Proof of Proposition 1 in
“A Joint Design Approach for Spectrum Sharing between Radar and Communication Systems”

We state Proposition 1 in [1] as follows:

**Proposition 1** ([1]). Suppose that \( \{ R_{xl} \} \) is initialized by \( \{ R_{xl} \} \equiv R_{x}^{0} \). Then, the optimal value of (P) in every iteration of the proposed algorithm could be achieved by \( \{ R_{xl}^{n} \} \) such that for any \( l, l' \in N_{L}^{+} \) (or \( l, l' \in N_{L}^{+} \setminus N_{L}^{+} \)), it holds that \( R_{xl}^{n} = R_{xl}^{n\prime} \).

**Proof:** The proposition can be proved using induction. We focus on the proof for \( l, l' \in N_{L}^{+} \) in the following. The proof for \( l, l' \in N_{L}^{+} \setminus N_{L}^{+} \) is similar.

From the proposition, we know that \( \{ R_{xl} \} \) is initialized such that \( R_{xl}^{0} = R_{xl}^{0\prime} = R_{x}^{0} \), \( \forall l, l' \in N_{L}^{+} \). We need to show that the optimal value of (P) in the \( n \)-th iteration is also achieved by \( \{ R_{xl}^{n} \} \) such that \( R_{xl}^{n} = R_{xl}^{n\prime} \), \( \forall l, l' \in N_{L}^{+} \). Because \( \{ R_{xl}^{n} \} \) is obtained via several inner iterations of solving (P) in [1], it suffices to show that the above property could be passed on between the iterations of solving (P).

Suppose that, in the \( (i - 1) \)-th inner iteration, the optimal value of (P) is achieved by \( \{ R_{xl}^{n(i-1)} \} \) such that \( R_{xl}^{n(i-1)} = R_{xl}^{n(i-1)} \), \( \forall l, l' \in N_{L}^{+} \). During the \( i \)-th iteration, \( \{ R_{xl}^{n(i)} \} \) is obtained by solving (P) with \( \{ R_{xl} \} = \{ R_{xl}^{n(i-1)} \} \). We will show that \( R_{xl}^{n(i)} \equiv 1/L \sum_{l=1}^{L} R_{xl}^{n(i-1)} \equiv R_{xl}^{n(i)} \), \( \forall l \in N_{L}^{+} \) is also feasible and achieves the same radar SINR as \( \{ R_{xl}^{n(i)} \} \). Based on the concavity of \( C_{l}(R_{xl}, \cdot) \), we have

\[
\sum_{l=1}^{L} C_{l}(R_{xl}^{n(i)}, \cdot) \leq LC_{l}(R_{xl}^{n(i)}, \cdot).
\]

For the communication transmission power, it trivially holds that \( \sum_{l=1}^{L} \text{Tr}(R_{xl}^{n(i-1)}) = L \text{Tr}(R_{xl}^{n(i)}) \). Therefore, \( \{ R_{xl}^{n(i)} \} \) is also feasible. The objective \( f(R_{xl}) \) of (P) in the \( i \)-th iteration is affine w.r.t. \( R_{xl} \) with coefficient depending on \( R_{xl} = R_{xl}^{n(i-1)} \). Given that \( R_{xl}^{n(i-1)} = R_{xl}^{n(i-1)} \), \( \forall l, l' \in N_{L}^{+} \), the affine functions \( f(\cdot) \) for are identical for any \( l \in N_{L}^{+} \). Therefore, \( \{ R_{xl}^{n(i-1)} \} \) achieves the same objective value as \( \{ R_{xl}^{n(i-1)} \} \) does.

Proposition 1 is proved.

**REFERENCES**