Variations on the Index Coding Problem: Pliable Index Coding and Caching

T. Liu
K. Wan
D. Tuninetti

University of Illinois at Chicago

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Motivation, NC and IC
Motivation

- Communications in complex wireless networks.
- Challenging: noise, multi-hop (relaying), multi-cast (compound setting), generalized feedback (overheard information interference or “side information”?), etc.
- Simplification: noiseless links without “multi access” but still with “broadcast”.

Future Wireless Networks

Wireless Internet access
Nth generation Cellular
Wireless Ad Hoc Networks
Sensor Networks
Wireless Entertainment
Smart Homes/Spaces
Automated Highways
All this and more…

Ubiquitous Communication Among People and Devices

My introduction slide for ECE 437 Wireless Communications.
Network Coding (NC)

- A network is a directed graph with capacitated edges connecting nodes/users.
- Source nodes have independent messages destined to subset of destination nodes; wlog multiple unicast.
- Links are noiseless, with broadcast but no multi-access interference.
- A node can implement *any* function of its received messages.

Example of a NC (Ahlswede, Cai, Li, Yeung [TIT2000]). If relay sends first A and then B, two transmission satisfies both receivers. If instead it sends A+B, one transmission satisfies both receivers, which results in 50% bandwidth savings!
Network Coding (NC)

- NC is **hard**!
- Non-linear strategies needed in general; related to entropic region / non-Shannon inequalities.
- NB: an **interference** channel with relays, i.e., a combination of two problems believed to be difficult in isolation. Hope to make progress because of its deterministic nature.

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Index Coding (IC)

- Special case of NC: one transmitter and no relays.
- The TX / server / base station has msgs $\mathcal{X} := \{x_1, \ldots, x_m\}$.
- $RX_j$ / client$j$ / user$j$ requires $x_j$ and knows $S_j \subseteq \mathcal{X} \setminus \{j\}$
- Goal: satisfy all clients with the minimum number of transmissions from TX.

Example of a IC. If the TX serves the requested messages one by one, 4 transmissions are necessary. If instead it uses coded transmission, 2 transmissions suffice.
Index Coding (IC)

- IC recently attracted lots of attention due to its theoretical significance and applications in wireless ad-hoc networks.
- IC is equivalent to NC (ask Salim) but simpler to describe.
- IC solved for \( m \leq 5 \) where linear codes suffice; some other classes too.
- General inner (graph theoretic quantities or random coding) and outer (cut-set type, but need non-Shannon inequalities) bounds have exponential complexity in \( m \).

Satellite transmission (Birk and Kol [INFOCOM1998]).
Pliable Index Coding (PICOD)
PICOD: Motivation

- Today’s communication networks increasingly deliver type of content rather than specific messages. Example: content distribution networks, advertisement networks, and social networks, etc.
- PICOD clients are flexible/pliable and happy to receive any one message they do not already have (for instance, any search results they have not already received, etc.).
- Model: Same as IC except that client $j$ is satisfied by decoding successfully any message not in its side information.
- Connection to IC: choose optimal demands; multi-casting allowed. Still side information sets fixed.
IC vs PICOD

$\mathcal{M} = \{b_1, b_2, b_3, b_4\}$

$\mathcal{I}_1 = \{b_2, b_4\}$  $\mathcal{I}_2 = \{b_1, b_3\}$  $\mathcal{I}_3 = \{b_2, b_4\}$  $\mathcal{I}_4 = \{b_1, b_3\}$

$\mathcal{R}_1 = \{b_1\}$  $\mathcal{R}_2 = \{b_2\}$  $\mathcal{R}_3 = \{b_3\}$  $\mathcal{R}_4 = \{b_4\}$

IC optimal transmission: $t_1 = b_1 + b_2$,  $t_2 = b_3 + b_4$.

PICOD optimal transmission: $t_1 = b_1 + b_2$. 50% bandwidth saving!
PICOD: Past Work

- PICOD is NP-hard, as general IC.
- Sufficiency of $O(\log n)$ transmissions for $|S_j| = \text{constant}, \forall j$.
- Sufficiency of $O(\min\{ (\log m)(1 + \log^+(\frac{n}{\log m})), m, n \})$ transmissions.
- There exists PICOD’s requiring $\Omega(\log n)$ transmissions.
- Achievability: Greedy algorithm with binary linear codes to satisfy the largest fraction of yet-to-be-satisfied clients.


PICOD: Our Work

- Refined analysis of the fraction of clients that can be satisfied by one transmission.
- Deterministic (not probabilistic) approach.
- Strictly better than the known lower bound.
- Capture effect of size of side information set (not seen in IC).
- No converse bounds.

Fraction of satisfied users with a single transmission vs. size of side information set [ITW2016].
Caching
High temporal variability of network traffic leads to high traffic in peak-traffic times and low traffic in peak-off times.

Idea: Caching (prefetching) can help to smooth the traffic in peak-traffic times by storing part of the contents in users’ local memories during peak-off periods.
Caching: Problem Formulation

- **Placement**: Cache content creation, without knowledge of later demands. Limit $M$.
- **Delivery**: Broadcast packets based on demands and cache contents. Limit $R$.
- **Goal**: Minimize the number of broadcast transmissions for the worst case demands.
- **Connection to IC**: *can choose the side side information* but measured against the *worst case multicast demands*.

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Maddah-Ali and Niesen [TIT2014], 2016 ITSoc Paper Award.
Caching: Past Work

Centralized vs Decentralized placement

- Centralized: Users in placement and delivery phases are the same.
- Decentralized: Otherwise.

Uncoded vs Coded placement

- Uncoded placement: Each user directly stores $MF$ out of $NF$ bits.
- Coded placement: Otherwise.

Maddah-Ali and Niesen (MAN) Coded Caching Schemes

- Decentralized: i.i.d. bit storage, coded delivery. Centralized outer bound. [MAN TNet2015]
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Caching: Our Work

- We showed optimality of MAN delivery for uncoded placement for \( N \geq K \). Key: IC acyclic outer bound.

- We have the state-of-the-art delivery for MAN+uncoded placement for \( N < K \), both centralized and decentralized. Key: several compression steps.

Decentralized system with \( N = 4 \) files and \( K = 20 \) users [ITW2016].
Conclusions
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- IC is a fundamental open problem in network information theory.
- It is deceivingly simple. It is representative of the general NC.
- It is the building block in several data delivery & management system problems.
- For IC and its variations, new directions needed for:
  - Inner bounds (right now mainly based on linear codes, or too complex / combinatorial).
  - Outer bounds (right now very few techniques but essentially all cut-set-based, and in general too complex / combinatorial).
- New ideas ...

THANK YOU