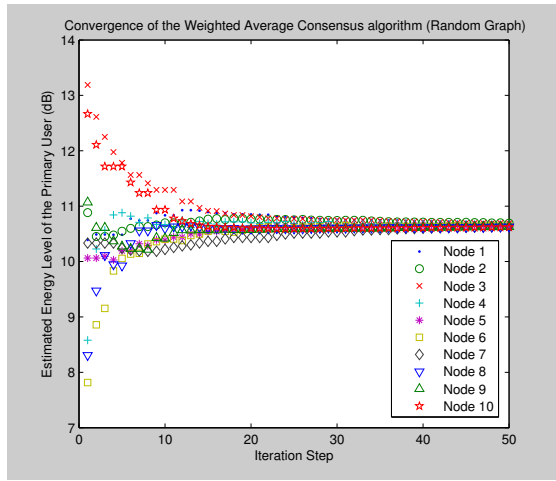
Fig. 3. Convergence of the network with a 10-node random graph, $\epsilon = 0.19$. The convergence failure probability is 0.
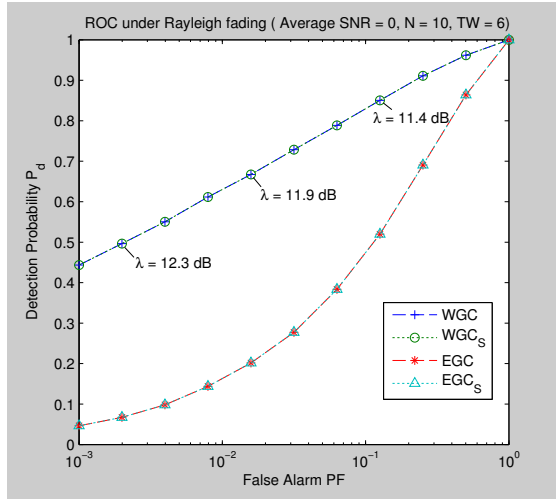

Fig. 4. Receiver operating characteristics under i.i.d Rayleigh fading (Each secondary user has the same average SNR, $\bar{\gamma}_i = 0$dB).

achieve high detecting performances under the condition of low average SNR.

The threshold λ is computed from the false alarm constraints and the hypothesis H_0 in (2). The two schemes share the same threshold, since the measurement output under H_0 is independent of the average SNR when the primary user signal is absent.

C. Performance Evaluation

Fig. 4 shows the receiver operating characteristics (ROC) of the two schemes when the average SNR is 0dB in each SU's measurement. We can see clearly from this figure, average consensus-based scheme (EGC and EGC_s) does not perform well in this case and the performance has been significantly improved by our proposed weighted average consensus scheme (WGC and WGC_s).

Fig. 5 shows the ROC of the two schemes when the average SNR of the SU's measurements vary from 0dB to

5dB. The proposed weighted combining scheme (WGC and WGC_s) achieves a satisfactory performance. When the false alarm is set to 10^{-2}, the detection probability reaches 0.8, while the equal gain scheme (EGC and EGC_s) is still below 0.5.
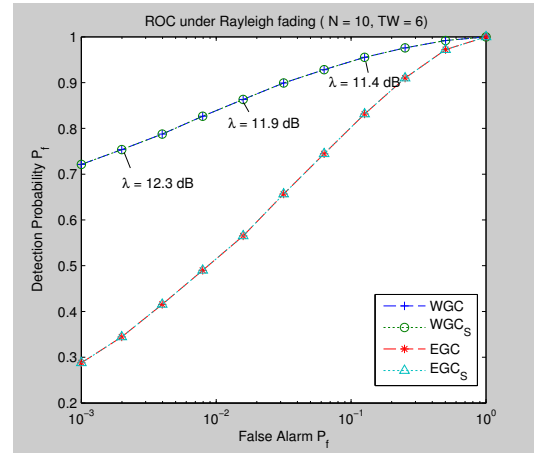

Fig. 5. Receiver operating characteristics under i.i.d Rayleigh fading (Each secondary user has different average SNR, $\bar{\gamma} = [5, 1, 2, 3, 4, 0, 4, 3, 2, 5]$).

Fig. 6 shows the detection probability curves with respect to the average SNR when the false alarm probability constraint is 0.005. We can see that when the average SNR is below 6dB, our proposed scheme (WGC and WGC_s) achieves much better performance than the equal gain combining (EGC and EGC_s). When the average SNR is higher than 6dB, the performance of these two schemes becomes closer, however, the weighted combining (WGC and WGC_s) still achieves higher detection probability.
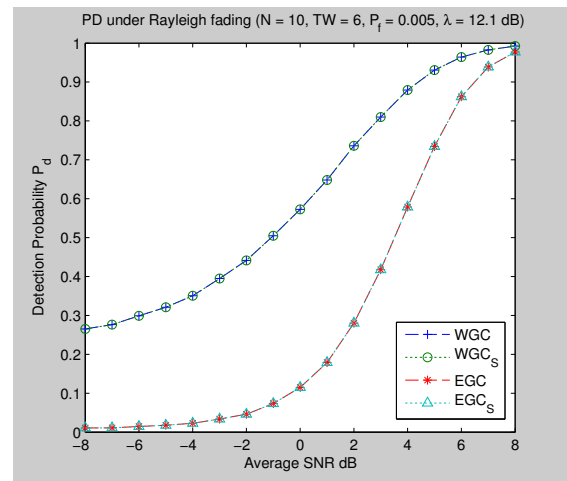

Fig. 6. Detection probability (P_d) with respect to average SNR under i.i.d. Rayleigh fading when the false alarm probability constraint is $P_f = 0.005$.

V. CONCLUSION AND FUTURE WORK

In this paper, we present a distributed weighted combining scheme for the cooperative spectrum sensing in cognitive radio networks. The proposed method is based on the weighted

average consensus algorithm for both fixed and time-varying graphs. Through weighted local fusion iteration, each SU holds the global decision statistic from the weighted soft measurement combining throughout the network. Simulation results show our approach outperforms the existing average consensus-based equal gain combining scheme under low SNR measurement conditions. Future work will pursue distributed asynchronous sensing and communciation.

ACKNOWLEDGMENT

APPENDIX

A. PROOF OF THEOREM 1

Proof of Theorem 1 shows that consensus iteration (18) actually is a paracontraction process and its fixed point is decided by the eigenspaces of the related Perron matrices.

For a connected undirected graph $\mathcal{G}$ and the associated Perron matrix $P = I - \epsilon\Sigma^{-1}L$ defined in (15), if we see each undirected link of the graph $\mathcal{G}$ as two directed links with opposite directions and different weights, then $\hat{L} = \Sigma^{-1}L$ is the Laplacian matrix of a strongly connected digraph $\hat{\mathcal{G}}$, and P is a nonnegative primitive matrix [11] having $\mathbf{1}$ as the right eigenvector associated with the eigenvalue 1. According to [11], P has the eigenvalue 1 as the maximum modulus, and all the other eigenvalues are in the unit circle, i.e., $|\lambda_i| < 1, i = 2, \ldots, n$. Therefore, we have $\| Px \| \leq \| P \| \| x \| = \| x \|$, $\forall x \in \mathbb{R}^n$, which proves P is a paracontracting matrix [15]. For a paracontracting matrix M, we denote $\mathcal{H}(M)$ as its fixed-point subspace, i.e., $\mathcal{H}(M) = \{x | x \in \mathbb{R}^n | Mx = x\}$. Apparently, $\mathcal{H}(M)$ is M's eigenspace associated with the eigenvalue 1.

If a collection of graphs $\{G_1, \ldots, G_p\}$ are jointly connected and occurs infinitely often, it is proved [13] that their corresponding Perron matrices satisfy $\bigcap_{i=1}^{p} \mathcal{H}(P_i) = \text{span}\{\mathbf{1}\} = \mathcal{H}\left(\frac{1}{p}\sum_{i=1}^{p}P_i\right) = \bigcap_{i=1}^{r}\mathcal{H}(P_i)$ for the set of all communication graphs $\{G_1, \ldots, G_r\}$, where $r \geq p$.

Denote $\bar{P} = \frac{1}{p}\sum_{i=1}^{p}P_i$, we know $\bar{P}$ is the primitive Perron matrix of a strongly connected graph, and $\bar{P}$ has $\mathbf{1}$ as the right eigenvector associated with the simple eigenvalue 1. We decompose the generalized eigenspace $\mathbb{E}^n$ of $\bar{P}$ as $E_1 \bigoplus E_s$ [16], where $E_1 = \text{span}\{\mathbf{1}\}$ associated with the simple eigenvalue 1, and E_s is the collection of all the other eigenspaces. For any initial value $x(0)$, we decompose $x(0) = u(0) + w(0)$, where $u(0) \in E_1$ and $w(0) \in E_s$. Because $u(0) \in \bigcap_{i=1}^{r}\mathcal{H}(P_i)$, the sequence given by $u(k+1) = P_{i(k)}u(k)$ is constant, and the limit $u^* = u(0)$. On the other hand, because E_s is invariant under all $P_{i(k)}$ [16], the sequence of vectors given by $w(k+1) = P_{i(k)}w(k)$ all belong to E_s which leads to the limit $w^* \in E_s$. From the properties of the infinite paracontracting process [15], w^* has to be in the fixed subspace of $\bigcap_{i=1}^{r}\mathcal{H}(P_i)$. It means

$w^* \in \text{span}\{\mathbf{1}\} = E_1$, which yields $w^* = 0$, because $E_1 \bigcap E_s = \varnothing$. Therefore, the iterative process (18) has the limit $x^* = u^*$, and u^* is given by the spectral projection of matrix $\bar{P}$, associated with the simple eigenvalue 1, as [17] $x^* = u^* = \frac{\mathbf{1}\sigma^T}{\sigma^T\mathbf{1}}x(0) = \frac{\sum_{i=1}^{n}\sigma_i x_i(0)}{\sum_{i=1}^{n}\sigma_i}\mathbf{1}$, where $\sigma = [\sigma_1, \ldots, \sigma_n]^T$ and σ_i is the element of the diagonal matrix Σ. This finishes the proof for Theorem 1.

B. PROOF OF COROLLARY 1

According to [18], balanced graph Laplacian matrix has $\mathbf{1}^T$ and $\mathbf{1}$ as the left and right eigenvectors associated with the eigenvalue zero, respectively. Therefore, every associated $P_{i(k)}$ has $\sigma^T = [\sigma_1, \ldots, \sigma_n]$ and $\mathbf{1}$ as the left and right eigenvectors, respectively, associated with the eigenvalue 1. With this property, the proof of Theorem 1 applies to Corollary 1. This finishes the proof of Corollary 1.

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